Model and optimal Call Admission Policy in Cellular Mobile Networks

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Abstract. For current cellular networks, two important Quality of Service (*QoS*) measures are the fractions of new and handoff calls that are blocked due to unavailability of channels. Based on these *QoS* measures, we propose a queuing network model with impatient users for handoff and new calls in cellular mobile networks. The number of simultaneous calls, that can be supported, is modeled by C identical servers with exponentially distributed session duration for each one of them. Priority is given to handoffs over new calls. We use for that a Guard Channel policy that reserves a set of C_H channels for handoff calls, new calls being served at their arrival if there are more than C_H available channels. We derive loss probabilities, mean waiting times in the queues, and we show that the Guard Channel policy is optimal for minimizing a linear objective function of the new and handoff calls. Simulation results illustrating the behavior of our system are given.

1 Introduction

During the last few years there is a great demand for PCS (Personal Communication Services) which will provide reliable voice and data communications anytime and anywhere. The service area in a PCS network is partitioned into cells. In each cell, a Base Station (BS) is allocated a certain number of channels which is assigned to the mobile terminal MT enabling the MT to communicate with other MT's. As the MT moves from one cell to another, any active call needs to be allocated a channel in the destination cell. This event is called a handoff. Allocating radio resources to users with minimum blocking of new calls and dropping of handoff calls has become a vital issue in a mobile communication system design. It is known that dynamic channel allocation schemes can reduce these blocking and dropping probabilities for a reasonable range of loadings [2, 9]. However, allocating channels to every user whenever the sources are available may not be the optimal strategy in term of system performance.

The cellular network PCS have to be designed to guarantee the required Quality of Service (*QoS*) to each service or type of call (call meaning voice or data). Several policies and analysis are proposed in [4, 6] to minimize blocking

of handoff calls. In our paper we investigate a call admission policy which gives priority to handoff attempts over new call attempts.

In this paper, we assume that the cellular network is homogeneous and thus, we can examine a single network cell in isolation. The radio ports are modeled by C identical servers with exponentially distributed session duration for each one of them. We use a Guard Channel policy that reserves a set of C_H channels for handoff calls, new calls being served at their arrival if there are more than C_H available channels. Cellular networks performance was analysed in [6] using an approach where new and handoff calls were assumed to be Poisson arrival Processes. This approach is acceptable for macro cellular networks where a call might request for handoff only a few times. In this paper, we propose a Markov Modulated Poisson Process (MMPP) to describe voice handoffs (VH) arrival process because the arrival rate may depend on the state of the environment. The data Handoffs (DH) (as files transfer) and new calls (NC) arrive according to two different Poisson processes. Two separate finite buffer size queues are considered, one for each type of calls (handoff or new). The handoff calls enter service if there are available channels or join its queue if no idle channels are available and the queue is not full. Otherwise, They are lost. An arriving new call first checks the servers. It enters service if the number of busy servers is less than $C - C_H$, otherwise it enters the queue if the buffer is not full. Both handoff and new calls are impatient. That is a handoff leaves its queue and will be lost if it fails to get a channel before a Time Out (TO). And, in the same way, if a new user cannot get a channel before the end of the Patience Waiting Time (PWT), it leaves the queue. We derive loss probabilities, mean waiting times in both queues and we show that the Guard Channel policy is optimal for minimizing a linear objective function of the new and handoff calls. This paper is organized as follows: the queueing model is precisely defined in section 2. Loss probabilities and mean waiting time in queues are derived in section 3. In section 4, we show that the Guard Channel policy is optimal for minimizing a linear objective function of the new and handoff calls. Simulation results illustrating the behavior of our system are given and discussed in section 4. Finally, we conclude with some remarks in section 5.

2 The model

We consider a system with two separate queues for handoff and new calls with priority given to handoff over new attempts.

