Probability Distribution of Index Distances in Normal Index Array for Normal Vector Compression

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Abstract. Shape models are in these days frequently transmitted over Internet and the research of their compression has been started. Considering the large portion of shape model can be normal vectors, a new scheme was recently presented to compress normal vectors using clustering and mixed indexing scheme. Presented in this paper is a mathematical investigation of the scheme to analyze the probability distribution of normal index distances in Normal Index array which is critical for the compression. The probability distribution is formulated so that the values can be easily calculated once the relative probabilities of C, R, E, S, and L op-codes in Edgebreaker are known. It can be shown that the distribution of index distances can be easily transformed into a few measures for the compression performance of the proposed algorithm.

1 Introduction

Being an Internet era, everybody transmits different kinds of files even though he or she does not notice. Just like ordinary text files, sound files, movies, etc., shape models are also transmitted frequently. Since, a shape model usually consists of topology, geometry, normal vectors, colors, and textures, the compression issue of each component has been separately studied since the first discussion on the compression of shape models [1], [2], [3], [4], [6], [9], [10], [11], [12], [13], [14].

Even though there were researches on the compression of normal vectors [3], [12], the relative file size of normal vectors compared to that of topology and/or geometry data in shape models has not been sufficiently addressed. Note that data size of normal vectors may take up to a half of the whole shape model file. Based on this point of view, we have recently presented an approach to compress the normal vectors of shape models represented in a simple mesh based on the clustering and mixed indexing scheme [8].

In our previous papers, however, only the algorithmic aspect of the scheme was presented with some experimental results. In this paper, therefore, we will

present mathematical analysis of the probability distribution of index distances of the mixed indexing scheme to compress normal vectors. It turns out that the distribution of index distances can be immediately transformed into a number of quantitative measures for the proposed compression scheme.

2 Related Works

Unlike the previous approaches [3], [12], the clustering and mixed indexing scheme starts with an explicit clustering of the normal vectors so that the distribution of model normal vectors is considered. The normal vectors are grouped in a few clusters with a unique mean normal vector so that normal vectors in the cluster are represented by the mean normal vector [8].

Given the clusters of model normal vectors, the normal vector compression starts with an initial design of data structure. It is assumed that normal vectors are assigned at vertices of each face through indices, and the topology of shape model is compressed by Edgebreaker. After a series of rearrangements, the relationship between mesh and normal index is represented in an array as illustrated in Figs. 1(a) and (b). In Fig. 1(a), a triangular face f_1 is related with three normal vectors n_1 , n_3 , and n_4 at its three appropriate vertices, and this fact is reflected in the first three integers, 1, 3, and 4, respectively, in Normal Index array of Fig. 1(b).

It turned out that a particular index in Normal Index array tends to appear quite shortly after the index was used. In Fig. 1(c), it is shown that the currently recurring index is not very far from the latest occurrence of the same index value. Hence, the recurring indices were replaced with *normal index distance* D , the relative distance between the recurring index and the latest occurrence of the index in Normal Index array, so that it took fewer bits than using the absolute values of the indices. Note that the bit saving was obtained by employing the mixed use of absolute and relative values for indices, *mixed indexing scheme* [8].

Fig. 1. Configuration of mesh and normal vectors. (a) normal vectors in mesh model, (b) Normal Index array (c) normal index distances of recurring normal indices

Edgebreaker compresses the topology of a mesh model by classifying a triangle into one of five op-codes, C, R, E, S, and L, depending on the states of two triangles neighboring to current one and a vertex commonly shared by these

neighboring triangles. Hence, Edgebreaker transforms a mesh model into a string consisting of five alphabets [9]. Through experiments on several large models of up to 200,000 triangles, Rossignac reports the relative frequencies, or probabilities, of these five alphabets as $P(C)=0.5$, $P(R)=0.363$, $P(E)=0.056$, $P(S)=0.056$, and $P(L)=0.025$ [9]. Even though the probability distribution depends on models, the descending order of the above probabilities seems to be true for most large models.

Fig. 2 introduces a few notions. When a triangle X is entered from another triangle through an edge E_G , the edge is called a *gate* as named by Edgebreaker. When the vertices of X are referenced, the order of the vertices is in CCW direction as the following: the left vertex of gate v_1 , the right vertex of gate v_2 , and the other vertex v_3 . The edge connecting v_1 and v_3 is called a left edge E_L of X , and the edge connecting v_2 and v_3 is called a right edge E_R of X . The triangle over E_L is called a left face F_L , and the one over E_R is called a right face F_R .

