An Efficient Load Balancing Algorithm Employing a Symmetric Balanced Incomplete Block Design

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Abstract. In order to maintain load balancing in distributed system, should obtain workload information from all the nodes on network. It requires $O(v^2)$ communication overhead, where v is the number of nodes. In this paper, we present a new synchronous dynamic distributed load balancing algorithm on a $(v, k + 1, 1)$ -configured network applying symmetric balanced incomplete block design, where $v = k^2 + k + 1$. Our algorithm needs only $O(v\sqrt{v})$ message overhead and each node receives workload information from all the nodes without redundancy. Therefore, load balancing is maintained since every link has the same amount of traffic for transferring workload information.

1 Introduction

In a distributed system it is likely that some processors heavily loaded while some are lightly loaded or idle. It is desirable that workload be balanced between these processors so that utilization of processors can be increased and response time can be reduced. A load balancing scheme [1]–[2] determines whether a task should be executed locally or by a remote processor. This decision can be made in centralized or distributed manner. In a distributed system, distributed manner is recommended. In order to make this decision, each node can be informed about workload information of other nodes. Also this information should be the latest because outdated information may cause an inconsistent view of system state from which adverse effect of load balancing. So disseminating load information may incur high link cost or significant communication traffic overhead.

In order to decrease communication overhead for obtaining workload information, the message is exchanged between adjacent nodes and local load-balancing process is performed repeatedly. Then it makes the whole system be

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load-balancing [3]–[4]. It is accomplished by employing CWA (Cube Walking Algorithm) on hypercube network, which utilizes workload information of all the nodes. This method requires $O(v^2)$ communication complexity and a communication path is $O(\log_2 v)$. To reduce communication cost, flooding scheme is applied. However, the overlap of transmitting information occurs [5]–[6]. Based on SBN (Symmetric Broadcast Networks), communication patterns between nodes are constructed. It also needs $O(v^2)$ communication complexity for collecting workload information from all the nodes and a communication path is $O(\log_2 v)$ [7]–[8].

In this paper we present the network consisting of v nodes and $v \times k$ links, each node of which is linked to $2k$ nodes. On this network, each node sends workload information of k nodes to k adjacent nodes periodically. In other words, each node receives information from k nodes and sends information to k nodes. Also there is no redundancy in this received information. So, each node receives workload information for $k^2 + k$ nodes with two round message interchange. Therefore load balancing in this algorithm is maintained so that every link has same amount of traffic.

2 About (v, k, λ) -Configuration

Let $V = \{0, 1, \dots, v-1\}$ be a set of v elements. Let $B = \{B_0, B_1, \dots, B_{b-1}\}$ be a set of b blocks, where B_i is a subset of V and $|B_i| = k$. For a finite incidence structure $\sigma = \{V, B\}$, if σ satisfies following conditions, then it is a balanced incomplete block design (BIBD) [9], which is called a (b, v, r, k, λ) -configuration.

1. B is a collection of b k -subsets of V and this k -subsets are called the blocks.
2. Each element of V is related with exactly r of b blocks.
3. Every two objects of V appears simultaneously in exactly λ of b blocks.
4. $k < v$

For a (b, v, r, k, λ) -configuration, if it satisfies $k = r$ and $b = v$, then it is a symmetric balanced incomplete block design (SBIBD) [10] and it is called a (v, k, λ) -configuration. There are some relations among parameters b, v, r, k, λ that are necessary conditions for existence of a corresponding (b, v, r, k, λ) -configuration.

1. In a (b, v, r, k, λ) -configuration, $bk = vr$ and $r(k-1) = \lambda(v-1)$
2. For a (b, v, r, k, λ) -configuration, $QQ^T = (r-\lambda)I + \lambda J$, $I : v \times v$ identity matrix, $J : v \times v$ matrix in which all the entities are 1's.
3. For a (b, v, r, k, λ) -configuration, $b \geq v$.
4. For a (v, k, λ) -configuration, every two blocs have exactly λ objects in common.

As shown above, it is not true that there exists a BIBD or SBIBD for arbitrary set of parameters b, v, r, k and λ . However there is no known sufficient condition on the existence of a certain (b, v, r, k, λ) -configuration or (v, k, λ) -configuration.

3 Generation of a $(v, k + 1, 1)$ -Configuration

We now present an algorithm to generate an incidence structure $\sigma = \{V, B\}$ satisfying the condition for a $(v, k + 1, 1)$ -configuration in the case that k is a prime number. This $(v, k + 1, 1)$ -configuration is employed for constructing network topology below.