It is assumed that voice handoffs arrive according to an MMPP process A^{VH} with intensity $\lambda(t) = \sum_{i=1}^n \lambda_i \times \mathbb{1}_{[Y(t)=i]}$ at time t where:

- $(Y(t))$ is an irreducible Markov chain with state space $\{1, 2, \dots, n\}$ and an infinitesimal generator denoted by Q_n ;

- $\lambda_i, i = 1..n$, are non-negative numbers;
- and

$$- \mathbb{1}_{[Y(t)=i]} = \begin{cases} 1 & \text{if } Y(t) = i \\ 0 & \text{otherwise.} \end{cases}$$

We suppose that the process of data handoff attempts is a Poisson process denoted by A^{DH} with arrival rate λ_{DH} and that mobile new users generate calls with a Poisson arrival denoted by A^{NC} with intensity λ_{NC} . The service is provided by C channels with exponentially distributed session duration for each one of them. We use a Guard Channel policy that reserves a set of C_H channels for handoff calls, new calls being served at their arrival if there are more than C_H available channels. The evolution of the system for both new

Fig. 1. The Model

- and handoff traffic flow is as follows :
- An arriving handoff call first checks the servers. It enters service if one of them is available. Otherwise, it is queued if the buffer is not full. A handoff in the queue will be lost if it fails to get a channel before a Time Out (TO).
 - An arriving new call first checks the servers. It enters service if the number of busy servers is less than $C - C_H$, otherwise it enters its queue if the buffer is not full. If a new user cannot get a channel before the end of the Patience Waiting

Time (PWT) it will be lost.

The Patience Waiting Time (PWT) and the Time Out (TO) are distributed exponentially with mean $1/\mu_N$ and $1/\mu_H$ respectively.

The queue buffer sizes are finite and denoted by K_H for handoff traffic and K_N for new calls.

Therefore, the new call generated by the mobile user calls may be lost at two cases: if the corresponding buffer is full at its arriving time or if its waiting time is greater than PWT. A handoff call may be lost if the corresponding buffer is full or if it fails to get a channel before that the TO expires.

The system is described by the multidimensional Markov Chain :

$$(X_{VH}(t), X_{DH}(t), X_{NC}(t), S(t), Y(t))$$

where :

- $X_{VH}(t)$ (respectively $X_{DH}(t)$) is the number of voice Handoffs (respectively data handoffs) in their queue at time t respectively,
- $X_{NC}(t)$ is the number of new calls at time t in their queue,
- $S(t)$ is the number of busy servers at time t ,

and

- $Y(t)$ is the phase of the MMPP representing the state of the environment.

In this paper, we analyse our system in the steady state.

3 Performance analysis

Let $B = \{(i, j, k, l, m) / 0 \leq i + j \leq K_H, 0 \leq k \leq K_N, 0 \leq l \leq C, 1 \leq m \leq n\}$ be the state space of the precedent multidimensional Markov Chain and $P(i, j, k, l, m)$ be the stationary probability that the system is in the state (i, j, k, l, m) .

The $P(i, j, k, l, m)$ $((i, j, k, l, m) \in B)$ verify the normalization condition :

$$\sum_{(i, j, k, l, m) \in B} P(i, j, k, l, m) = 1 \quad (1)$$

A VH or DH call is lost if the corresponding buffer is full or if its waiting time in the queue exceeds a Time Out (TO).

The probability that a VH mobile user attempt finds the buffer full is given by :

$$\tilde{P}_{VH} = \frac{\sum_{i+j=K_H} \sum_{k=0}^{K_N} \sum_{m=1}^n \lambda_m P(i, j, k, C, m)}{\sum_{m=1}^n \lambda_m \pi_m} \quad (2)$$

where $\pi_Y = (\pi_1, \dots, \pi_n)$ is the unique vector probability solution of $\pi_Y Q_n = 0$

(where Q_n is the infinitesimal generator of Y).