Suppose that F_R is being parsed to be compressed after X was compressed. Then, we call F_R a current face CF and X a previous face PF, respectively. When a face is of type S, in Edgebreaker notion, we denote the face SF for short. When it is necessary to refer to the vertices, or the normal vectors at those vertices, of a face, they will be referred to in CCW order starting from the left vertex of a gate.

Fig. 2. Notions for topological elements

3 Analysis of Normal Index Array

In our previous papers [8], it was observed that approximately 80 % of the normal indices are recurring. Hence, it can be said that the Normal Index array may be better compressed if the recurring normal indices can be more efficiently encoded. If we have mathematical justification about the distribution of recurring normal indices, it may help to develop a more competitive compression scheme. Note that the bit saving by mixed indexing scheme depends on the distribution of normal index distances and therefore, the investigation on the distribution of D is necessary.

To analyze Normal Index array, we rearranged the configurations of triangles produced by Edgebreaker and assumed that each vertex has a unique normal

vector associated with. CF and PF may be connected through an edge or a vertex. In some cases, however, CF and PF may not be directly connected by sharing either an edge or a vertex. When CF and PF are connected through an edge, they are called *edge-connected*. Similarly, it is called *vertex-connected* if there is a vertex shared by both CF and PF. When CF and PF are separate, it is called *disconnected*. Shown in Fig. 3 illustrates examples for these cases: Fig. 3(a) is a case that CF and PF are edge-connected, and Fig. 3(b) illustrates a case that two faces are vertex-connected. In the example of Fig. 3(c), there are two intermediate faces separating CF from PF. For the simplification of analysis, it is assumed that each vertex has a unique normal vector associated with.

Fig. 3. Face configurations. (a) edge-connected cases, (b) a vertex-connected case, (c) a disconnected case

3.1 Edge-Connected Cases

Shown in Fig. 4 are edge-connected cases denoted as *Econ*. Example in Fig. 4(a) is a case when PF is of either type C, L, or S. In this case, CF is always F_R of PF by sharing E_R of PF by Edgebreaker rule and the corresponding Normal Index array is always given as shown in the right column of Fig. 4(a). By CCW order starting from the left vertex of a gate as discussed in Section 2, normal index order of PF in Fig. 4(a) is defined as 1, 2, and 3 and that of CF is defined as 3, 2, and 4. In these cases, i th index, the first index of CF, is always identical to $i-1$ th index, the last index of PF, and therefore D of i th index is 1. Similarly, $i+1$ th index, the second index of CF, is always identical to $i-2$ th index, the second index of PF, and D of $i+1$ index is always 3. On the other hand, $i+2$ th index, the last index of CF, does not have any index with identical value in PF. Therefore, $i+2$ th index has D of ∞ . The normal index with $D = \infty$ is the index with a normal index distance greater than 4 or not a recurring index. Note that normal index distances of 2 or 4 cannot occur at all.

When PF is of type R, CF has to be F_L of PF as shown in Fig. 4(b). Note that normal index order of PF is defined as 1, 2, and 3 (the same order in Fig. 4(a)) but that of CF is defined as 1, 3, and 4. In this case, the indices for CF can have two distinct normal index distances: 2 and 3. The i th index, the first index of CF, is always identical to $i-3$ th index, the first index of PF, and $i+1$ th index, the second index of CF, should be identical to $i-1$ th index, the last index of PF, in the array. Normal index distances of 1 and 4 cannot occur at all and D of $i+2$ th

index, the last index of CF, is ∞ . Two cases in the above are the only possible configurations that CF and PF share an edge.

Fig. 4. Face configuration and Normal Index array in edge-connected cases. (a) PF of op-codes C, S, and L, (b) PF of op-codes R

3.2 Vertex-Connected Cases

On the other hand, a case that CF shares a vertex with PF in common forms a vertex-connected case denoted as *Vcon*. If this case occurs as illustrated in Fig. 5, there should be a SF similarly to a disconnected case that will be elaborated shortly and SF shares its left edge with CF. In addition, SF, PF and CF shares a vertex in common. Since PF should be of type E in this case, Edgebreaker pops a face, which will be CF in the next moment, from a stack and the process jumps to CF.

Let α , β , and γ be three edges of PF as shown in the figure. Since a gate to PF can be either one of the three edges, there can be different path sets leading to α , β , and γ gates of PF and accordingly there are three different representation sets, different normal index order, of Normal Index array depending on the paths to the gates of PF. Note that normal index order of CF is independent of any gate to PF and is defined as 1, 3, and 6 as shown in Fig. 5.

