An Axiomatic Semantics for the Synchronous Language *Gentzen* [★]

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Abstract. We propose an axiomatic semantics for the synchronous language *Gentzen*, which is an instantiation of the paradigm *Timed Concurrent Constraint Programming*. We view *Gentzen* as a prototype of the class of state-oriented synchronous languages, since it offers the basic constructs that are shared by the languages in the class. Since synchronous concurrency cannot be simulated by arbitrary interleaving, we cannot exploit “head normal forms”, on which axiomatic theories for asynchronous process calculi are based.

1 Introduction

Synchronous languages [4,11] have been proposed for programming *reactive systems* [14], i.e. systems which maintain an ongoing interaction with their environment at a rate controlled by it. These languages are based on the *synchronous hypothesis* [8], which states that a reactive system is able to react instantaneously and in no time to stimuli from the external environment, so that outputs from the system are available as soon as inputs from the environment are. The synchronous hypothesis is indeed an abstraction and amounts to requiring that the system is able to react to an input before the subsequent input arrives.

One of the main features of the synchronous hypothesis is that “it reconciles concurrency with determinism” [8], in the sense that system components running in parallel are perfectly synchronized and cannot arbitrarily interleave, because each of them reacts whenever an input from the environment arrives.

In [8,12,16,21,3] several state-oriented synchronous languages have been proposed for programming reactive systems where control handling aspects are predominant. These languages have been endowed with operational semantics [3, 7,13,16,22] and with denotational semantics [10,20,22,23]. We believe that it is worth developing also axiomatic semantics for such languages, in order to characterize behavioral equivalent programs. Possible applications of such semantics are, among others, transformation of programs and proof by rewriting.

Axiomatic theories are well established in the field of asynchronous process description languages since the eighties [6,17], and, for languages in suitable classes, algorithms have been developed to obtain axiomatizations in a syntax-driven way (as examples, see [1,2]). A central idea in the mentioned papers is

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that concurrency can be simulated by interleaving: concurrent processes can be reduced to *head normal forms*, i.e. nondeterministic choices of sequential processes.

Since in the synchronous setting processes running in parallel are synchronized and run at the same rate, concurrency cannot be simulated by interleaving and head normal forms for synchronous languages have no meaning. So, developing axiomatic theories for synchronous languages cannot be done by trivially adapting what has been already done in the field of asynchronous languages.

In this paper we develop an axiomatic theory for the language *Gentzen*, which is an instantiation of the paradigm *Timed Concurrent Constraint Programming* [22]. We view *Gentzen* as a prototype of the class of state-oriented synchronous languages, because it offers the basic constructs that are shared by all languages in this class and that are not offered by asynchronous languages. Axiomatic theories for any other state-oriented synchronous language can be obtained by extending our work to deal with constructs that characterize such a language.

1.1 *Gentzen*: A Prototype of State-Oriented Synchronous Languages

A *Gentzen* program maintains an ongoing interaction with its environment by communicating through *binary signals*. No distinction between input and output signals is done: a set of signals Π is assumed and signals in Π can be sensed and broadcast both by the program and by the environment. *Gentzen* assumes a discrete notion of time, i.e. time is a sequence of instants. At every instant the external environment stimulates the program by broadcasting a set of signals; the program reacts immediately and broadcasts a set of signals to the environment.

We assume the syntax for *Gentzen* given in [21]:

$$A ::= \text{skip} \mid \text{tell } a \mid \text{if } a \text{ then } A \mid \text{if } \bar{a} \text{ then } A \mid \text{next } A \mid A \parallel A \mid \text{rec } P. A \mid P$$

where a , A and P range over binary signals, *agents* and *recursion variables*, respectively. Notice that we do not use the same notation of [21]. We use **rec** instead of “ μ ”, and we write **if** $\bar{a}$ **then** A for **if** a **else** A .

We will denote by $\equiv$ the syntactic identity over agents.

Agent **skip** does nothing at each instant: it reacts to every input by responding with the empty output. Agent **tell** a broadcasts the signal a and then terminates, i.e. it behaves as **skip**. Agent **if** a **then** A behaves as A if the environment broadcasts a , whereas **if** $\bar{a}$ **then** A behaves as A if the environment does not broadcast a . Note that if a is present (resp. absent) then **if** a **then** A (resp. **if** $\bar{a}$ **then** A) starts A immediately. Agent **next** A will start A at the next instant. Agent $A_1 \parallel A_2$ is the synchronous parallel composition of A_1 and A_2 . Signals broadcast by A_1 (resp. A_2) can be detected immediately by A_2 (resp. A_1). Construct **rec** is the recursion construct. To be sure that at each instant the computation is finite, it is required that recursion be *guarded*, i.e. that recursion variables appear in bodies of **next** (see [22] for further discussion).

According to the synchronous hypothesis, Gentzen constructs take no time, except **next** which takes precisely one unit of time. So, Gentzen offers the constructs that are shared by all synchronous languages: a mechanism to detect the instantaneous presence/absence of signals (**if** - **then** -), a mechanism to broadcast instantaneously signals (**tell** -), a mechanism to delay the computation for one instant (**next** -), synchronous parallel composition (with instantaneous communication between parallel components) and guarded recursion.

Despite its simplicity, Gentzen is sufficiently expressive to embed compositionally the state-oriented synchronous language Argos [16] and the dataflow synchronous language Lustre [9] (see [25] for the proof).

1.2 Technical Development of the Paper

Technically, we give a *structural operational semantics* [19] for Gentzen in terms of a *labeled transition system* (LTS), with LTS states corresponding to agents and LTS transitions corresponding to agent reactions, and then we consider the *bisimulation* on the LTS as a behavioral equivalence over Gentzen agents. Bisimulation on our LTS is a congruence and our LTS reflects the input/output behavior of agents (in the sense that the LTS reflects the operational semantics of Gentzen given in [22]). So, bisimilar agents are distinguished neither by any Gentzen context nor by the external environment. Therefore, we believe that bisimulation is a reasonable notion of behavioral equivalence.

To axiomatize Gentzen, we provide a system of axioms defining an equality relation over agents. We prove that this equality relation is *sound* and *complete* modulo bisimulation, in the sense that two arbitrary agents are equated if and only if they are bisimilar. Our axiom system consists of a *finite* set of *unconditional* axioms plus the *recursive specification principle* [5,17]. (Since concurrency cannot be simulated by interleaving, to have a finite axiomatization we do not need the auxiliary operator “left merge” [6], which, as it has been proved in [18], is needed to have finite axiomatizations of asynchronous process algebras.)