A DH call finds the buffer full upon arrival with probability $\tilde{P}_{DH}$ given by :

$$\tilde{P}_{DH} = \sum_{i+j=K_H} \sum_{k=0}^{K_N} \sum_{m=1}^n P(i, j, k, C, m) \quad (3)$$

If the time out expires before that a handoff gets a channel, the call is lost. The loss probability due to time out is given by :

$$P_{VTO} = \frac{\sum_{(i,j,k,l,m) \in B/i>0} i\mu_H P(i, j, k, l, m)}{(\sum_{m=1}^n \lambda_m \pi_m)(1 - \tilde{P}_{VH})} \quad (4)$$

for the voice handoffs

$$P_{DTO} = \frac{\sum_{(i,j,k,l,m) \in B/j>0} j\mu_H P(i, j, k, l, m)}{\lambda_{DH}(1 - \tilde{P}_{DH})} \quad (5)$$

for the data handoffs

So, the total loss probability for voice and data handoffs is respectively :

$$P_{VH} = \tilde{P}_{VH} + (1 - \tilde{P}_{VH})P_{VTO} \quad (6)$$

for the voice handoffs

$$P_{DH} = \tilde{P}_{DH} + (1 - \tilde{P}_{DH})P_{DTO} \quad (7)$$

for the data handoffs

In the same way, a new call is lost if at its arrival the corresponding buffer is full or if its waiting time in the queue exceeds the Patience Waiting Time (PWT).

The probability that a NC finds the buffer full is given by :

$$\tilde{P}_{NC} = \sum_{i+j=0}^{K_H} \sum_{m=1}^n \sum_{l=C_H}^C P(i, j, K_N, l, m) \quad (8)$$

The loss probability $\tilde{P}_{NC}$ due to PWT is given by :

$$\tilde{P}_{PWT} = \frac{\sum_{(i,j,k,l,m) \in B/k>0} k\mu_N P(i, j, k, l, m)}{\lambda_{NC}(1 - \tilde{P}_{NC})} \quad (9)$$

So, the total loss probability for new calls is given by :

$$P_{NC} = \tilde{P}_{NC} + (1 - \tilde{P}_{NC})P_{PWT} \quad (10)$$

The mean number of VH in the queue is given by :

$$E(VH) = \sum_{(i,j,k,l,m) \in B} iP(i,j,k,l,m) \quad (11)$$

In the same way, we obtain the mean number of DH and NC in the queue :

$$E(DH) = \sum_{(i,j,k,l,m) \in B} jP(i,j,k,l,m) \quad (12)$$

$$E(NC) = \sum_{(i,j,k,l,m) \in B} kP(i,j,k,l,m) \quad (13)$$

and

$$E(DN) = \sum_{(i,j,k,l,m) \in B} lP(i,j,k,l,m) \quad (14)$$

The mean waiting time in the queue W , for voice handoffs, data handoffs and new calls can be obtained using Little's formula [1].

So, we have :

$$W(VH) = \frac{E(VH)}{\left(\sum_{m=1}^n \lambda_m \pi_m \right) (1 - \tilde{P}_{VH})} \quad (15)$$

$$W(DH) = \frac{E(DH)}{\lambda_{DH} (1 - \tilde{P}_{DH})} \quad (16)$$

and

$$W(NC) = \frac{E(NC)}{\lambda_{NC} (1 - \tilde{P}_{NC})} \quad (17)$$

The probabilities $P(i,j,k,l,m)$ are numerically obtained using DNAmaca [5] which provides performance analysis sequence including steady state solution. In section 4, we prove that the Guard Channel policy is an optimal policy.

4 Optimal Admission Policy

In this section, we consider the problem of finding an optimal admission policy that determines the acceptance or rejection of new and handoff calls. For the cellular system described in section 3, let $S(t)$ be the Markov chain representing the number of busy channels at time t . If, at steady state, that Markov chain is in state l , the policy consists then to choose an action $a(l)$ in a set A of acceptance/reject actions depending on the state l , as follows : a call will be rejected due to impatience time or if there is no available channel and the buffer is full, otherwise it will be accepted. When the system is in state l and action a is chosen, then we incur a cost $C(l, a)$ which will be L_h or L_n according to the action is a handoff or a new call reject. Moreover, the next state of the system is chosen according to the transition probabilities $P_{lj}(a)$. Our problem is to find a policy π^* over all call admission control policies π such that :

$$\phi_{\pi^*} = \min_{\pi} \phi_{\pi} \tag{18}$$

where

$$\phi_{\pi} = \lim_{N \rightarrow +\infty} E_{\pi} \big(\sum_{k=0}^{N-1} L_h \pi_{hk} + \sum_{k=0}^{N-1} L_n \pi_{nk} \big)$$

