AN ALGEBRAIC STRUCTURE OF PETRI NETS

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Abstract.

The paper concerns algebraic properties of Petri nets. A wide class of nets, called simple nets, is introduced and a lattice of these nets is defined. It turns out that nets representing sequential systems and processes are atoms of this lattice, and this fact provides the natural way of building nets representing concurrent systems as the superposition of nets representing sequential system components.

The notion of concurrency relation for large class of nets including cyclic nets is precisely defined.

An influence of static, i.e. unmarked, structure of nets on the class of "proper" markings is discussed. The notion of natural markings, i.e. markings defined by the static (unmarked) structure of nets is introduced.

Properties of safeness, compactness, fireability and K-density of marked nets are discussed. A classification of nets is proposed and an attempt of the algebraic definition of net with properties required from "well defined" dynamic concurrent system is given.

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1. Introduction.

Petri nets theory constitutes an axiomatic approach towards the phenomena of concurrent systems and processes (see Petri(1977), Petri(1978), Mazurkiewicz(1977) and others). Properties of Petri nets were tryed to prove by means of different methods with different results.

One of the basic research methods of science is the partition into indivisible components (atoms). Then we can describe properties of the whole structure by means of the properties of components. Our approach is the following: we construct a special class of Petri nets (called simple nets), rules of decomposition into subnets, and we describe a class of indivisible nets - called atoms. We also define the operation which makes possible a construction of the more complicated structures from atoms.

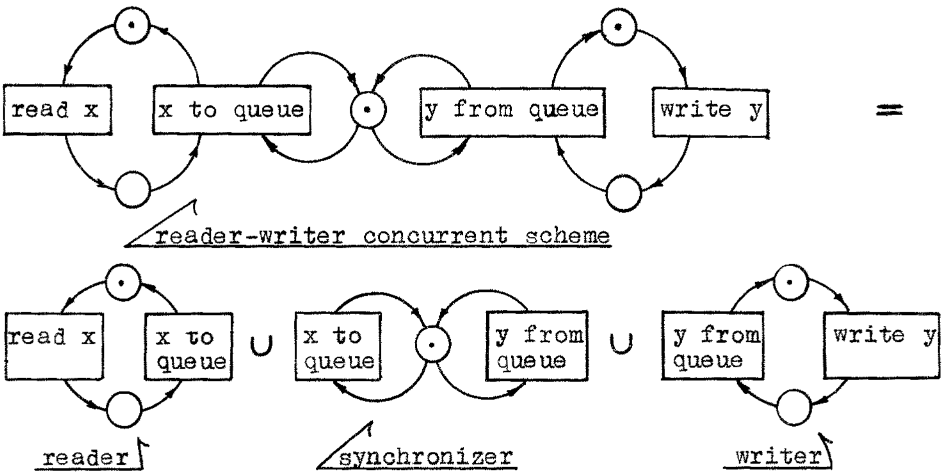
It turns out that nets representing sequential systems and processes, called elementary in the paper, are atoms, and that nets created by elementary nets (called proper in the paper) have much properties required from "well defined" concurrent systems (safeness, fireability and so on).

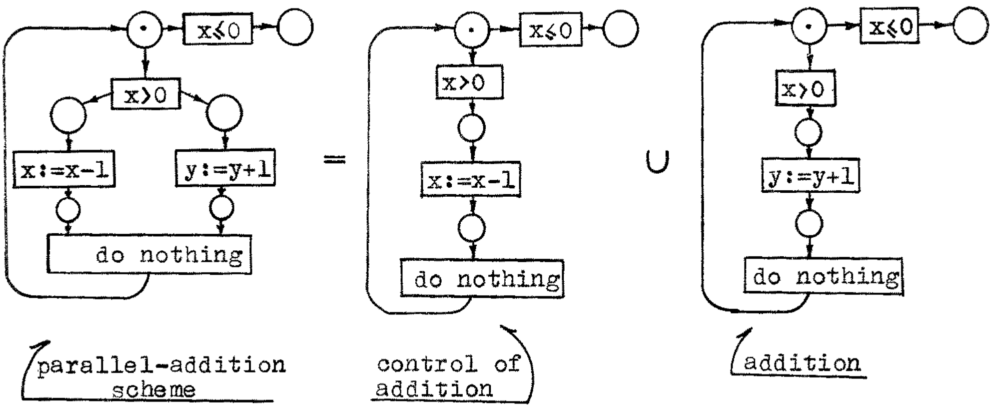
In the paper a lattice of unmarked nets is defined and different notions of concurrency relation (see Petri(1977), Petri(1978), Janicki(1979)) for large class of nets including cyclic nets are introduced.