To prove the completeness of our axiomatization, we introduce a notion of normal form of agents, we prove that any agent can be transformed into a bisimilar normal form by applying our axioms, and we prove that bisimilar normal forms are equated by our axioms. The idea of exploiting normal forms to prove completeness of axiomatizations is well established since the work of Milner [17]. But, contrarily to those of Milner, our normal forms contain the construct of parallel composition. Our normal forms are of the form $A_1 \parallel \dots \parallel A_n$, where $A_1, \dots, A_n$ are sequential agents (“ $\parallel$ ” is commutative and associative). The syntactic structure of our normal forms is such that, given a normal form $A_1 \parallel \dots \parallel A_n$ and an arbitrary input from the environment, $A_1, \dots, A_n$ react, but at most one of them will be able to react to the next input.

2 The Labeled Transition System

In this section we propose an LTS as an operational semantic model for Gentzen. LTS states correspond to agents, LTS transitions correspond to agent reactions,

and LTS labels carry information on *causality* among signals, i.e. labels highlight *causes* of signals broadcast by agents. These causes consist in the presence/absence of other signals.

We will prove that our LTS permits to recover the input/output behavior of agents (see Prop. 1, Sect. 2.2) and to detect whether instantaneous communications give rise to *nonreactivity*, i.e. the inability of an agent to react to a given input, and *nondeterminism* (see Prop. 2, Sect. 2.2).

Example 1. The agent $A \equiv \text{if } \bar{a} \text{ then tell } a$ is non-reactive if the environment does not supply a . In fact, A broadcasts a iff a is absent at the same instant, so no reaction is admissible. The agent $B \equiv \text{if } \bar{a} \text{ then tell } b \parallel \text{if } \bar{b} \text{ then tell } a$ is nondeterministic if the environment supplies neither a nor b . In fact, either the left branch of B broadcasts b and the right branch does not broadcast a , or the right branch of B broadcasts a and the left branch does not broadcast b .

Both reactivity and determinism are decidable (see [22] or Prop. 2, Sect. 2.2). Only reactive and deterministic agents are accepted in [22]. (Note that [22] requires only reactivity and nondeterminism; it does not require the more restrictive property of *constructiveness* recently defined in [7].) Reactivity and determinism are required also in all state-oriented synchronous languages except the specification language Statecharts [12], which admits nondeterminism.

2.1 The Definition of the LTS

We let a, b range over the set of signals Π and π range over subsets of Π . We denote with $\bar{a}$ the absence of a and, with abuse of notation, we define $\bar{\bar{a}} = a$. We denote with $\overline{\Pi}$ the set $\{\bar{a} \mid a \in \Pi\}$, and we let γ range over $\Pi \cup \overline{\Pi}$.

An *event* E (over Π) is a subset of $\Pi \cup \overline{\Pi}$ such that there is no signal a such that both $a \in E$ and $\bar{a} \in E$. It is interpreted as the assumption that every signal a with $a \in E$ (resp. $\bar{a} \in E$) is present (resp. absent).

Definition 1. Given an event E and a symbol $u \in \Pi \cup \{n\}$, the pair (E, u) is a causality term with E as cause and u as action.

Let us explain Def. 1. Given a signal $a \in \Pi$, the action a refers to the act (by some agent **tell** a) of broadcasting the signal a . The action n refers to the act (by some agent **next**) of pausing for one instant of time. Now, given a causality term (E, u) , the action $u \in \Pi \cup \{n\}$ is performed by an agent that is embedded in the body of some agent **if** b **then** $_$ (resp. **if** $\bar{b}$ **then** $_$) for every $b \in E$ (resp. $\bar{b} \in E$). So, if $u = a$ then (E, u) reflects causality among signals, i.e. it reflects that a is broadcast because each $b \in E$ (resp. $\bar{b} \in E$) is present (resp. absent).

Definition 2. A (LTS) label is a pair $l = \langle \mathcal{E}_l, \mathcal{N}_l \rangle$ such that:

- $\mathcal{E}_l$ is a set of causality terms s.t. $\bigcup_{(E,u) \in \mathcal{E}_l} E \cup \bigcup_{(E,a) \in \mathcal{E}_l, a \in \Pi} \{a\}$ is an event;
- $\mathcal{N}_l$ is a set of causality terms s.t. for no $(E', u) \in \mathcal{N}_l$ we have that $E' \subseteq \bigcup_{(E,u) \in \mathcal{E}_l} E \cup \bigcup_{(E,a) \in \mathcal{E}_l, a \in \Pi} \{a\}$.

The set of labels is denoted by $\mathcal{L}$. A transition $A \xrightarrow{l} A'$ corresponds to the reaction of the agent A which performs the actions represented by the causality terms in $\mathcal{E}_l$ and does not perform the actions represented by the causality terms in $\mathcal{N}_l$. Causality terms in $\mathcal{E}_l$ are needed to recover the input/output behavior of A and to discover whether A is reactive and deterministic. The rôle of $\mathcal{N}_l$ will be explained in the following.

Table 1. The transition system specification for Gentzen.

$\mathbf{skip} \xrightarrow{\langle \emptyset, \emptyset \rangle} \mathbf{skip}$	(skip)	$\mathbf{next} A \xrightarrow{\langle \{(\emptyset, n)\}, \emptyset \rangle} A$	(next)
$\mathbf{tell} a \xrightarrow{\langle \{(\emptyset, a)\}, \emptyset \rangle} \mathbf{skip}$	(tell)		
$\frac{A \xrightarrow{l} A', a(l) \in \mathcal{L}}{\mathbf{if} a \mathbf{then} A \xrightarrow{a(l)} A'} \quad (\mathit{then_0})$		$\frac{A \xrightarrow{l} A'}{\mathbf{if} a \mathbf{then} A \xrightarrow{N(a, l)} \mathbf{skip}} \quad (\mathit{then_1})$	
$\frac{A \xrightarrow{l} A', \bar{a}(l) \in \mathcal{L}}{\mathbf{if} \bar{a} \mathbf{then} A \xrightarrow{\bar{a}(l)} A'} \quad (\mathit{else_0})$		$\frac{A \xrightarrow{l} A'}{\mathbf{if} \bar{a} \mathbf{then} A \xrightarrow{N(\bar{a}, l)} \mathbf{skip}} \quad (\mathit{else_1})$	
$\frac{A[\mathbf{rec} P. A/P] \xrightarrow{l} A'}{\mathbf{rec} P. A \xrightarrow{l} A'} \quad (\mathit{rec})$		$\frac{A_1 \xrightarrow{l_1} A'_1, A_2 \xrightarrow{l_2} A'_2, l_1 \otimes l_2 \in \mathcal{L}}{A_1 \parallel A_2 \xrightarrow{l_1 \otimes l_2} A'_1 \parallel A'_2} \quad (\mathit{par})$	

The LTS is defined by the transition system specification in Table 1.