3.1 Design of an Algorithm to Construct $(v, k + 1, 1)$ -Configuration

Algorithm 1 for Generating an incidence structure.

Incidence structure $T = \{V, B\}$, where $V = \{0, 1, \dots, v - 1\}$,
 $B = \{B_0, B_1, \dots, B_{b-1}\}$,
 $|B_i| = k + 1$. $B_{i,j}$ is the j^{th} element of B_i

1. Select a prime number k and compute $v = k^2 + k + 1$.
2. Construct two incidence structures $X = \{V, C\}$ and $Y = \{V, D\}$.
 - a) $C_{i,j}$, $0 \leq i, j \leq k$, has a value as following .

$$C_{i,j} = \begin{cases} 0 & \text{if } j = 0 \\ t & \text{if } j \geq 1 \end{cases}$$
 where $t = i \times k + j$
 - b) $D_{i,j}$, $0 \leq i \leq (k^2 - 1)$, $0 \leq j \leq k$, has a value as following .

$$D_{i,j} = \begin{cases} C_{0,t}, t = \lfloor i/k \rfloor + 1 & \text{if } j = 0 \\ C_{j,t}, t = (i + (j - 1) \times \lfloor i/k \rfloor) \bmod k + 1 & \text{if } j \geq 1 \end{cases}$$
3. Generate $Z = \{V, B\}$ from X and Y .

$$B_i \leftarrow C_i$$

$$B_{i+k+1} \leftarrow D_i$$

The table below illustrates how to create $Z = \{V, B\}$, $V = \{0, 1, \dots, 6\}$. We now prove that this structure satisfies the conditions of a $(v, k + 1, 1)$ -configuration.

Table 1. A set of blocks on Z generated from Algorithm 1

X	Y	Z
$C_0 = \{0, 1, 2\}$	$D_0 = \{1, 3, 5\}$	$B_0 = \{0, 1, 2\}$
$C_1 = \{0, 3, 4\}$	$D_1 = \{1, 4, 6\}$	$B_1 = \{0, 3, 4\}$
$C_2 = \{0, 5, 6\}$	$D_2 = \{2, 3, 6\}$	$B_2 = \{0, 5, 6\}$
	$D_3 = \{2, 4, 5\}$	$B_3 = \{1, 3, 5\}$
		$B_4 = \{1, 4, 6\}$
		$B_5 = \{2, 3, 6\}$
		$B_6 = \{2, 4, 5\}$

Definition 1: On incidence structure Y , Sector S_i is the i^{th} family of k blocks, $D_j \in S_i, i = \lfloor j/k \rfloor$.

For example, If k equals 3, then $\lfloor 0/k \rfloor = \lfloor 1/k \rfloor = \lfloor 2/k \rfloor = 0$. So, $S_0 = \{D_0, D_1, D_2\}$. There are k sectors in Y .

Lemma 1: For two elements $D_{i1,j1}$ and $D_{i2,j2}$, $D_{i1,j1} \neq D_{i2,j2}$, if $j1 \neq j2$.

Proof: From Algorithm 1-2-(a), if $0 < j \leq k$, $0 \leq i \leq k$ then $C_{i,j} = i \times k + j$. This means if $j > 0$ then all the elements are distinct. And as shown in Algorithm 1-2-(b), an element of C_j is placed on j^{th} element of a certain block of Y if $D_{i,j} = C_{j,t}, t \neq 0$.

Lemma 2: For a sector consisting of k blocks, the first element of each block has the same value and the other k^2 elements are equal to $V - C_0$.

Proof: In the case that $D_{i,0} = C_{0,\lfloor i/k \rfloor + 1}$, the first element of k blocks on a sector have the same value. According to Algorithm 1-2-(b), $D_{i,j} = C_{j,t}, t = (i + (j - 1)\lfloor i/k \rfloor) \bmod k + 1$. Since k is a prime number, each element except the first element of each block is distinct and these distinct k^2 elements are equal to $V - C_0$.

