Affine Structure from Translational Motion in Image Sequences

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Abstract. *In this paper a method for obtaining affine structure from an image sequence taken by a translating camera with constant intrinsic parameters is presented. A general geometric constraint, expressed using the camera matrices, is derived and this constraint is used in a least squares solution of the problem. The first step is to obtain a projective reconstruction, in the form of a sequence of camera matrices (and a sparse set of feature points), and then these constraints are used to upgrade to an affine reconstruction. The proposed algorithm extends previous results of affine structure recovery from two images with a translating camera to the general case of a sequence of images. The proposed method is illustrated in both simulated and real experiments.*

Keywords: structure and motion estimation, affine camera, translating motion, stratification

1 Introduction

One of the main goals in computer vision is to recover the Euclidean 3D structure of a scene, where the projective to affine reconstruction upgrade is an essential step [1]. Much work has been done in recovering both structure and motion from image sequences where the motion of the camera as well as the structure of the scene are unknown [2–4]. For these algorithms, it is assumed and often required the camera motion is unconstrained (i.e. the camera movement contains both translation and rotation). In many applications of computer vision, such as car crash tests, vehicle navigation and monitoring of industrial assembly lines, the relative motion of the camera is restricted to a translation. This information can be used in the reconstruction process. In [5] a method is presented to obtain an affine reconstruction from one pair of images taken with a translating camera with constant intrinsic parameters. We will now extend these previous results by showing that given a sequence of two or more images from a translating camera with constant intrinsic parameters, a projective reconstruction of the scene can be upgraded to an affine reconstruction. We will also present an algorithm for doing this.

Three-dimensional reconstruction from a purely translating camera has some inherent limitations. In the case of a translating camera where all the intrinsic parameters are varying and unknown, an affine reconstruction is impossible [6, 7]. It is also well-known that the affine to Euclidean upgrade from translational motion is critical in the

sense that some extra information is needed in order to get a unique Euclidean structure [8]. However, as such motions are frequent in practice, we need to be able to deal with them.

Several authors have exploited the special form of the camera matrices for translational motion and applications to projective reconstruction. In [9] it is shown that the camera matrices have simple forms when assuming translational motion and also when co-planar points can be utilized, e.g. the projectively reduced setting. Also in [10] co-planar structures are used to simplify the reconstruction algorithm. However, neither of these papers utilizes translating cameras and affine reconstruction from constant intrinsic parameters.

The purpose of this paper is to present an algorithm for affine reconstruction from an image sequence taken by a translating camera. The first step is to estimate the projective structure and motion using standard techniques. The obtained sequence of camera matrices (determined up to an unknown projective transformation) is then upgraded to an affine motion, i.e. a sequence of camera matrices determined up to an affine transformation. The constraints needed to perform this upgrade is derived and a least squares solution is adopted. The paper is organized as follows: In Section 2 some notation and background about projective reconstruction are given. The constraints imposed by assuming translational motion are derived in Section 3 and the least squares solution is presented in Section 4. In Section 5 experiments on both synthetic and real data are presented and in Section 6 we will give some conclusions.

2 Background and Notation

We will use the standard pin-hole camera model:

$$\lambda \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} \gamma f & sf & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\underbrace{K}_{P}} \underbrace{\begin{bmatrix} R & -Rt \end{bmatrix}}_{\mathbf{x}} \underbrace{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}_{\mathbf{X}}. \quad (1)$$

Here, f denotes the focal length, γ and s the aspect ratio and the skew and (x_0, y_0) the principal point. These are called the **intrinsic parameters**, and they are contained in the upper-triangular **calibration matrix** K . Furthermore, R and t denote the relation between the camera coordinate system and the object coordinate system, where R is a rotation matrix and t a translation vector, i.e. a Euclidean transformation. The object points in homogeneous coordinates are denoted by $\mathbf{X}$ and the image points in homogeneous coordinates by $\mathbf{x}$.

In a sequence with several images we will use the notation

$$\lambda_i \mathbf{x}_i = P_i \mathbf{X}, \quad i = 1, \dots, m, \quad (2)$$

where m denotes the number of images.