Rule *skip* states that **skip** performs no action at every instant.

Rule *next* states that **next** A performs the pausing action n and will behave as A at the next instant.

Rule *tell* states that **tell** a broadcasts a and will behave as **skip**. The causality term $\langle \emptyset, a \rangle$ expresses that the action of broadcasting a is performed independently of the presence/absence of signals in the environment.

Before explaining rules for **if** $_$ **then** $_$, we introduce some more notations.

Given a label $l \in \mathcal{L}$ and a symbol $\gamma \in \Pi \cup \overline{\Pi}$, we denote with $\gamma(l)$ the pair:

$$\gamma(l) = \{ \langle (E \cup \{\gamma\}, b) \mid (E, b) \in \mathcal{E}_l \rangle, \{ \langle (E \cup \{\gamma\}, b) \mid (E, b) \in \mathcal{N}_l \rangle \}$$

and with $N(\gamma, l)$ the pair:

$$N(\gamma, l) = \langle \emptyset, \{ \langle (E \cup \{\gamma\}, b) \mid (E, b) \in \mathcal{E}_l \cup \mathcal{N}_l \rangle \} \rangle.$$

Rule *then_0* states that if a is present then **if** a **then** A reacts as A . The label $a(l)$ highlights that actions performed by agents in the body of A require the presence of a . Rule *then_1* states that if a is absent then **if** a **then** A terminates.

Example 2. Given $A \equiv \mathbf{if} a \mathbf{then} \mathbf{tell} b$, rule *then_0* implies $A \xrightarrow{l_0} \mathbf{skip}$, $l_0 = \langle \{ \langle \{a\}, b \rangle \}, \emptyset \rangle$, and rule *then_1* implies $A \xrightarrow{l_1} \mathbf{skip}$, $l_1 = \langle \emptyset, \{ \langle \{a\}, b \rangle \} \rangle$.

Now, we are able to explain the rôle of the second component of labels. Let us consider the label l_1 of Example 2. The causality term $(\{a\}, b)$ in $\mathcal{N}_{l_1}$ keeps track of the fact that the reaction represented by the transition labeled by l_1 is performed because a is absent. In principle, one could keep track of this information in other ways. As an example, it could seem to be reasonable to remove $\mathcal{N}_{l_1}$ and to add the causality term $(\{\bar{a}\}, s)$ to $\mathcal{E}_{l_1}$, with s denoting some idle action. But, with a similar choice, agents **if** a **then** **if** b **then** A and **if** b **then** **if** a **then** A would not be bisimilar, despite they are intuitively equivalent. On the contrary, our choice guarantees that these agents are bisimilar.

Rules *else_0* and *else_1* are analogous to *then_0* and *then_1*, respectively.

Rule *par* states that $A_1 \parallel A_2$ performs both reactions of A_1 and A_2 . The partial function $\otimes : \mathcal{L} \times \mathcal{L} \rightarrow \mathcal{L}$ is defined as follows:

$$l_1 \otimes l_2 = \langle \mathcal{E}_{l_1} \cup \mathcal{E}_{l_2}, \mathcal{N}_{l_1} \cup \mathcal{N}_{l_2} \rangle.$$

Finally, rule *rec* is a standard recursion rule.

2.2 Correspondence with the Operational Semantics of [22]

We say that a set of signals $\pi \subseteq \Pi$ *triggers* a transition $A \xrightarrow{l} A'$ if the transition represents the reaction of A to an environment prompting π . Formally, let us assume a label l such that the causality terms in $\mathcal{E}_l$ having actions in Π are $(E_1, a_1), \dots, (E_n, a_n)$. We say that π triggers $A \xrightarrow{l} A'$ iff there exists a sequence of sets of signals $\pi_1 \subseteq \dots \subseteq \pi_{n+1}$ such that:

1. $\pi_1 = \pi$;
2. $\pi_{i+1} = \pi_i \cup \{a_i\}$, $1 \leq i \leq n$;
3. $(E_i \cap \Pi) \subseteq \pi_i$ and $\overline{E_i \cap \Pi} \cap \pi_{n+1} = \emptyset$, $1 \leq i \leq n$;
4. both $E \cap \Pi \subseteq \pi_{n+1}$ and $\overline{E \cap \Pi} \cap \pi_{n+1} = \emptyset$, for every $(E, u) \in \mathcal{E}_l$;
5. either $E \cap \Pi \not\subseteq \pi_{n+1}$ or $\overline{E \cap \Pi} \cap \pi_{n+1} \neq \emptyset$, for every $(E, u) \in \mathcal{N}_l$.

Intuitively, π triggers $A \xrightarrow{l} A'$ if π enables a chain of agents **tell** a_i , $1 \leq i \leq n$, so that signals in $\pi_{n+1} = \pi \cup \bigcup_{1 \leq i \leq n} \{a_i\}$ are in the environment (cond. 1–3), all pausing actions represented by causality terms (E, n) in $\mathcal{E}_l$ are enabled by π_{n+1} (cond. 4), and no action represented by a causality term (E, u) (with $u \in \{n\} \cup \Pi$) in $\mathcal{N}_l$ is enabled by π_{n+1} (cond. 5).

The following propositions have been proved in [26].