Case α is a case that the edge α is used as the gate of PF and index order of PF is defined as 4, 5, and 3. Similarly, Case β , and Case γ can be defined. Even though there can be several paths from SF to PF in Case α , the resulting Normal Index array between CF and PF remains similar as shown in the figure. In this case, the second index of CF has always normal index distance of 2 while the other two indices have distances higher than 4. Note that the number of intermediate faces between SF and PF does not affect the relationship between CF and PF in Normal Index array. Similarly, in Case β , the second index of CF has a normal index distance 3 and the other indices have distances higher than 4. Case γ can also have several paths from SF to PF, and the middle index of CF has always normal index distance of 4 and the other two indices have distances higher than 4. As before, the number of intermediate faces does not affect the relationship between CF and PF in Normal Index array.

Fig. 5. Face configurations and Normal Index arrays of vertex-connected cases

3.3 Disconnected Cases

Fig. 6 shows a disconnected case denoted as *Dcon*. Note that there should be at least two faces in-between CF and PF and these two faces are edge-connected. Similarly to vertex sharing cases, this case occurs when PF was arrived after an SF, a face of type S. PF should be always of type E and CF is a face popped from a stack.

In this case, there has to be at least another intermediate face between PF and SF. Note that the intermediate face, in this example, can be L type. Of course, there can be several other configurations of faces that will result in disconnected cases. In the given example, the normal index distances of CF are always greater than 4.

Fig. 6. Face configurations and Normal Index arrays of disconnected cases

4 Probability Distributions

4.1 Probabilities of Face Configurations

The probability distribution of the above cases is of interest for the analysis of bit size of Normal Index array. It is assumed that the probability distribution of op-codes C, R, E, S, and L is known a priori, and denoted as $P(C)$, $P(R)$, $P(E)$, $P(S)$, and $P(L)$, respectively. In the following discussions of probabilities, random variables are occasionally ignored intentionally for the conciseness of the presentation.

Since the edge-connected cases happen when PF is of type either C, S, L, or R, the probability of edge-connected cases, $P(\text{Econ})$, is given by the following equation.

$$P(\text{Econ}) = P(C) + P(S) + P(L) + P(R) . \quad (1)$$

The exact probability of vertex-connected case, $P(\text{Vcon})$, cannot be known immediately. However, we make a few observations that can provide an approach to estimate the probability in a reasonable manner. Since PF has three possible gates, α , β , and γ , all of the paths from SF to PF can be grouped into three groups passing through each gate. Hence, $P(\text{Vcon})$ can be estimated if the probabilities of the gates $P(\alpha)$, $P(\beta)$, and $P(\gamma)$ are appropriately estimated. Therefore,

$$P(\text{Vcon}) = P(\alpha) + P(\beta) + P(\gamma) . \quad (2)$$

The exact probability of disconnected case cannot be known immediately as well. However, this case occurs only when there is a SF and PF is of type E. Since vertex-connected cases can also happen in this face configuration, the following equation can be deduced.

$$P(S) = P(E) = P(\text{Vcon}) + P(\text{Dcon}) . \quad (3)$$

Let a path π be a sequence of faces between two particular faces including both faces. Then, $P_\alpha(\pi)$ denotes the probability of path which enters through the gate α . If a path π_1 is a path with maximum probability among all possible paths leading to the gate α , we call $P_\alpha(\pi_1)$ the majorizing probability of the gate α . Therefore $P_\alpha(\pi_1)$ can be safely interpreted as a lower-bound of $P(\alpha)$. $P_\beta(\pi_1)$, $P_\gamma(\pi_1)$ and $P_{\text{Dcon}}(\pi_1)$ are defined similarly. From the above discussions, the following system can be deduced.

$$\begin{aligned} P(E) &= P(S) \\ P(E) &= P(\text{Vcon}) + P(\text{Dcon}) \\ P(\text{Vcon}) &= P(\alpha) + P(\beta) + P(\gamma) \\ P(C) + P(R) + P(E) + P(S) + P(L) &= 1 . \end{aligned} \quad (4)$$

