Formal Pipeline Design

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Abstract. The action systems formalism has recently been applied to the area of asynchronous and synchronous VLSI design. In this paper, we study formal aspects of synchronous pipelining. We show how the framework of synchronous action systems can be used to derive a pipelined structure from a non-pipelined specification in a correctness-preserving manner.

1 Introduction

Pipelining is a very common technique used in digital systems design in order to increase parallelism. It has an important impact on the rate at which data is produced and consumed, a pipelined version allowing a higher speed than the non-pipelined version of the same circuit [7].

Formal methods of concurrent programming become increasingly important in design of complex VLSI systems. In our earlier work, we have shown how the *action systems formalism* [1] can be applied in design of both self-timed [8] and synchronous circuits [9,10]. In this paper, we continue our work on synchronous modeling by focusing on the formal aspects of clocked pipelining. The actions composing a synchronous action system are high-level representations of circuit functionality. One single action may actually map on a very complex hardware implementation, in terms of the time required to perform. Therefore, the device should operate on a large clock period, situation which, often, does not represent a solution. Thus, a further splitting of the execution of the initial action becomes necessary, reflected further in a pipelined realization of the digital device. Here, we present a stepwise procedure by which a non-pipelined synchronous action system specification is transformed into a pipelined form, in a correctness-preserving manner.

2 Synchronous Action Systems

The action systems formalism is based on an extended version of Dijkstra's language of *guarded commands* [5]. A *synchronous action system* [9] $\mathcal{A}$ has the form $\text{sys } \mathcal{A}(g) :: [\text{var } l; \text{init } g, l := g_0, l_0; \text{do } A_1 \nabla A_2 \text{ od }]$, where g and l are lists of global and local variables initialised to g_0 and l_0 , respectively, and $A_1 \nabla A_2$ is the *synchronous composition of the actions* A_1 and A_2 which access the variables g and l .

Any of the actions of the synchronous system $\mathcal{A}$ is defined as a non-deterministic assignment, $x := x'.Q$. Here, Q is a boolean relation which determines the value(s) x' assigned to the (list of) variable(s) x . For example: $Q \hat{=} x' = x + y$, where y is another variable. The synchronous composition operator ' ∇ ' between the actions A_i in the above system $\mathcal{A}$ indicates that the write variables of the involved actions are updated simultaneously in an atomic manner. At a lower abstraction level, a synchronous composition consists of a *read phase* and a *write phase* which are executed sequentially one after another.

The hardware representation of a synchronous action is represented by a set of registers (D flip-flops), each associated with one of the write variables of the action, and the corresponding combinational logic (specified, under all input situations, by the predicate Q).

Two synchronous action systems $\mathcal{A}_1$ and $\mathcal{A}_2$ can be composed using the operator ' ∇ '. The composition $\mathcal{A}_1 \nabla \mathcal{A}_2$ is defined to be a synchronous action system which is composed of the actions of the constituent systems. This system merges the global variables of the components $\mathcal{A}_i$ keeping the local variables distinct.

Actions and action systems are intended to be developed in a stepwise manner within the *refinement calculus* [2]. A *correct* transformation of a given action A into the action A' is denoted as $A \leq A'$. Refinement of synchronous action systems is mainly based on the theory of *trace refinement* [3]. A *trace*, or a sequence of values of the global variables, represents an observable behaviour of a system. An abstract action system $\mathcal{A}$ is said to be (*trace*) *refined* by the concrete system $\mathcal{C}$, denoted $\mathcal{A} \sqsubseteq \mathcal{C}$, if the traces of $\mathcal{A}$ and $\mathcal{C}$ are equivalent. Notice that, in a trace, several successive equivalent states are considered as a single one.