An influence of static, unmarked structure of nets on the class of "proper" markings, and properties of natural markings, i.e. markings defined by the static net structure, are discussed.

The motivation of the approach we have presented is the observation that all Petri nets representing real, well defined concurrent systems or processes can be treated as the superposition of sequential components.

Consider two very simple examples (compare Mazurkiewicz(1977)), namely, a reader-writer concurrent scheme and a parallel-addition scheme. Note that the reader-writer scheme is the superposition of three sequential components: the reader, the synchronizer and the writer; and the parallel-addition scheme consists of two sequential schemes: the control of addition and the addition.





The paper is an attempt to prove some fundamental properties of net superposition, net decomposition, and nets created by these operations.

In the paper we shall use the standard mathematical notation ($|X|$ denote the cardinality of X, $\emptyset$ denote the empty set, and so on)

2. Simple nets.

In this section we recall from Janicki (1978) the basic notion of this paper, namely the notion of simple net. This definition and the notation used in this section are base for the further considerations.

For every set X, let $left: X \times X \mapsto X$, $right: X \times X \mapsto X$ be the following functions:

$$(\forall (x,y) \in X \times X) \quad left((x,y))=x, \ right((x,y))=y.$$

By a <u>simple net</u> (abbr. <u>s-net</u>) we mean any pair

$$N = (T,P),$$

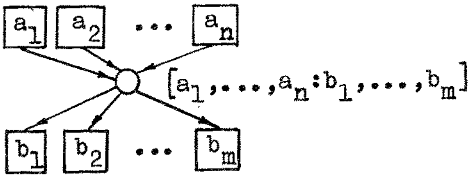
where: T is a set (of <u>transitions</u>),

$P \subseteq 2^T \times 2^T$ is a relation (also interpreted as a set of <u>places</u>),

$(\forall a \in T)(\exists p,q \in P) \quad a \in left(p) \cap right(q)$,

$P = \emptyset \iff T = \emptyset$.

In the paper we restrict our attention to finite s-nets. Instead of $(\{a_1,\ldots,a_n\},\{b_1,\ldots,b_m\}) \in P$ we shall write $[a_1,\ldots,a_n : b_1,\ldots,b_m] \in P$. Every s-net $N=(T,P)$ can be graphically represented using the graph



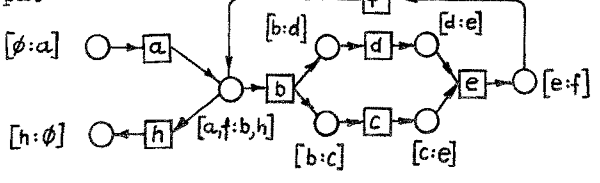
to denote the fact that $[a_1,\ldots,a_n : b_1,\ldots,b_m] \in P$.

<u>Example 2.1.</u>

Let $N=(T,P)$, where $T=\{a,b,c,d,e,f,h\}$,

$P = \{[\emptyset:a],[a,f:b,h],[h:\emptyset],[c:e],[b:c],[b:d],[d:e],[e:f]\}$.

The pair $N=(T,P)$ is a s-net and it can be represented by the following graph.



□

In the literature nets are usually defined differently, starting with two disjoint sets transitions and places, and introducing a flow-relation between them (compare Petri(1977), Petri(1978)). This approach is luckier in the sense that it makes more easy to handle operation among nets.

3. A lattice of simple nets.

In this section we shall describe the algebraic structure of simple nets.

Let SNETS denote the family of all s-nets.