Lemma 3: For incidence structure Y , $D_{a,j} = D_{b,j}, j \geq 1$, if

$$b = ((a - c(j - 1)) \bmod k + k(\lfloor a/k \rfloor + c)) \bmod k^2.$$

Proof: From Algorithm 1-2-(b), $D_{a,j} = C_{j,t}$. We now prove that $D_{b,j} = C_{j,t}$. t can be calculated from parameters b, j below. Then t obtained on this lemma is equal to that from Algorithm 1-2-(b). Therefore, $D_{a,j} = D_{b,j}$.

$$\begin{aligned} t &= (b + (j - 1) \times \lfloor b/k \rfloor) \bmod k + 1 \\ &= (((a - c(j - 1)) \bmod k + k(\lfloor a/k \rfloor + c)) + (j - 1)((a - c(j - 1)) \bmod k + k(\lfloor a/k \rfloor + c))/k) \bmod k + 1 \\ &= (((a - c(j - 1)) + (j - 1) \times (\lfloor a/k \rfloor + c)) \bmod k + 1 \\ &= (a + (j - 1)\lfloor a/k \rfloor) \bmod k + 1 \end{aligned}$$

Here, if $D_{a,j}$ is in sector S_s then $D_{b,j}$ is in $S_{(s+c) \bmod k}$. In case of $c \equiv 0 \pmod k$, then $a = b$.

Lemma 4: Each element of V appears in exactly $k + 1$ times in Z .

Proof: According to Algorithm 1-2-(a), $C_{i,0} = 0$. Since $0 \leq i \leq k$, 0 appears $k + 1$ times. The other $v - 1$ elements, $V - \{0\}$, appear exactly once on X . From Lemma 3, each element of $C_{0,j}, 1 \leq j \leq k$, appears k times in a sector of Y and the rest k^2 elements appear once in every sector of Y . Therefore, each element appears $k + 1$ times in Z .

Lemma 5: Any pair of elements of V appears in exactly only once in Z .

Proof: The first element of V makes a pair with all the other elements and this pair appears once by designing rule of incidence structure (see Algorithm 1-2-(a)). Each element of $C_{0,j}, 1 \leq j \leq k$ makes a pair with $V - C_0$ elements and it also appears once proven by Lemma 3. The rest k^2 elements are now considered. For an arbitrary pair $D_{a,j1} = D_{a,j2}, j1, j2 \geq 1$, in order to

make the same pair on other block D_b , the two elements should be on the same block. According to Lemma 4, if $j1 = j2$, then they are located on D_b . However, this case does not occur since $j1 \neq j2$. Therefore, any pair of elements of V appears in exactly only one time in Z .

Theorem 1: Z designed by Algorithm 1 satisfies the conditions of a $(v, k+1, 1)$ -configuration.

Proof: Z satisfied the conditions of the SBIBD by emplying Lemma 4 and Lemma 5.

3.2 Design of Network Configuration

In order to construct a network topology which have minimum link cost and traffic overhead, we imported $(v, k+1, 1)$ -configuration. An incidence structure $Z = \{V, B\}$ satifies the conditions for a $(v, k+1, 1)$ -configuration and M be a binary incidence matrix of Z . Then this matrix M can be transformed to an adjacent matrix of a graph $G = \{V, E\}$. Based on this idea, network topology can be designed as follows.

Algorithm 2 for Design of Network Configuration.

1. Create an incidence structure $Z = \{V, B\}$ by Algoritm 1.
2. Generate $L = \{V, E\}$ from Z by exchanging blocks so that every block i includes object i .

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 $E_0 \leftarrow B_0$ 
for (  $i = 1$  ;  $i < v$  ;  $i = i + 1$  ) {
  if (  $i \leq k$  ) {  $j \leftarrow i \times k + 1$  ;  $t \leftarrow B_{j,i}$  }
  else if (  $i \% k = 1$  ) {  $t \leftarrow B_{i,0}$  }
  else {  $j \leftarrow \lceil i/k \rceil - 1$  ;  $t \leftarrow B_{i,j}$  }
   $E_t \leftarrow B_i$ 
}

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3. Create an adjacent matrix $A = (a_{ij})$ for graph G from L , where G is a network topology containing v processors.

$$a_{ij} = \begin{bmatrix} 1 & \text{if } i \neq j, \text{ and if } i \in E_j \text{ or } j \in E_i \\ 0 & \text{otherwise} \end{bmatrix}$$

G has v nodes since G is created from $(v, k+1, 1)$ -configuration. Each block $L[i]$ is composed of $k+1$ elements and i is the one of them. Each node obtains $2k$ links from Step 3 of Algorithm 2. So, G become a $2k$ -regular graph. therefore there are $(2k \times v)/2 = vk$ links in G . Given $Z = \{V, B\}$ described on Fig.1, performance of Algorithm 2 is shown on Table 2 and Fig.1.