Fig. 1. The four phase handshake

The design of synchronous action systems [10] is based on correctly deriving synchronous representations from asynchronous action systems. An asynchronous system interacts with its environment via an asynchronous *communication (handshake) channel*, which is composed of two boolean variables [6], denoted *req* and *ack* (*request*, *acknowledge*), and *data variables*. The handshake variable *req* is updated by the *master* system which is the active party of communication requesting tasks from the *server* system which is the passive party of communication. The variable *ack* is updated by the server whenever it has completed a task requested by the master. In our approach, the synchronous system is the server component of an asynchronous description and a system

that models the environment acts as the master. We build our design flow on the four-phase signalling protocol [4] (Fig. 1).

3 Pipelined Synchronous Action Systems

In order to obtain a pipelined version of an initially non-pipelined system, we have to introduce / eliminate local variables of the specific system. We do this according to the methods presented in [11], accommodating for our purposes the rules presented in [2]. We continue with the introduction of the pipelining procedure. For performing the translation from an asynchronous to a synchronous representation, the system under analysis has to comply with a set of *synchronization requirements* [10]. Thus, one of the requirements specifies that it is possible to transform a sequence (for instance $A_1; \dots; A_n$) into a simultaneous assignment ($A_1 \star \dots \star A_n$), and from here, into a synchronous composition, if actions in the sequence do not read what other actions, positioned earlier in the sequence, have just updated. The process preserves the existence of the communication variables *req* and *ack* in the synchronous description.

Pipelining Procedure. Let us consider now a synchronous action system, $\mathcal{A}$, obtained from an initial asynchronous system. Thus, $\mathcal{A}$ contains the action *ACK* that updates the acknowledge signal *ack* :

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sys  $\mathcal{A}(req, ack : bool; x, y, a, b : data) ::$ 
 $[[$  var  $l;$  init  $l, x, \dots, b := l_0, x_0, \dots, b_0; req, ack := false$ 
 $\mathbf{do} \ A \ \nabla \ B \ \nabla \ \mathbf{ACK} \ \mathbf{od}$ 
 $]],$ 

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(1)

$$\begin{aligned}
A &\hat{=} a := a'.Q_A, & Q_A &\hat{=} (req \Rightarrow Q_a) \wedge (\neg req \Rightarrow a' = a), \\
B &\hat{=} b := b'.Q_B, & Q_B &\hat{=} (req \Rightarrow Q_b) \wedge (\neg req \Rightarrow b' = b), \\
ACK &\hat{=} ack := ack'.Q_K, & Q_K &\hat{=} (req \Rightarrow ack' = true) \wedge (\neg req \Rightarrow ack' = false)
\end{aligned}$$

Let us now assume that the direct circuit implementation of the actions A and B of the system $\mathcal{A}$ is estimated to be so complex that it cannot be operated with the specified clock frequency. Then the computations of A and B have to be divided into n ($n > 1$) phases each of which is simple enough to be executed in one clock cycle. A set of $n - 1$ new local variables is introduced for every action we split, in order to store the results of each phase. The combinational logic that updates the variable *ack* is a very simple construct. Consequently, the update is possible to be completed in a single clock step. Therefore, the update on the variable *ack* is only delayed but not split over n stages.

Pre-analysis Step. Intuitively, the first step in dividing the action A into n pipelined actions is to transform it locally into the atomic n -element sequence:

$$\begin{aligned}
A &\leq A', A' \hat{=} A_0; \dots; A_{n-1}, \\
A_j &\hat{=} a_j := a'_j.Q_a^j, j = 0, \dots, n-1, a_{n-1} = a
\end{aligned}$$

The above refinement requires that

$$(Q_a^0 \wedge Q_a^1[a'_0/a_0] \wedge \dots \wedge Q_a^{n-1}[a'_{n-2}/a_{n-2}]) \equiv Q_A$$

As each action of the sequence A' reads the values of the variables updated by the previous action in the sequence, it is not possible to obtain a synchronous representation of the action A' (for instance $A_0 \nabla \dots \nabla A_{n-1}$). However, this study is necessary as it indicates what the relations Q_a^j , $j = 0 \dots n-1$ are (in other words, it shows *how* to split the execution of action A). They will be used in the following steps, when we describe the transformation of the system $\mathcal{A}$ into an equivalent pipelined system. Observe that a similar procedure is to be applied to action B , too.