In the above linear system, there are five unknowns: $P(\text{Vcon})$, $P(\text{Dcon})$, $P(\alpha)$, $P(\beta)$, and $P(\gamma)$. Note that $P(C)$, $P(R)$, $P(E)$, $P(S)$ and $P(L)$ are known. Since there is no way to calculate the exact probabilities of $P(\alpha)$, $P(\beta)$, $P(\gamma)$, $P(\text{Vcon})$ and $P(\text{Dcon})$, we have to estimate the probabilities of $P(\alpha)$, $P(\beta)$, $P(\gamma)$ and $P(\text{Dcon})$ using known probability distributions. Among several ways, we have chosen to allocate $P(E)$ to $P(\alpha)$, $P(\beta)$, $P(\gamma)$ and $P(\text{Dcon})$ proportionally to the ratios of the majorizing probabilities $P_\alpha(\pi_1)$, $P_\beta(\pi_1)$, $P_\gamma(\pi_1)$, and $P_{\text{Dcon}}(\pi_1)$. In other words,

$$\begin{aligned} P(\alpha) &= P(E) P_\alpha(\pi_1) / \{P_\alpha(\pi_1) + P_\beta(\pi_1) + P_\gamma(\pi_1) + P_{\text{Dcon}}(\pi_1)\} \\ P(\beta) &= P(E) P_\beta(\pi_1) / \{P_\alpha(\pi_1) + P_\beta(\pi_1) + P_\gamma(\pi_1) + P_{\text{Dcon}}(\pi_1)\} \\ P(\gamma) &= P(E) P_\gamma(\pi_1) / \{P_\alpha(\pi_1) + P_\beta(\pi_1) + P_\gamma(\pi_1) + P_{\text{Dcon}}(\pi_1)\} \\ P(\text{Dcon}) &= P(E) P_{\text{Dcon}}(\pi_1) / \{P_\alpha(\pi_1) + P_\beta(\pi_1) + P_\gamma(\pi_1) + P_{\text{Dcon}}(\pi_1)\} . \end{aligned} \quad (5)$$

Then, $P(\text{Vcon})$ can be also calculated by another simple arithmetic. Finding better approaches to allocate these probabilities is another issue to be pursued in the future.

4.2 Probabilities of Face Configurations

When the op-code for PF is C, S, or L, CF is F_R of PF and Normal Index array is as shown in Fig. 4. Note that this is an edge-connected case. In this case, $D_i = 1$, $D_{i+1} = 3$, and $D_{i+2} = \infty$. This fact was discussed earlier. Let $P(\text{CSL}) = P(\text{C}) + P(\text{S}) + P(\text{L})$. Then, $P(D = 1 \mid \text{PF} = \text{CSL}) = \frac{1}{3}$, $P(D = 3 \mid \text{PF} = \text{CSL}) = \frac{1}{3}$, $P(D = \infty \mid \text{PF} = \text{CSL}) = \frac{1}{3}$, and $P(D = 2 \mid \text{PF} = \text{CSL}) = P(D = 4 \mid \text{PF} = \text{CSL}) = 0$. Therefore, the following equation holds.

$$P(D = 1 \cap \text{PF} = \text{CSL}) = P(D = 1 \mid \text{PF} = \text{CSL})P(\text{CSL}) = \frac{1}{3}P(\text{CSL}). \quad (6)$$

Similarly to Eq. (6), it can be shown that $P(D = 3 \cap \text{PF} = \text{CSL}) = \frac{1}{3}P(\text{CSL})$, $P(D = \infty \cap \text{PF} = \text{CSL}) = \frac{1}{3}P(\text{CSL})$, $P(D = 2 \cap \text{PF} = \text{CSL}) = 0$, and $P(D = 4 \cap \text{PF} = \text{CSL}) = 0$.

When the op-code for PF is R, similar observation can be made as

$$P(D = 2 \cap \text{PF} = \text{R}) = P(D = 2 \mid \text{PF} = \text{R})P(\text{R}) = \frac{1}{3}P(\text{R}). \quad (7)$$

Similarly to Eq. (7), it can be also shown that $P(D = 3 \cap \text{PF} = \text{R}) = \frac{1}{3}P(\text{R})$, $P(D = \infty \cap \text{PF} = \text{R}) = \frac{1}{3}P(\text{R})$, $P(D = 1 \cap \text{PF} = \text{R}) = 0$, and $P(D = 4 \cap \text{PF} = \text{R}) = 0$.

In the vertex-connected case, op-code of PF should be always E and CF is the top face in the stack pushed into by the most recent face of type S. In this case, the second index of CF only has a relative index value smaller than five. In the case of gate α , it turns out that there can be only two values of normal index distances: $D = 2$ or $D = \infty$, since it is always guaranteed that $D_i = \infty$, $D_{i+1} = 2$, and $D_{i+2} = \infty$. If gate β , similar observation yields $D = 3$ or $D = \infty$ since $D_i = \infty$, $D_{i+1} = 3$, and $D_{i+2} = \infty$. If gate γ , $D_i = \infty$, $D_{i+1} = 4$, and $D_{i+2} = \infty$ and therefore it can be deduced that $D = 4$ or ∞ . Hence, the normal index distance value D which is not infinity always only depends on the gate to PF, and can have value of either 2, 3, 4 or ∞ . Besides, $P(\text{Vcon}) = P(\alpha) + P(\beta) + P(\gamma)$.