Proposition 1. *A set of signals π triggers a transition $A \xrightarrow{l} A'$ if and only if the agent A reacts to the set of signals π so that it will behave as A' at the next instant and the resulting environment contains signals $\pi \cup \bigcup_{(E,a) \in \mathcal{E}_l, a \in \Pi} \{a\}$.*

Proposition 2. *An agent A is reactive and deterministic if and only if, for any agent B such that $A \xrightarrow{l_1} \dots \xrightarrow{l_n} B$, $n \geq 0$, every set of signals π triggers precisely one transition $B \xrightarrow{l} B'$.*

Example 3. Given the agent A of Example 1, we have that $A \xrightarrow{l} \text{skip}$, with $l = \langle \emptyset, \{(\{\bar{a}\}, a)\} \rangle$, is the unique transition from A . (Note that $A \not\xrightarrow{l'}$ for $l' = \langle \{(\{\bar{a}\}, a)\}, \emptyset \rangle$, since l' violates the first condition of Def. 2 and is not in $\mathcal{L}$.) No set $\pi \subseteq \Pi$ with $a \notin \pi$ triggers the transition. It follows that A is non-reactive. Given the agent B of Example 1, we have that $B \xrightarrow{l_1} \text{skip}$ and $B \xrightarrow{l_2} \text{skip}$, with $l_1 = \langle \{(\{\bar{a}\}, b)\}, \{(\{\bar{b}\}, a)\} \rangle$ and $l_2 = \langle \{(\{\bar{b}\}, a)\}, \{(\{\bar{a}\}, b)\} \rangle$. The empty set of signals triggers both transitions. It follows that B is nondeterministic.

Note that the part of LTS reachable from a given state A is finite. This fact and Prop. 2 imply that reactivity and determinism of agents are decidable.

2.3 Removing Redundant Information from Labels

Although the LTS we have defined serves our purpose in reflecting the operational semantics of agents given in [22], we are not satisfied with it. In fact, it discriminates agents that are intuitively equivalent. As an example, agents A and $A \parallel \text{if } a \text{ then } A$ are not bisimilar. Our aim is now to remedy this situation.

Let $\text{del} : \mathcal{L} \rightarrow \mathcal{L}$ be the function removing redundant information from labels such that:

- del removes from $\mathcal{E}_l \cup \mathcal{N}_l$ each (E, u) s.t. $(E', u) \in \mathcal{E}_l \cup \mathcal{N}_l$ and $E \supset E'$;
- del removes from $\mathcal{E}_l \cup \mathcal{N}_l$ every pair of causality terms $(E \cup \{a\}, n)$ and $(E \cup \{\bar{a}\}, n)$ and adds the causality term (E, n) either to $\mathcal{E}_l$, if $(E \cup \{a\}, n)$ or $(E \cup \{\bar{a}\}, n)$ were in $\mathcal{E}_l$, or to $\mathcal{N}_l$, if both $(E \cup \{a\}, n)$ and $(E \cup \{\bar{a}\}, n)$ were in $\mathcal{N}_l$.
- del replaces each causality term $(E \cup \{\gamma\}, n) \in \mathcal{E}_l \cup \mathcal{N}_l$ by (E, n) , provided that $(E' \cup \{\bar{\gamma}\}, n) \in \mathcal{E}_l \cup \mathcal{N}_l$ and $E' \subset E$.

If both (E, u) and (E', u) appear in l and $E \supset E'$, then the former causality term carries redundant information. In fact, it is sufficient that the environment prompts signals as assumed by E' to have the action represented by u . Intuitively, this corresponds to have that A is equivalent to $A \parallel \text{if } \gamma \text{ then } A$.

If both $(E \cup \{a\}, n)$ and $(E \cup \{\bar{a}\}, n)$ appear in l , then they are replaced by (E, n) because it is sufficient that the environment prompts signals as assumed by E to have the pausing action represented by n . Intuitively, this corresponds to have that $\text{next } A$ is equivalent to $\text{if } a \text{ then next } A \parallel \text{if } \bar{a} \text{ then next } A$.

Note that $A \equiv \text{if } a \text{ then tell } b \parallel \text{if } \bar{a} \text{ then tell } b$ is not equivalent to $A' \equiv \text{tell } b$. In fact, agent $B \equiv \text{if } b \text{ then tell } a$ discriminates them, since $A' \parallel B$ is deterministic and reactive, whereas $A \parallel B$ is not. For this reason, given $b \in \Pi$, del does not replace $(E \cup \{a\}, b)$ and $(E \cup \{\bar{a}\}, b)$ by (E, b) .

If both $(E \cup \{\gamma\}, n)$ and $(E' \cup \{\bar{\gamma}\}, n)$ appear in l and $E' \subset E$, then the former causality term is replaced by (E, n) , because it is sufficient that the environment prompts signals as assumed by E to have the action represented by n .

Now, we will consider the LTS obtained by applying function del to every label. This corresponds to replace $a(l)$ by $\text{del}(a(l))$, $\bar{a}(l)$ by $\text{del}(\bar{a}(l))$, $N(a, l)$ by $\text{del}(N(a, l))$, $N(\bar{a}, l)$ by $\text{del}(N(\bar{a}, l))$ and $l_1 \otimes l_2$ by $\text{del}(l_1 \otimes l_2)$ in Table 1.

It is immediate that Prop. 1 and Prop. 2 are still valid.

2.4 Bisimulation

In order to relate agents having the same input/output behavior, we consider the bisimulation on the states of the LTS. Given agents A_1 and A_2 such that $A_1 \approx A_2$, the external environment is not able to distinguish between them. In fact, at each instant, if A_1 and A_2 are stimulated with the same set of signals, then they respond by broadcasting the same signals. Moreover, A_1 is reactive and deterministic if and only if A_2 is. (These facts are consequences of Prop. 1 and Prop. 2.) We show now that no Gentzen context is able to discriminate them, i.e. Gentzen constructs preserve bisimulation.

Theorem 1. *The bisimulation on Gentzen agents is a congruence.*

Proof. Table 1 contains a recursion rule and only rules in the well known *de Simone format* [24]. So, Gentzen is a *de Simone language* and, as it has been proved in [24], this guarantees that bisimulation is a congruence. $\square$

2.5 Some Remarks

The structure of our LTS labels could seem too heavy. One may wonder whether LTS labels consisting of pairs $\langle \text{“received signals”}, \text{“produced signals”} \rangle$ would be sufficient. Unfortunately, such a simple solution must be rejected, since labels do not carry information on signal causality, which is needed to define a semantics of a synchronous language in a compositional way (see [15] for a detailed discussion).