Step 1. Buffering. We start the transformation of the action system (1) by introducing $3 \cdot (n-1)$ variables. Consequently, we also have $3 \cdot (n-1)$ new actions. Initially we only modify the original actions so that they have as input the new local variables, instead of the initial global variables:

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sys  $\mathcal{A}^1$  ( $req, ack : bool; x, y, a, b : data$ ) ::
|| [ var  $l, a_0, \dots, a_{n-2}, b_0, \dots, b_{n-2}, req_0, \dots, req_{n-2}$ ;
init  $l, x, \dots, b := l_0, x_0, \dots, b_0; req, ack, req_0, \dots, req_{n-2} := false$ ;
 $a_0, b_0, \dots, a_{n-2}, b_{n-2} := a_{0_0}, b_{0_0}, \dots, a_{n-2_0}, b_{n-2_0}$ ;
do  $A' \nabla A_0 \dots \nabla A_{n-2} \nabla B' \nabla B_0 \dots \nabla B_{n-2}$ 
 $\nabla ACK' \nabla ACK_0 \dots \nabla ACK_{n-2}$  od
||,

 $A' \hat{=} A[a_{n-2}, req_{n-2}/x, req], \quad B' \hat{=} B[b_{n-2}, req_{n-2}/y, req],$ 
 $A_0 \hat{=} a_0 := x, \quad B_0 \hat{=} b_0 := y,$ 
 $ACK' \hat{=} ACK[req_{n-2}/req], \quad ACK_0 \hat{=} req_0 := req,$ 

 $S_j \hat{=} s_j := s_{j-1}, s \in \{a, b, req\}, S \in \{A, B, ACK\}, j = 1 \dots n-2$ 

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The hardware implementation of the system $\mathcal{A}^1$ is shown in Fig. 2.

Fig. 2. Intermediate stage in the pipelining process

The result of **Step 1** is the refinement $\mathcal{A} \sqsubseteq \mathcal{A}^1$. Observe that until $req_{n-2} = true$, the system $\mathcal{A}^1$ does not modify its observable state (it stutters). Also, after req becomes *false*, until $req_{n-2} = false$, $\mathcal{A}^1$ stutters again.

Step 2. Auxiliary Actions. Next, we introduce $2 \cdot n$ additional actions $D_0, \dots, D_{n-1}$ and $C_0, \dots, C_{n-1}$, that synchronously update the new local variables $d_0, \dots, d_{n-1}$ and $c_0, \dots, c_{n-1}$, respectively. We have

$$\begin{aligned}
 D_0 &\hat{=} d_0 := d'_0.Q_D^0, & Q_D^0 &\hat{=} Q_a^0[d'_0/a'_0], \\
 C_0 &\hat{=} c_0 := c'_0.Q_C^0, & Q_C^0 &\hat{=} Q_b^0[c'_0/b'_0]
 \end{aligned}$$

For $j = 0 \dots n-1$, the actions D_j and C_j update the corresponding output variables as specified by the predicates selected in the pre-processing step:

$$\begin{aligned} D_j &\hat{=} d_j := d'_j, Q_D^j, Q_D^j \hat{=} (req_{j-1} \Rightarrow Q_a^j[d'_j/a'_j]) \wedge (\neg req_{j-1} \Rightarrow d'_j = d_j), \\ C_j &\hat{=} c_j := c'_j, Q_C^j, Q_C^j \hat{=} (req_{j-1} \Rightarrow Q_b^j[c'_j/b'_j]) \wedge (\neg req_{j-1} \Rightarrow c'_j = c_j) \end{aligned}$$

At the system level we have $\mathbf{sys} \mathcal{A}^1 \sqsubseteq \mathbf{sys} \mathcal{A}^2$, where $\mathcal{A}^2$ is described as