Let $\sqsubseteq$ be the relation in SNETS defined as follows:

$$N_1=(T_1,P_1) \sqsubseteq N_2=(T_2,P_2) \iff P_1 \subseteq P_2.$$

Note that $\sqsubseteq$ is a partial order relation and $N_1 \sqsubseteq N_2$ implies $T_1 \subseteq T_2$. Let $\sup\{N_1,N_2\}$, $\inf\{N_1,N_2\}$ denote respectively the least upper bound and the greatest lower bound with respect to the relation $\sqsubseteq$.

Theorem 3.1.

For every $N_1=(T_1,P_1)$, $N_2=(T_2,P_2) \in$ SNETS :

$$\sup\{N_1,N_2\} = (T_1 \cup T_2, P_1 \cup P_2),$$

$$\inf\{N_1,N_2\} = (\mathrm{left}(P),P),$$ where P is the greatest set fulfilling the condition $P \subseteq P_1 \cap P_2$ & $\mathrm{left}(P)=\mathrm{right}(P)$. ■

Define the following operations:

$$N_1 \cup N_2 = \sup\{N_1,N_2\} \;, \quad N_1 \cap N_2 = \inf\{N_1,N_2\} \;,$$

$$\bigcup_{N \in S} N = \sup\{N \mid N \in S\} \;, \quad \bigcap_{N \in S} N = \inf\{N \mid N \in S\} \;.$$

Theorem 3.2.

The algebra (SNETS,$\cup$,$\cap$) is a lattice with the greatest lower bound $(\emptyset,\emptyset)$. ■

It turns out that the lattice (SNETS,$\cup$,$\cap$) is not distributive. Now we introduce the atomic structure of simple nets.

A simple net N=(T,P) is said to be an atom iff:

1. $N \neq (\emptyset,\emptyset)$,
2. $(N' \sqsubseteq N) \implies (N' = N$ or $N' = (\emptyset,\emptyset))$.

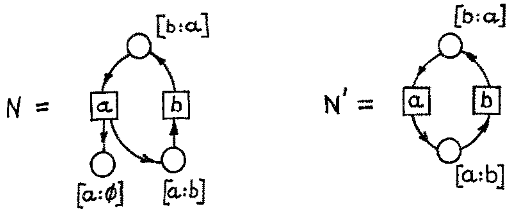
For every s-net, let <u>atoms(N)</u> denote the set of all atoms contained in N, i.e.

$$\text{atoms}(N) = \{N' \mid N' \subseteq N \ \& \ N' \text{ is an atom}\}.$$

A simple net N is said to be <u>atomic</u> iff: $N = \bigcup_{N' \in \text{atoms}(N)} N'$.

<u>Example 3.1.</u>

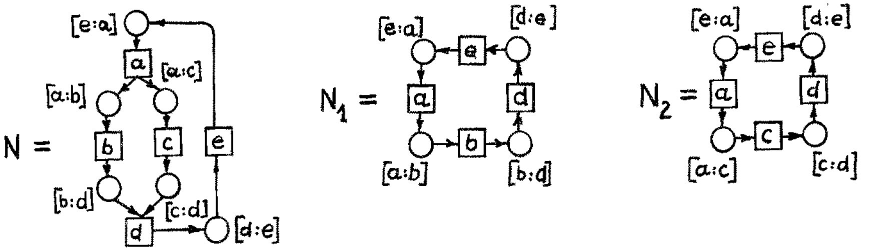
Let N, N′ be nets defined below. Note that atoms(N)=$\{N'\}$ and $N' \neq N$, so the net N is not atomic.



□

<u>Example 3.2.</u>

Let N, N_1, N_2 be nets defined below. In this case atoms(N)=$\{N_1, N_2\}$, and $N = N_1 \cup N_2$, thus N is the atomic net.



□

Let N=(T,P) be a simple net. To simplify the considerations, we shall use the following well known notation:

1. $(\forall p \in P)\ \ p^{\bullet} = \text{right}(p),\ {}^{\bullet}p = \text{left}(p)$,
2. $(\forall a \in T)\ \ a^{\bullet} = \{p \in P \mid a \in \text{left}(p)\},\ {}^{\bullet}a = \{p \in P \mid a \in \text{right}(p)\}$.