E_{π} represents the conditional expectation given that the policy π is employed

and

the binary mapping π_{hk}, π_{nk} describing the actions for each state of the sytem are defined by :

$$\pi_{hk} = \begin{cases} 1 & \text{if } k\text{'th handoff is rejected} \\ 0 & \text{otherwise.} \end{cases}$$

$$\pi_{nk} = \begin{cases} 1 & \text{if } k\text{'th new call is rejected} \\ 0 & \text{otherwise.} \end{cases}$$

we need the following :

Lemma 1. *If there exists a bounded function $h(l)$, $l=0,1,...$, and a constant g such that :*

$$g + h(l) = \min_a \{ C(l, a) + \sum_{j=0}^{\infty} P_{lj}(a) h(j) \}; l \geq 0 \tag{19}$$

then there exists a stationary policy π^ such that :*

$$g = \phi_{\pi^*}(l) = \min_{\pi} \phi_{\pi}(l) \tag{20}$$

For the proof of lemma 2, see [8], page 144.

We obtain the following :

Theorem 2. *an optimal policy for the problem (18) is a Guard Channel policy.*

Now, we prove the existence of a bounded function $h(l)$ and a constant g using the following construction:

For a discount factor $\alpha \in (0, 1)$ and any policy π , we define :

$$V_\pi(l) = E_\pi \left[\sum_{n=0}^{\infty} \alpha^n C(l_n, a_n) | l_0 = l \right], l \geq 0 \quad (21)$$

where E_π represents the conditional expectation given that the policy π is employed and the initial state is l .

Let,

$$V_\alpha(l) = \inf_\pi V_\pi(l) \quad (22)$$

A policy π^* is said to be α -optimal if :

$$V_{\pi^*}(l) = V_\alpha(l) \forall l \geq 0 \quad (23)$$

For the proof of the theorem 2, we use the lemma 1.

We denote $V_\alpha^k(l)$ the optimal cost for the k -stage starting in state l .

The α -optimal cost function $V_\alpha(\cdot)$ for $0 \leq l \leq C$ satisfies :

$$\begin{aligned} V_\alpha^k(l) = & \\ & \lambda_h \min(V_\alpha^{k-1}(l+1), L_h + V_\alpha^{k-1}(l)) \\ & + \lambda_n \min(V_\alpha^{k-1}(l+1), L_n + V_\alpha^{k-1}(l)) \\ & + l\mu V_\alpha^{k-1}(l-1) + (C-l)\mu V_\alpha^{k-1}(l) \end{aligned}$$

Where $\lambda_h = \lambda_{VH} + \lambda_{DH}$ and $\lambda_n = \lambda_{NC}$.

We consider that $V_\alpha^k(C+1) = \infty$ and $V_\alpha^k(-1) = V_\alpha^k(0)$.

We define:

$$V_\alpha(l) = \lim_{N \rightarrow +\infty} V_\alpha^N(l) \quad (24)$$

It is shown in [10] that this function is non-decreasing and convex. Moreover, since the state space B of our system defined in section 3 is finite and the Markov chain induced by the control policy is irreducible, $V_\alpha(l) - V_\alpha(0)$ is bounded uniformly according to [8], page 146.

Let us define :

$$h_\alpha(l) = V_\alpha(l) - V_\alpha(0) \quad (25)$$

to be the α -cost of state l relative to state 0.