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sys  $\mathcal{A}^2$  ( $req, ack : bool; x, y, a, b : data$ ) ::
[[ var  $l, a_0, b_0, req_0, d_0, c_0 \dots, a_{n-2}, b_{n-2}, req_{n-2}, d_{n-1}, c_{n-1}$ ;
init  $l, x, \dots, b := l_0, x_0, \dots, b_0; req, ack, req_0, \dots, req_{n-2} := false$ ;
 $a_0, b_0 \dots, a_{n-2}, b_{n-2} := a_{0_0}, b_{0_0} \dots, a_{n-2_0}, b_{n-2_0}$ ;
 $d_0, c_0 \dots, d_{n-2}, c_{n-2} := d_{0_0}, c_{0_0} \dots, d_{n-2_0}, c_{n-2_0}$ ;
do  $A' \nabla A_0 \dots \nabla A_{n-2} \nabla B' \nabla B_0 \dots \nabla B_{n-2}$ 
 $\nabla ACK' \nabla ACK_0 \dots \nabla ACK_{n-2}$ 
 $\nabla C_0 \dots \nabla C_{n-1} \nabla D_0 \dots \nabla D_{n-1}$  od
]]
```

Step 3. Removal of Auxiliary Variables. The Final Description. Next, notice that

$$\begin{aligned} req \wedge req_{n-2} &\Rightarrow (Q_D^{n-1}[a/d_{n-1}] \equiv Q_A[a_{n-2}, req_{n-2}/x, req]) \\ &\wedge (Q_C^{n-1}[b/b_{n-1}] \equiv Q_B[b_{n-2}, req_{n-2}/y, req]) \end{aligned}$$

The interpretation of the above relation is as follows. The update on variable a , either according to Q_A , or to Q_D^{n-1} , leads to the same result. Therefore, we can write

$$\begin{aligned} A' &\leq A'', A'' \hat{=} D_{n-1}[a/d_{n-1}], \\ B' &\leq B'', B'' \hat{=} C_{n-1}[b/c_{n-1}] \end{aligned}$$

Now we can safely eliminate the variables a_{n-2}, b_{n-2} from the system description, together with the actions that update them. We repeat this procedure for the variables a_j, b_j ($j = n-3 \dots 0$) and for the corresponding actions. Eventually we come to the system ($\mathcal{A}^2 \sqsubseteq \mathcal{A}^3$):

```

sys  $\mathcal{A}^3$  ( $req, ack : bool; x, y, a, b : data$ ) ::
[[ var  $l, req_0, d_0, c_0 \dots, req_{n-2}, d_{n-1}, c_{n-1}$ ;
init  $l, x, \dots, b := l_0, x_0, \dots, b_0; req, ack, req_0, \dots, req_{n-2} := false$ ;
 $d_0, c_0 \dots, d_{n-2}, c_{n-2} := d_{0_0}, c_{0_0} \dots, d_{n-2_0}, c_{n-2_0}$ ;
do  $A'' \nabla B'' \nabla D_0 \dots \nabla D_{n-2} \nabla C_0 \dots \nabla C_{n-2}$ 
 $\nabla ACK' \nabla ACK_0 \dots \nabla ACK_{n-2}$  od
]]
```

The implementation of the system $\mathcal{A}^3$ is shown in Fig. 3.

4 Conclusions

We introduced an action systems-based method to transform a non-pipelined synchronous system description into a pipelined structure within a correctness preserving formal framework. The pipelining procedure was viewed as a four-step refinement which started from atomic sequencing of an action and was completed by splitting the action in question into separate synchronous components each

Fig. 3. The system $\mathcal{A}^3$ in hardware

of which constituted a stage of the created pipeline. The work presented in this paper was motivated by importance and generality of pipelining in contemporary VLSI design and by the view that the need for formal design methods is becoming more and more obvious with increasing complexity of digital VLSI systems.

Several issues are still to be addressed in further work studies, such as the elimination of the communication variables and its implication in pipelined-system modelling, as well as the subject of pipeline control.

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