Example 4. Assume agents $A \equiv \text{if } a \text{ then tell } b \parallel \text{if } c \text{ then tell } d$ and $A' \equiv \text{if } a \text{ then if } c \text{ then (tell } b \parallel \text{tell } d)$. Let $A \xrightarrow{l} \text{skip} \parallel \text{skip}$ and $A' \xrightarrow{l} \text{skip} \parallel \text{skip}$, with $l = \langle \{a, c\}, \{b, d\} \rangle$, be the transitions representing the reactions of A and A' to input $\{a, c\}$, respectively. Let $B \equiv \text{if } b \text{ then tell } c$ and $B \xrightarrow{l'} \text{skip}$, with $l' = \langle \{b\}, \{c\} \rangle$, be the transition representing the reaction of B to input $\{b\}$. The transition $A \parallel B \xrightarrow{l''} (\text{skip} \parallel \text{skip}) \parallel \text{skip}$, with $l'' = \langle \{a\}, \{b, c, d\} \rangle$, must be obtained by combining $A \xrightarrow{l} \text{skip} \parallel \text{skip}$ with $B \xrightarrow{l'} \text{skip}$. Therefore, also the transition $A' \parallel B \xrightarrow{l''} (\text{skip} \parallel \text{skip}) \parallel \text{skip}$ is in the LTS. This is wrong, since $A' \parallel B$ reacts to $\{a\}$ by broadcasting no signal.

We have said in the introduction of this paper that synchronous concurrency cannot be reduced to sequentiality. If this sentence were wrong, it would be possible to simulate agent A of Example 4 by a sequential agent $\hat{A}$. Now, $\hat{A}$ should broadcast both b and d iff both a and c are present. So, both $\text{tell } b$ and $\text{tell } d$ should appear in the body of both $\text{if } a \text{ then } _$ and $\text{if } c \text{ then } _$. In this case, the Gentzen context $[- \parallel B]$, with B as in Example 4, discriminates A and $\hat{A}$, thus proving that $\hat{A}$ cannot simulate A .

3 The Axiomatization

In this section we present an axiomatization over Gentzen which is sound and complete modulo bisimulation.

3.1 The Axiom System

Let us denote by “=” the least reflexive, symmetric and transitive congruence over Gentzen agents satisfying the axioms in Table 2.

Table 2. An axiom system for Gentzen.

$A_1 \parallel A_2 = A_2 \parallel A_1$ ($\parallel_1$)	$A_0 \parallel (A_1 \parallel A_2) = (A_0 \parallel A_1) \parallel A_2$ ($\parallel_2$)
$A \parallel \text{skip} = A$ ($\parallel_3$)	$A \parallel A = A$ ($\parallel_4$)
$\text{if } \gamma \text{ then } (A_1 \parallel A_2) = \text{if } \gamma \text{ then } A_1 \parallel \text{if } \gamma \text{ then } A_2$	(<i>if</i> _1)
$\text{if } \gamma \text{ then if } \gamma' \text{ then } A = \text{if } \gamma' \text{ then if } \gamma \text{ then } A$	(<i>if</i> _2)
$\text{if } \gamma \text{ then if } \gamma \text{ then } A = \text{if } \gamma \text{ then } A$	(<i>if</i> _3)
$A = A \parallel \text{if } \gamma \text{ then } A$	(<i>if</i> _4)
$\text{if } \gamma \text{ then skip} = \text{skip}$	(<i>if</i> _5)
$\text{next } A = \text{if } a \text{ then next } A \parallel \text{if } \bar{a} \text{ then next } A$	(<i>if</i> _6)
$\text{next } A_1 \parallel \text{next } A_2 = \text{next } (A_1 \parallel A_2)$	(<i>next</i>)
$\text{rec } P. A = A[\text{rec } P. A/P]$	(<i>rec</i> _1)
$B = A[B/P]$ and P guarded in A imply $B = \text{rec } P. A$	(<i>rec</i> _2)

Axioms $\parallel_1 - \parallel_3$, *if*_1 – *if*_5, *next* and *rec*_1, *rec*_2 are very intuitive.

Axiom $\parallel_4$ deserves attention. Given transitions $A \xrightarrow{l_1} A_1$ and $A \xrightarrow{l_2} A_2$ with $l_1 \neq l_2$, it holds that $l_1 \otimes l_2 \notin \mathcal{L}$ (this fact can be proved by induction over A). Moreover, given a label $l \in \mathcal{L}$, it holds that $l \otimes l = l$. From these facts it follows that $A \parallel A \approx A$, i.e. axiom $\parallel_4$ is sound.

We have explained in Section 2 that one expects axiom *if*_6, and that one does not expect the following axiom:

$$\text{tell } b = \text{if } a \text{ then tell } b \parallel \text{if } \bar{a} \text{ then tell } b.$$

The following result follows from definitions of $\gamma(l)$, $N(\gamma, l)$, $l_1 \otimes l_2$ and $\text{del}(l)$.

Lemma 1 (Soundness). *Given agents A and A' , $A = A'$ implies $A \approx A'$.*

Our axiom system is *finite*; all axioms except the *recursion specification principle* [5,17] (i.e. axiom *rec*_2) are *unconditional*. We have not enriched the language by any auxiliary operator. In particular, it is worth noting that we do not need the auxiliary operator “left merge” [6], which, on the contrary, is needed to have finite axiomatizations for asynchronous process algebras (see [18]).

3.2 Normal Forms

Given a string $\vartheta \in (\Pi \cup \overline{\Pi})^*$, we denote by **if** ϑ **then** A the agent:

$$\text{if } \vartheta \text{ then } A \equiv \begin{cases} A & \text{if } \vartheta = \epsilon \text{ } (\epsilon \text{ denotes the empty string}) \\ \text{if } \gamma \text{ then if } \phi \text{ then } A & \text{if } \vartheta = \gamma\phi, \phi \in (\Pi \cup \overline{\Pi})^* \end{cases}$$

Given a string $\vartheta \in (\Pi \cup \overline{\Pi})^*$, we denote by $|\vartheta|$ the set $\{\gamma \mid \gamma \text{ appears in } \vartheta\}$.

Definition 3. *An agent A is a normal form if either $A \equiv \text{skip}$ or there exist agents $A_1, \dots, A_n$ such that:*

1. $A \equiv A_1 \parallel \dots \parallel A_n$;
2. $A_i \equiv \text{if } \vartheta_i \text{ then } B_i$, and either $B_i \equiv \text{tell } a_i$, or $B_i \equiv \text{next } A'_i$, or $B_i \equiv P_i$, with a_i a signal, A'_i an arbitrary agent and P_i a recursion variable;
3. if $B_i \equiv \text{next } A'_i$ then, for each $a \in \Pi$, either a or $\bar{a}$ appears in ϑ_i ;
4. if $B_i \equiv \text{next } A'_i$ then every symbol $\gamma \in \Pi \cup \overline{\Pi}$ appears at most once in ϑ_i ;
5. if $B_i \equiv \text{next } A'_i$ and $B_j \equiv \text{next } A'_j$ then there exists some $\gamma \in \Pi \cup \overline{\Pi}$ such that γ appears in ϑ_i and $\bar{\gamma}$ appears in ϑ_j ;
6. if $B_i \equiv \text{tell } a$ then there exists no j such that $B_j \equiv \text{tell } a$ and $|\vartheta_j| \subseteq |\vartheta_i|$.