Let's now compute the probability of $D = 2$. Since $D = 2$ can only occur when the gate α is used to enter to PF of type E, it is necessary to know $P(\alpha)$. Therefore, $P(D = 2 \cap \text{Connectivity} = \text{Vcon}) = P(D = 2 \cap \text{Gate} = \alpha) = P(D = 2 \mid \text{Gate} = \alpha)P(\alpha) = \frac{1}{3}P(\alpha)$. Similarly, $P(D = 3 \cap \text{Connectivity} = \text{Vcon}) = \frac{1}{3}P(\beta)$, $P(D = 4 \cap \text{Connectivity} = \text{Vcon}) = \frac{1}{3}P(\gamma)$ and $P(D = 1 \cap \text{Connectivity} = \text{Vcon}) = 0$. In addition, it can be shown that $P(D = \infty \cap \text{Connectivity} = \text{Vcon}) = \frac{2}{3}P(\text{Vcon})$.

In the disconnected case, on the other hand, there is no relative index with distance less than 5. All three indices will have distances of ∞ . Table 1 summarizes the probability distribution of normal index distances, 1, 2, 3, 4, and ∞ for all possible face configurations.

Table 1. Probability distribution of normal index distances for Econ, Vcon, and Dcon

D	Econ		Vcon			Dcon	Sum
	CSL	R	E			E	
			α	β	γ		
1	$\frac{1}{2} P(\text{CSL})$	0	0	0	0	0	$\frac{1}{2} P(\text{CSL})$
2	0	$\frac{1}{2} P(\text{R})$	$\frac{1}{2} P(\alpha)$	0	0	0	$\frac{1}{2} P(\text{R}) + \frac{1}{2} P(\alpha)$
3	$\frac{1}{2} P(\text{CSL})$	$\frac{1}{2} P(\text{R})$	0	$\frac{1}{2} P(\beta)$	0	0	$\frac{1}{2} P(\text{CSL}) + \frac{1}{2} P(\text{R})$ $+ \frac{1}{2} P(\beta)$
4	0	0	0	0	$\frac{1}{2} P(\gamma)$	0	$\frac{1}{2} P(\gamma)$
∞	$\frac{1}{2} P(\text{CSL})$	$\frac{1}{2} P(\text{R})$	$\frac{2}{3} P(\alpha)$	$\frac{2}{3} P(\beta)$	$\frac{2}{3} P(\gamma)$	$P(\text{Dcon})$	$1 - \{\frac{2}{3} P(\text{CSL}) + \frac{2}{3} P(\text{R})$ $+ \frac{1}{3} P(\text{Vcon})\}$
Sum	$P(\text{C}) + P(\text{R}) + P(\text{S}) + P(\text{L})$		$P(\text{Vcon})$			$P(\text{Dcon})$	1

We have counted the actual frequencies of five op-codes from the cow model which has 14,508 recurring indices (83.32 %) among 17,412 normal indices and applied this probability distribution to the appropriate equations. Since $P(\text{C}) = 0.5$, $P(\text{R}) = 0.4288$, $P(\text{E}) = 0.0276$, $P(\text{S}) = 0.0274$, and $P(\text{L}) = 0.0162$, we have calculated $P(\alpha) = 0.0046$, $P(\beta) = 0.0010$, $P(\gamma) = 0.0216$, and $P(\text{Dcon}) = 0.0004$ from Eq. (5), and the probability distribution for normal index distance turned out as Table 2.

Table 2. Probability distribution of normal index distances for the cow model

D	CSL	R	E	Sum
1	0.1812	0	0	0.1812
2	0	0.1429	0.0015	0.1445
3	0.1812	0.1429	0.0003	0.3245
4	0	0	0.0072	0.0072
∞	0.1812	0.1429	0.0185	0.3426
Sum	0.5436	0.4288	0.0276	1

5 Conclusions

Since compression is one of the core technologies for a seamless transmission through the Internet, the compression of topology, geometry, etc. for 3D shape models have been extensively studied. Considering the fact that normal vectors may take almost a half of the whole model file, we have recently presented a new scheme to compress normal vectors of 3D mesh models using the clustering and the relative indexing algorithm. Since the performance analysis of the proposed

algorithm was not discussed in the earlier papers, we have presented in this paper a mathematical analysis of the index distances in Normal Index array. It can be easily shown that the information presented in this article can be transformed into a few measures which indicate the performance of the proposed compression scheme.

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