The initial structure and motion is obtained from the following steps:

Extract and track feature points through the image sequence:

The standard Harris corner detector, cf. [11] is used together with a correlation-based tracker, similar to the KLT tracker, cf. [12], [13].

Use a robust method to estimate fundamental matrices and trifocal tensors:

The RANSAC method method is used to estimate the fundamental matrix, F , relating corresponding image coordinates in two images, and the trifocal tensor, T , relating corresponding image coordinates in three images. In practice, feature points are tracked until the estimated fundamental matrix has a smaller error than an estimated homography according to an information criterion and similarly for the trifocal tensor.

Iteratively, use resection and intersection:

Once a structure is obtained from three key-frames, the camera position for additional frames can be estimated using resection and new feature points can be reconstructed using intersection. Also in this stage a robust method has to be applied to remove false matches and outliers.

Bundle adjustment:

The maximum likelihood estimate of the projective structure and motion is the solution to the optimization problem

$$f = \sum_{i,j \in I} (\mathbf{x}_{i,j} - g(P_i, \mathbf{X}_j))^2, \quad (3)$$

where $g(P_i, \mathbf{X}_j)$ denotes the re-projected feature points. The minimization of f can be done by iterative methods as described for the bundle adjustment method, [14].

3 Translational Motion

In a Euclidean reconstruction, the camera matrices P_i^E are of the form

$$P_i^E = K_i R_i [I \mid -\mathbf{t}_i].$$

Knowing that the camera is a pure translation, one can assume that $R_i = I$. Further, assuming constant intrinsic parameters we have

$$P_i^E = K [I \mid -\mathbf{t}_i].$$

A coordinate transformation, represented by a 4×4 H , of projective 3D-space is an affine transformation iff it has the following form:

$$H = \begin{bmatrix} A & \mathbf{u} \\ 0 & 1 \end{bmatrix},$$

where A is a nonsingular 3×3 matrix. Given the Euclidean motion, P_i^E , and 3D-points $\mathbf{X}_j$ which satisfies the camera equations

$$\lambda_{ij} \mathbf{x}_{ij} = P_i^E \mathbf{X}_j = K [I \mid -\mathbf{t}_i] \mathbf{X}_j, \quad \text{for all } i, j,$$

we can change the coordinate system with the transformation

$$H = \begin{bmatrix} K^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

so that the cameras are changed to

$$P_i^E H = K [I \mid -\mathbf{t}_i] H = [K K^{-1} \mid -K \mathbf{t}_i] = [I \mid \mathbf{b}_i].$$

and the points are transformed by $\mathbf{X}_j \rightarrow H^{-1} \mathbf{X}_j$. Notice that this is also a valid reconstruction since the projection equations are still satisfied. Therefore, since K is unknown (and constant), it is only possible to reconstruct the scene up an unknown affine transformation and one can without loss of generality assume that an object coordinate system has been chosen such

$$P_1^A = [I \mid 0] \text{ and } P_i^A = [I \mid \mathbf{b}_i], \quad i > 1. \quad (4)$$

Given the projective structure and motion, obtained as described above, in the form of a sequence of camera matrices $\{P_i^P\}_{i=1\dots n}$, $P_i^P = [A_i \mid \mathbf{p}_i]$, known up to an unknown scale factor and reconstructed feature points $\mathbf{X}_j$, our task is now to find a projective transformation, represented by a general non-singular 4×4 matrix, such that the transformed camera matrices $P_i^P H$ take the canonical form in (4). Again, without loss of generality we can assume that the projective coordinate system is chosen such that $P_1^P = [I \mid 0]$. Let

$$H = \begin{bmatrix} A & \mathbf{b} \\ \mathbf{v}^T & s \end{bmatrix}$$

where A is a 3×3 matrix, $\mathbf{b}$, $\mathbf{v}$ are 3-vectors and s is a scalar. The first camera matrix gives