Let $A \equiv A_1 \parallel \dots \parallel A_n$ be a normal form. Conditions 1, 2 and 3 of Def. 3 will be exploited in proving the completeness of our axiomatization.

Condition 4 is a reasonable request and can be easily respected by transforming agents by axioms *if* 2 and *if* 3.

Condition 5 implies that if $A_i \equiv \text{if } \vartheta_i \text{ then next } A'_i$ and each signal a such that $a \in |\vartheta_i|$ (resp. $\bar{a} \in |\vartheta_i|$) is present (resp. absent), then A will behave as A'_i at the next instant. More precisely, at the next instant the agent $\text{skip} \parallel \dots \parallel \text{skip} \parallel A'_i \parallel \text{skip} \parallel \dots \parallel \text{skip}$ will run.

Condition 6 implies that redundant agents broadcasting signals do not appear as parallel components of A . Moreover, if $A_i \equiv \text{if } \vartheta_i \text{ then tell } a$, then $(|\vartheta_i|, a)$ appears in any label l such that $A \xrightarrow{l}$, since it is not removed by function *del*.

We prove now that each agent can be transformed into a normal form, having only normal forms as derivatives, by applying axioms of Table 2.

Lemma 2 (Reducibility to normal forms). *Given an agent A , there exist normal forms $A_1, \dots, A_m$ such that:*

- $A = A_1$;
- either $A_i = \text{skip}$, or $A_i = A_{i,1} \parallel \dots \parallel A_{i,n_i}$, $A_{i,j} \equiv \text{if } \vartheta_{i,j} \text{ then } B_{i,j}$, and, if $B_{i,j} \equiv \text{next } A_{f(i,j)}$, then $f(i,j) \in \{1, \dots, m\}$.

Proof. By structural induction on A .

Base case: if either $A \equiv \text{skip}$ or $A \equiv \text{tell } a$ or $A \equiv P$ then the thesis is immediate, because A is a normal form.

Induction step: Assume that, given agents A' and A'' , there exist normal forms $A_1', \dots, A_{m'}'$ and $A_1'', \dots, A_{m''}''$, either $A_{i'}' = \text{skip}$ or $A_{i'}' = A_{i',1}' \parallel \dots \parallel A_{i',n_{i'}}'$, either $A_{i''}'' = \text{skip}$ or $A_{i''}'' = A_{i'',1}'' \parallel \dots \parallel A_{i'',n_{i''}}''$, $A_{i',j}' \equiv \text{if } \vartheta_{i',j}' \text{ then } B_{i',j}'$, $A_{i'',j}'' \equiv \text{if } \vartheta_{i'',j}'' \text{ then } B_{i'',j}''$, $A' = A_1'$, $A'' = A_1''$.

We consider the following cases:

- $A \equiv \text{next } A'$: Since $A' = A_{1'}$ and $=$ is a congruence, we can rewrite A into $\text{next } A_{1'}$, which does not satisfy Condition 3 of Def. 3. Now, $\text{next } A_{1'}$ is rewritten into a normal form by axiom *if_6*, and the thesis follows.
- $A \equiv A' \parallel A''$: Since $A' = A_{1'}$ and $A'' = A_{1''}$, we infer $A = A_{1'} \parallel A_{1''}$ by the fact that $=$ is a congruence. Let us denote with $\mathcal{I} \subseteq \{1', \dots, m'\} \times \{1'', \dots, m''\}$ the least set such that:

- $(1', 1'') \in \mathcal{I}$;
- if $(i', i'') \in \mathcal{I}$, $A_{i', j'} \equiv \text{if } \vartheta_{i', j'} \text{ then next } A_{f(i', j')}$,
 $A_{i'', j''} \equiv \text{if } \vartheta_{i'', j''} \text{ then next } A_{f(i'', j'')}$ and $|\vartheta_{i', j'}| = |\vartheta_{i'', j''}|$
then $(f(i', j'), f(i'', j'')) \in \mathcal{I}$.

The set $\mathcal{I}$ contains (i', i'') if A reaches a state in which it behaves as $A_{i'} \parallel A_{i''}$. The thesis follows if we infer $A_{i'} \parallel A_{i''} = A_{i', i''}$, with $A_{i', i''}$ an arbitrary normal form having only normal forms as derivatives, for every $(i', i'') \in \mathcal{I}$.

If $A_{i'} = \text{skip}$ (resp. $A_{i''} = \text{skip}$) then we delete it by axiom $\parallel_3$, and the thesis follows. Otherwise, agent $A_{i'} \parallel A_{i''}$ satisfies Cond. 1–4 of Def. 3. Let us assume that it does not satisfy Cond. 5, i.e. $A_{i', j'} \equiv \text{if } \vartheta_{i', j'} \text{ then next } A_{f(i', j')}$, $A_{i'', j''} \equiv \text{if } \vartheta_{i'', j''} \text{ then next } A_{f(i'', j'')}$ and $|\vartheta_{i', j'}| = |\vartheta_{i'', j''}|$, for some $1 \leq j' \leq n_{i'}$, $1 \leq j'' \leq n_{i''}$. By axioms $\parallel_1, \parallel_2$ we rewrite $A_{i'} \parallel A_{i''}$ so that $A_{i', j'}$ and $A_{i'', j''}$ appear as adjacent. Now, by axiom *if_2* we rewrite $\text{if } \vartheta_{i'', j''} \text{ then next } A_{f(i'', j'')}$ into $\text{if } \vartheta_{i', j'} \text{ then next } A_{f(i'', j'')}$, and, by axioms *if_1* and *next*, we rewrite $A_{i', j'} \parallel \text{if } \vartheta_{i', j'} \text{ then next } A_{f(i'', j'')}$ into $\text{if } \vartheta_{i', j'} \text{ then next } (A_{f(i', j')} \parallel A_{f(i'', j'')})$. Since this rewriting preserves conditions 1–4 of Def. 3 and can be applied to every pair $(A_{i', j'}, A_{i'', j''})$ such that $|\vartheta_{i', j'}| = |\vartheta_{i'', j''}|$, we can transform $A_{i'} \parallel A_{i''}$ into an agent satisfying Cond. 1–5 of Def. 3. If the agent so obtained violates Cond. 6 of Def. 3, we can remedy to this situation by applying axioms $\parallel_1, \parallel_2$ as before, and then axioms *if_2* and *if_4*.