Table 2. Blocks of L generated from Z of Table 1

L
$E_0 = \{0, 1, 2\}$
$E_1 = \{1, 3, 5\}$
$E_2 = \{2, 3, 6\}$
$E_3 = \{0, 3, 4\}$
$E_4 = \{1, 4, 6\}$
$E_5 = \{2, 4, 5\}$
$E_6 = \{0, 5, 6\}$

Fig. 1. (7,3,1)-configured network obtained from L

4 Design of an Efficient Load Balancing Algorithm on (v,k+1,1)-Configured Networks

An efficient load balancing algorithm is now constructed on (v,k+1,1)-configured networks generated by Algorithm 2.

Definition 2 : Construct two sets S_i and R_i consisting of adjacent k nodes, where S_i is a set of nodes to which node i sends workload information and R_i is a set of nodes to receive i 's workload information.

$$S_i = \{v \mid v \in E_i - i\}$$

$$R_i = \{v \mid i \in E_v \text{ and } i \neq v\}$$

Definition 3 : Generate two sets SF_i and RF_i , where $SF_i(j)$ is a set of workload information for i 's adjacent nodes transmitted from node i to node j at time T_{2t} and $RF_i(j)$ is i 's workload information transmitted from node i to node j at time T_{2t+1} .

$$SF_i = \{SF_i(j) \mid j \in S_i, SF_i(j) = \{E_i - \{j\}\}\}.$$

$$RF_i = \{RF_i(j) \mid j \in R_i, RF_i(j) = i\}.$$

Algorithm 3 for Construction of an Efficient Load Balancing Algorithm.

1. Node i sends a set of workload information $SF_i(j)$ to node $j \in S_i$ at T_{2t} and renews a table of workload information.

2. Node i sends a set of workload information $RF_i(j)$ to node $j \in R_i$ at T_{2t+1} and renews a table of workload information.
3. Repeat the first step.

The following table indicates that node i sends workload information $SF_i(j)$ and $RF_i(j)$ to node j at times T_{2t} and T_{2t+1} , respectively. So every node can obtain workload information for all the nodes at T_{2t+2} and this fact is proven in Theorem 2.

Table 3. Two steps for sending workload information from Node i

Node ID	T_{2t}	T_{2t+1}
0	$SF_0(1) = \{ 0, 2 \}$ $SF_0(2) = \{ 0, 1 \}$	$RF_0(3) = \{ 0 \}$ $RF_0(6) = \{ 0 \}$
1	$SF_1(3) = \{ 1, 5 \}$ $SF_1(5) = \{ 1, 3 \}$	$RF_1(0) = \{ 1 \}$ $RF_1(4) = \{ 1 \}$
2	$SF_2(3) = \{ 2, 6 \}$ $SF_2(6) = \{ 2, 3 \}$	$RF_2(0) = \{ 2 \}$ $RF_2(5) = \{ 2 \}$
3	$SF_3(0) = \{ 3, 4 \}$ $SF_3(4) = \{ 3, 0 \}$	$RF_3(1) = \{ 3 \}$ $RF_3(2) = \{ 3 \}$
4	$SF_4(1) = \{ 4, 6 \}$ $SF_4(6) = \{ 4, 1 \}$	$RF_4(3) = \{ 4 \}$ $RF_4(5) = \{ 4 \}$
5	$SF_5(2) = \{ 5, 4 \}$ $SF_5(4) = \{ 5, 2 \}$	$RF_5(1) = \{ 5 \}$ $RF_5(6) = \{ 5 \}$
6	$SF_6(0) = \{ 6, 5 \}$ $SF_6(5) = \{ 6, 0 \}$	$RF_6(2) = \{ 6 \}$ $RF_6(4) = \{ 6 \}$

Theorem 2: According to Algorithm 3, every node obtains workload information for all the nodes at T_{2t+2} .

Proof: At T_{2t} , node i sends workload information for $SF_i(j)$ to node j . On an arbitrary pair $(SF_{i_1}(j), SF_{i_2}(j))$, $i_1 \neq i_2$, intersection of these sets is empty since on $(v, k+1, 1)$ -configuration, every two objects appears simultaneously in exactly one of v blocks and node j is an element of S_{i_1} and S_{i_2} , respectively. So node j obtains workload information for k^2 nodes. And at T_{2t+1} , node i transmits its workload information to node j by Algorithm 3-2. Then, node j receives k workload information. Therefore, node j receives workload information for $k^2 + k$ nodes at T_{2t+2} .

5 Conclusion

In order for the system to increase utilization and to reduce response time, workload should be balanced. In this paper, we present an efficient load balancing algorithm on $(v, k+1, 1)$ -configured networks consisting of v nodes and vk links. Our algorithm needs only $O(v\sqrt{v})$ message overhead and each node receives workload information from all the nodes without redundancy and load balancing is maintained so that every link has same amount of traffic for transferring workload information.

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