$$P_1^P H = P_1^A \Leftrightarrow [I \mid 0] \begin{bmatrix} A & \mathbf{b} \\ \mathbf{v}^T & s \end{bmatrix} = [I \mid 0] \Leftrightarrow [A \mid \mathbf{b}] = [I \mid 0],$$

which implies that $A = I$ and $\mathbf{b} = 0$, giving

$$H = \begin{bmatrix} I & 0 \\ \mathbf{v}^T & s \end{bmatrix}$$

For the other cameras $P_i^P = [A_i \mid \mathbf{p}_i]$ we get

$$P_i^P H \sim P_i^A \Leftrightarrow [A_i \mid \mathbf{p}_i] \begin{bmatrix} I & 0 \\ \mathbf{v}^T & s \end{bmatrix} \sim [I \mid \mathbf{b}_i] \Leftrightarrow [A_i + \mathbf{p}_i \mathbf{v}^T \mid s \mathbf{p}_i] \sim [I \mid \mathbf{b}_i],$$

where $\sim$ means equality up to scale, implying

$$A_i + \mathbf{p}_i \mathbf{v}^T \sim I \quad \text{and} \quad \mathbf{b}_i \sim \mathbf{p}_i.$$

Thus we obtain a linear constraint on the vector $\mathbf{v}$ in the transformation matrix H , of the form $A_i + \mathbf{p}_i \mathbf{v}^T \sim I$. If we can calculate $\mathbf{v}$ we also have H and we can upgrade the projective reconstruction to affine by multiplying P_i^P with

$$H_a = \begin{bmatrix} I & 0 \\ \mathbf{v}^T & 1 \end{bmatrix}.$$

In summary, we have the following algorithm for affine reconstruction from translational motion:

1. Projective reconstruction $P_i^P = [A_i | \mathbf{p}_i]$ using traditional techniques.
2. Estimate $\mathbf{v}$ using $A_i + \mathbf{p}_i \mathbf{v}^T \sim I$.
3. Form $H_a = \begin{bmatrix} I & 0 \\ \mathbf{v}^T & 1 \end{bmatrix}$
4. Upgrade the reconstruction to affine, $P_i^A = P_i^P H_a = [I | \mathbf{b}_i]$.

In the next section we show how $\mathbf{v}$ can be estimated using a least squares estimate of $\mathbf{v}$.

4 A Least Squares Solution

The second step in the algorithm presented above requires us to solve for $\mathbf{v}$ in the over-determined linear system of equations $A_i + \mathbf{p}_i \mathbf{v}^T \sim I$. We can rewrite this as

$$A_i + \mathbf{p}_i \mathbf{v}^T = \lambda_i I,$$

where λ_i denote unknown scale factors. We now have n equations of the form

$$\begin{bmatrix} A_i^{11} & A_i^{12} & A_i^{13} \\ A_i^{21} & A_i^{22} & A_i^{23} \\ A_i^{31} & A_i^{32} & A_i^{33} \end{bmatrix} + \begin{bmatrix} p_i^1 v_1 & p_i^1 v_2 & p_i^1 v_3 \\ p_i^2 v_1 & p_i^2 v_2 & p_i^2 v_3 \\ p_i^3 v_1 & p_i^3 v_2 & p_i^3 v_3 \end{bmatrix} = \lambda_i I, \quad (5)$$

where A_i^{jk} denote element (j, k) of A_i and similarly for p_i^k . Eliminating λ_i from (5) we obtain the following equations

$$\begin{cases} A_i^{jk} + p_i^j v_k = A_i^{(j+1)(k+1)} + p_i^{j+1} v_{k+1}, & j = k, \quad j \neq 3, \\ A_i^{jk} + p_i^j v_k = 0, & j \neq k, \end{cases} \quad (6)$$

containing 8 linearly independent equations. Rewriting these into one matrix equation we get