- $A \equiv \text{if } \gamma \text{ then } A'$: Since $A' = A_{1'}$, we infer $A = \text{if } \gamma \text{ then } A_{1'}$ by the fact that $=$ is a congruence. If $A_{1'} = \text{skip}$ then we rewrite $\text{if } \gamma \text{ then } A_{1'}$ into skip by axiom *if_5* and the thesis follows. Otherwise, since $A_{1'}$ is a normal form, we apply axiom *if_1* to $\text{if } \gamma \text{ then } A_{1'}$ and we obtain an agent satisfying conditions 1, 2, 3 and 5 of Def. 3. By axioms *if_2* and *if_3* we can rewrite every parallel component of this agent so that it satisfies also Cond. 4, while preserving other conditions. The agent so obtained can be transformed into an agent satisfying Cond. 6 of Def. 3 as in the previous case, and the thesis follows.
- $A \equiv \text{rec } P.A'$: If $A' \equiv \text{skip}$ then $A = \text{skip}$ by axiom *rec_1* and the thesis follows. Otherwise, let $H_{1'}, \dots, H_{m'}$ be agents s.t. $H_{i'} \equiv A_{i'}[A/P]$. We have $H_{i'} \equiv A_{i'}[A/P] \equiv (A_{i', 1} \parallel \dots \parallel A_{i', n_{i'}})[A/P] \equiv$
 $A_{i', 1}[A/P] \parallel \dots \parallel A_{i', n_{i'}}[A/P] \equiv$
 $A_{i', 1}[H_{f(i', 1)}/A_{f(i', 1)}][A/P] \parallel \dots \parallel A_{i', n_{i'}}[H_{f(i', n_{i'})}/A_{f(i', n_{i'})}][A/P] =$
 $A_{i', 1}[H_{f(i', 1)}/A_{f(i', 1)}][A'[A/P]/P] \parallel \dots$
 $\dots \parallel A_{i', n_{i'}}[H_{f(i', n_{i'})}/A_{f(i', n_{i'})}][A'[A/P]/P] =$
 $A_{i', 1}[H_{f(i', 1)}/A_{f(i', 1)}][A_{1'}[A/P]/P] \parallel \dots$
 $\dots \parallel A_{i', n_{i'}}[H_{f(i', n_{i'})}/A_{f(i', n_{i'})}][A_{1'}[A/P]/P] \equiv$
 $A_{i', 1}[H_{f(i', 1)}/A_{f(i', 1)}][H_{1'}/P] \parallel \dots \parallel A_{i', n_{i'}}[H_{f(i', n_{i'})}/A_{f(i', n_{i'})}][H_{1'}/P],$

where the equalities are inferred by axiom *rec_1* and by the fact that $=$ is a congruence. Now, $H_{i'}$ has been rewritten as a parallel composition of normal forms having only normal forms as derivatives. Let us call $K_{i'}$ such agent. Every $K_{i'}$ can be transformed in a normal form as in the case for “ $\parallel$ ”. Finally, the thesis follows since $A = A'[A/P] = A_{1'}[A/P] \equiv H_{1'} = K_{1'}$. $\square$

3.3 The Proof of Completeness

First of all we recall the notion of guarded recursive specification and we prove that every guarded recursive specification has a solution which is unique modulo $=$. Then, we exploit this result and Lemma 2 to prove that two arbitrary agents A and A' such that $A \approx A'$ are equated by our axioms. In fact, we prove that $A = B$ and $A' = B'$, where A' and B' are normal forms and solutions of the same guarded recursive specification.

A *recursive specification* over variables $\vec{P} = P_1, \dots, P_m$ is a set of equations

$$P_i = B_i \quad 1 \leq i \leq m \quad (1)$$

where B_i is an agent, $1 \leq i \leq m$. It is *guarded* if and only if $P_1, \dots, P_m$ are guarded in $B_1, \dots, B_m$, i.e. $P_1, \dots, P_m$ appear in $B_1, \dots, B_m$ in bodies of **next** ..

A *solution* of a recursive specification as (1) is a set of agents $\vec{A} \equiv A_1, \dots, A_m$ such that $A_i = B_i[\vec{A} / \vec{P}]$, $1 \leq i \leq m$.

The following result, which states that every guarded recursive specification has a solution unique modulo “ $=$ ”, has been presented in [17]. (Note that axiom *rec_2* plays a central rôle in the proof.)

Lemma 3. *Every guarded recursive specification*

$$P_i = B_i \quad 1 \leq i \leq m$$

has a solution $\vec{A} \equiv A_1, \dots, A_m$. Given any other solution $\vec{A'} \equiv A'_1, \dots, A'_m$, it holds that $A'_i = A_i$, $1 \leq i \leq m$.

We show now that two arbitrary bisimilar agents are equated by our axioms.

Lemma 4 (Completeness). *Given agents A and A' , $A \approx A'$ implies $A = A'$.*

Proof. By Lemma 2, there are normal forms $A_1, \dots, A_m$ and $A_{1'}, \dots, A_{m'}$ s.t.:

- $A = A_1$, $A' = A_{1'}$
- either $A_i = \mathbf{skip}$ or $A_i = A_{i,1} \parallel \dots \parallel A_{i,n_i}$, $1 \leq i \leq m$
- either $A_{i'} = \mathbf{skip}$ or $A_{i'} = A_{i',1} \parallel \dots \parallel A_{i',n_{i'}}$, $1 \leq i' \leq m'$
- $A_{i,j} \equiv \mathbf{if } \vartheta_{i,j} \mathbf{ then } B_{i,j}$, $1 \leq i \leq m$, $1 \leq j \leq n_i$
- $A_{i',j'} \equiv \mathbf{if } \vartheta_{i',j'} \mathbf{ then } B_{i',j'}$, $1 \leq i' \leq m'$, $1 \leq j' \leq n_{i'}$.

Moreover, by axioms $\parallel_4$ we can assume that $A_{i,j} \not\equiv A_{i,h}$ for $1 \leq j < h \leq n_i$ and $A_{i',j'} \not\equiv A_{i',h'}$ for $1 \leq j' < h' \leq n_{i'}$.

Now, to prove the thesis it is sufficient to prove that $A_1 = A_{1'}$.