Then, from equations (24) and (25), we obtain :

$$\begin{aligned}
 (1 - \alpha)V_\alpha(0) + h_\alpha(l) = \\
 \lambda_h \min(h_\alpha(l + 1), L_h + h_\alpha(l)) \\
 + \lambda_n \min(h_\alpha(l + 1), L_n + h_\alpha(l)) \\
 + l\mu h_\alpha(l - 1) + (C - l)\mu h_\alpha(l)
 \end{aligned}$$

Now, for some sequence $\alpha_n \rightarrow 1$:

h_{α_n} converges to a function $h(l)$ and $(1 - \alpha_n)V_{\alpha_n}(0)$ converges to a constant g , then we obtain that :

$$\begin{aligned}
 g + h(l) = \lambda_h \min(h(l + 1), L_h + h(l)) \\
 + \lambda_n \min(h(l + 1), L_n + h(l)) \\
 + l\mu h(l - 1) + (C - l)\mu h(l)
 \end{aligned}$$

Now, we are able to apply lemma 1 to say that a stationary policy $a(l)$ verifying

$$a(l) = \phi_{\pi^*}(l) = \min_{\pi} \phi_{\pi}(l) \quad (26)$$

exists, and to give a:

Proof of Theorem 2

The optimal policy $a(l)$ that satisfies (26) is given by :

$$a(l) = \begin{cases} 1 & \text{if } h(l + 1) - h(l) \leq L_n \\ 0 & \text{otherwise.} \end{cases}$$

Since $h(l)$ is non-decreasing and convex, $h(l + 1) - h(l)$ is non-decreasing in l . Hence, there exist integers i_0 and i_1 where :

$$i_0 = \inf\{l : h(l + 1) - h(l) > L_n\} \quad (27)$$

and

$$i_1 = \inf\{l : h(l + 1) - h(l) > L_h\} \quad (28)$$

we note that $i_1 > i_0$ since $L_h > L_n$.

So, the Guard channel policy that reserves $C - i_0$ channels for handoff calls is the optimal policy for our problem.

5 Numerical and Simulation results

In this section, we present numerical and simulation results. For all experiments we consider $C = 18$ servers with exponentially distributed session duration of parameter $\mu = 1$. Figures Fig.2 and Fig.3 represent loss probabilities versus Handoff and new calls buffer size respectively. In both cases, the parameters C_H , PWT and TO are respectively 5, 40 and 30. When the buffer size increases, we note that the loss probability of both Handoff and new calls decreases, and the handoffs loss probability is lower. Figures Fig.4 and Fig.5 depict loss probabilities of handoff

Fig. 2. handoffs loss probabilities vs. buffer size, $C = 18, C_H = 5, PWT = 40, TO = 30$

Fig. 3. new call loss probabilities vs. buffers size, $C = 18, C_H = 5, PWT = 40, TO = 30$

and new calls versus number of guard channels C_H . Loss probability decreases

for Handoff mobile users when we increase the number of guard channels. The Loss probability of new calls increases when we increase the number of guard channels.

In Figures Fig.4 and Fig.5 we see that if we reserve 3 or 4 channels from 18 channels, the Handoff loss probability is lower than 10^{-6} , and the new call loss probability does not exceed the value 10^{-4} . So, we propose to reserve 20% of channels to Handoffs.

The effect of channel reservation on the response times is depicted in Fig.6.

Fig. 4. handoffs loss probabilities vs.guard channels, $C = 18, PWT = 40, TO = 30$

Fig. 5. new call loss probabilities vs.guard channels, $C = 18, PWT = 40, TO = 30$

response time

Fig. 6. response time vs. number of guard channels

6 Conclusion

In this paper, we have considered a Guard channel policy that gives priority to handoff over new calls in a cellular mobile network. We show that this policy is optimal for a linear objective function of the new and handoff call blocking probabilities. Under this policy, performance for handoffs are optimal without serious degradation for new calls. Moreover, blocking probabilities and response times for both types of calls are obtained and numerical results show the impact of the buffer size and the number of guard channels on the system behaviour. Finally Simulation results are conducted to validate our results.

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