$$M \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ 1 \end{bmatrix} = 0 \quad \text{where} \quad M = \begin{bmatrix} M_2 \\ M_3 \\ \vdots \\ M_i \\ \vdots \\ M_n \end{bmatrix} \quad \text{and} \quad M_i = \begin{bmatrix} p_i^1 - p_i^2 & 0 & A_i^{11} - A_i^{22} \\ 0 & p_i^2 - p_i^3 & A_i^{22} - A_i^{33} \\ p_i^1 & 0 & -p_i^3 & A_i^{11} - A_i^{33} \\ 0 & p_i^1 & 0 & A_i^{12} \\ 0 & 0 & p_i^1 & A_i^{13} \\ p_i^2 & 0 & 0 & A_i^{21} \\ 0 & 0 & p_i^2 & A_i^{23} \\ p_i^3 & 0 & 0 & A_i^{31} \\ 0 & p_i^3 & 0 & A_i^{32} \end{bmatrix},$$

where three equations have been used for the first constraint in (6) - even if there exists only two linearly independent constraints - because of symmetry and numerical stability. Note that we do not include M_1 in the construction of M since all the coefficients in M_1 will be zero. Let $\mathbf{x}^T = (v_1, v_2, v_3, 1)$. The least squares solution to the above optimization problem

$$\min_{\|\mathbf{x}\|=1} \|M\mathbf{x}\|$$

is given by $\mathbf{x} =$ (the last column in V), where V is the right unitary matrix in the singular value decomposition of M , given by $M = U\Sigma V^T$. Since

$$\mathbf{x} = x_4 [\mathbf{v}^T 1]^T,$$

we can now determine $\mathbf{v}$ in a least squares sense from the n camera matrices.

5 Experiments

To verify the theoretical results, a simulated experiment was performed by generating a sequence of 10 camera matrices $P_i^P = K[I \mid \mathbf{t}_i]H$, where K are random but constant intrinsic camera parameters, $\mathbf{t}_i$ represents a translational motion in a random direction with a randomly varying speed, and H is a random nonsingular projective transformation matrix. The camera matrices P_i^P now represent a reconstruction of a translating object up to an unknown projective transformation. After normalization to $P_i^P = [A_i \mid \mathbf{p}_i]$ where $P_1^P = [I \mid 0]$, the presented method was successfully used to obtain an affine reconstruction, resulting in $P_i^A = [I \mid \mathbf{b}_i]$.

In a second experiment, a sequence of 20 images of a stationary scene were captured using a translating camera. Figure 1 shows two images from the image sequence and the affine reconstruction.

Fig. 1. Two images from an image sequence obtained by a translating camera (left) and some of the reprojected points from the affine reconstruction (right).

A projective reconstruction of the scene were obtained using a standard reconstruction algorithm, and the resulting camera matrices were upgraded from projective to affine using the algorithm derived above. The reprojected 3D points are shown in Figure 1, and we note that parallelism seems to be preserved, a characteristic of affine transformations. Figure 2 shows a vrml object from a Euclidean reconstruction, which we obtained by assuming reasonable intrinsic parameters for the camera. It was created from the reprojected image points and texture mapped using a Delaunay triangulation of the image points and the texture from one of the images.

A standard measure of the quality of the reconstruction is the root mean square (RMS) of the reprojection errors (which we will denote by e) an image or in this case in an image sequence. For the projective reconstruction we had $e_1 = 0.2448$ pixels, while after the affine reconstruction the RMS of the reprojection error was $e_2 = 0.3403$ pixels. After applying a bundle adjustment changing only the structure, we had $e_3 = 0.2406$ pixels, showing the stability in the presented algorithm.

Fig. 2. The reconstructed scene.

6 Conclusions

In this paper we have shown that it is possible to make an affine reconstruction from a translating camera. We have presented an algorithm that utilizes all images in a translating image sequence, not only a stereo pair. The algorithm is based on a standard projective reconstruction, followed by a least squares solution based on the constraints evolving from the assumption of a translating camera. The proposed algorithm has been successfully demonstrated on both synthetic and real data. Future research would focus on implementing a bundle adjustment algorithm that gives the optimal affine reconstruction and investigate constraints needed to upgrade to a Euclidean structure.

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