Let us denote by $\mathcal{I} \subseteq \{1, \dots, m\} \times \{1', \dots, m'\}$ the set of pairs (i, i') such that $A_i \approx A_{i'}$. Since $A \approx A'$, $A = A_1$ and $A' = A_{1'}$, and $=$ is sound modulo $\approx$, we have $(1, 1') \in \mathcal{I}$.

We prove now that, given a pair $(i, i') \in \mathcal{I}$, and an arbitrary index $1 \leq j \leq n_i$, there exists an index $1 \leq j' \leq n_{i'}$ such that:

- $\vartheta_{i',j'} = \vartheta_{i,j}$;
- either $B_{i,j} \equiv B_{i',j'}$, or $B_{i,j} \equiv \text{next } A_{f(i,j)}$, $B_{i',j'} \equiv \text{next } A_{f(i',j')}$ and $(f(i,j), f(i',j')) \in \mathcal{I}$.

We begin with proving that there is a $1 \leq j' \leq n_{i'}$ such that $|\vartheta_{i,j}| = |\vartheta_{i',j'}|$ and the second condition above is satisfied. We consider the following cases:

- $B_{i,j} \equiv \text{tell } a$: Since A_i is a normal form and, in particular, it satisfies Cond. 6 of Def. 3, $(|\vartheta_{i,j}|, a)$ appears in any label l such that $A \xrightarrow{l}$. Since $A_i \approx A_{i'}$, we have $A_{i'} \xrightarrow{l}$. Since $(|\vartheta_{i,j}|, a)$ appears in l , there exists $1 \leq j' \leq n_{i'}$ such that $|\vartheta_{i',j'}| = |\vartheta_{i,j}|$ and $B_{i',j'} \equiv \text{tell } a$.
- $B_{i,j} \equiv \text{next } A_{f(i,j)}$: Since A_i is a normal form and, in particular, it satisfies Cond. 5 of Def. 3, we have $A \xrightarrow{l} A_{f(i,j)}$ for some label l . Since $A \approx A'$, there exists an agent $A_{f(i',j')}$ such that $A \xrightarrow{l} A_{f(i',j')}$ and $A_{f(i,j)} \approx A_{f(i',j')}$.

Now, if $|\vartheta_{i,j}| = |\vartheta_{i',j'}|$ but $\vartheta_{i,j} \neq \vartheta_{i',j'}$, we can apply axiom *if*_2 to $A_{i',j'}$ while preserving conditions of Def. 3. So, we can rewrite **if** $\vartheta_{i',j'}$ **then** $B_{i',j'}$ into **if** $\vartheta_{i,j}$ **then** $B_{i',j'}$. Now, we reapply our reasoning from the beginning.

Let us consider now the recursive specification

$$P_{i,i'} = A_{i,i'} \quad (i, i') \in \mathcal{I}$$

where $A_{i,i'}$ is the agent such that $A_{i,i'} \equiv H_{i,i',1} \parallel \dots \parallel H_{i,i',n_i}$, with

either $H_{i,i',j} \equiv \text{if } \vartheta_{i,j} \text{ then next } P_{f(i,j),f(i',j')}$,

if both $A_{i,j} \equiv \text{if } \vartheta_{i,j} \text{ then next } A_{f(i,j)}$ and $A_{i',j'} \equiv \text{if } \vartheta_{i',j'} \text{ then next } A_{f(i',j')}$,

or $H_{i,i',j} \equiv A_{i,j} \equiv A_{i',j'}$, otherwise.

This recursive specification is guarded and has both $\vec{G}$ and $\vec{G}'$ as solutions, where $G_{i,i'} \equiv A_i$ and $G'_{i,i'} \equiv A_{i'}$ for each $(i, i') \in \mathcal{I}$. Now, by Lemma 3 it follows that $A_i = A_{i'}$, $(i, i') \in \mathcal{I}$. Since $(1, 1') \in \mathcal{I}$, we have $A_1 = A_{1'}$, as required. $\square$

The following theorem states that axioms in Table 2 give an axiomatization sound and complete modulo bisimulation on Gentzen agents.

Theorem 2 (Soundness and completeness). *Given arbitrary agents A and A' , $A = A'$ if and only if $A \approx A'$.*

Proof “If”: by Lemma 4. “Only if”: by Lemma 1.

4 Conclusions and Future Work

We have presented an axiomatization of the synchronous language *Gentzen*, in order to characterize behaviorally equivalent agents.

The axiomatization of *Gentzen* cannot be done by trivially adapting techniques developed in the field of asynchronous concurrency. Axiomatizations of asynchronous process description languages are based on the transformation of processes into “head normal forms”. In order to transform concurrent processes into head normal forms, concurrency must be simulated by interleaving. This is impossible in the synchronous setting, because processes running in parallel are perfectly synchronized and cannot interleave.

The axiomatization of *Gentzen* should be intended as an initial step toward the development of axiomatizations for state-oriented synchronous languages. In fact, *Gentzen* offers all basic constructs that are shared by the languages in this class and that distinguish synchronous languages w.r.t. asynchronous ones.

Given any other state-oriented synchronous language, we believe that it can be axiomatized by simply extending our work to deal with those constructs that are typical in such a language and that are not offered by *Gentzen*. In particular, we believe that such a language can be axiomatized by exploiting the notion of normal form proposed in this paper. To support this affirmation, we have developed an axiomatic semantics for the language Esterel [8] in [26]. The axiomatization of Esterel is derived by extending the technique proposed in the present paper to deal with the interruption mechanism “**trap**”, the suspension mechanism “**suspend**”, the construct of instantaneous sequencing “;” and the property of *constructiveness* [7]. The technique adopted to axiomatize these constructs is to express them in terms of constructs of *Gentzen*.

The approach of [26] can be generalized to any synchronous language: given a superset of *Gentzen*, it is sufficient to express any construct in terms of constructs of *Gentzen*. So, *Gentzen* could play in the synchronous setting the rôle of T_{fintree} (i.e. a subset of CCS suitable for expressing finite LTSs) in the asynchronous setting, where an axiomatization of T_{fintree} has been proposed in [17], and supersets of T_{fintree} are axiomatized by expressing any construct in terms of constructs of T_{fintree} . Following [1,2], where algorithmic techniques to compute axiomatizations of classes of supersets of T_{fintree} have been proposed, one could develop algorithmic techniques to compute axiomatizations of classes of supersets of *Gentzen*.

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