Of course, $(\forall p \in P)\ \ p = ({}^{\bullet}p, p^{\bullet})$. Note that the above operations are correctly defined for every pair (T,P), where $P \subseteq 2^T \times 2^T$.

<u>Lemma 3.3.</u>

A pair (T,P), where $P \subseteq 2^T \times 2^T$ is a simple net iff:

$$(\forall a \in T)\ \ a^{\bullet} \neq \emptyset \ \ \& \ \ {}^{\bullet}a \neq \emptyset . \ ■$$

Let $F \subseteq T \times P \cup P \times T$ be the following relation:

$$(\forall x, y \in T \cup P)\ \ (x,y) \in F \iff x \in \text{left}(y) \text{ or } y \in \text{right}(x).$$

Note that for every s-net (T,P), the triple (T,P,F) is a standard representation of Petri net (see Petri(1977), Petri(1978)).

A simple net N is said to be <u>connected</u> iff
$(\forall x,y \in T \cup P) \quad (x,y) \in (F \cup F^{-1})^*$.
Other words, a net is connected if its graph is connected.

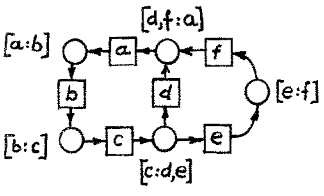
<u>Theorem 3.4.</u>
Every atom is connected. ■

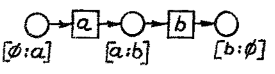
4. Elementary nets.

Now we define simple nets which represent sequential systems and processes. We shall prove that these nets are special kind of atoms.

A simple net N=(T,P) is said to be <u>elementary</u> iff
1. $(\forall a \in T) \quad |{}^\bullet a| = |a^\bullet| = 1$,
2. N is cannected.

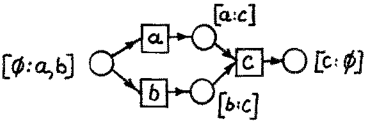
Examples of elementary nets are given below.





<u>Theorem 4.1.</u>
Every elementary net is an atom. ■

Of course, not every atom is an elementary net. For example, the below simple net is the atom, but it is not elementary.



For every s-net N, let
$\text{elem}(N) = \{N' \mid N' \sqsubseteq N \ \& \ N' \text{ is elementary}\}$.

Of course, $\text{elem}(N) \subseteq \text{atoms}(N)$, and generally this inclusion is proper one. Note thet every elementary net is equivalent witn a totally labelled state machine.

5. Proper nets.

In this section we introduce a class of nets generated by superposition of sequential, i.e. elementary, nets. We shall prove in the sequel that this class of s-nets defines marked nets which have much properties required from "well defined" concurrent dynamic systems.

A s-net N is said to be proper iff: $N = \bigcup \{N' \mid N' \in elem(N)\}$. Of course, every proper net is atomic, but not every atomic net is proper.

Corollary 5.1.
A s-net N=(T,P) is proper $\Longleftrightarrow$ there is a set $\{N_1, \dots, N_m\}$ of elementary nets and $N = N_1 \cup \dots \cup N_m$. ■

There are such proper nets that elem(N)≠atoms(N). Let PNETS denote the family of all proper nets. It can be easily proved that the family PNETS is closed under the operation "$\cup$", but it is not closed under the operation "$\cap$". The net from Example 3.2 is proper, the net considered in Section 6 is not proper.

6. Marked nets.

In this section we shall extend the approach we have presented to marked nets. Unmarked nets represent the static structure of dynamic systems, while marked nets represent the dynamic structure of those systems. We aim to show in which way the static structure describes the dynamic structure of concurrent systems and vice versa.

Let N=(T,P) be a simple net.
Let $Rl \subseteq 2^P \times 2^P$ be the following relation:

$$(M_1, M_2) \in Rl \Longleftrightarrow (\exists a \in T)\ M_1 - {}^\bullet a = M_2 - a^\bullet \ \&\ {}^\bullet a \subseteq M_1 \ \&\ a^\bullet \subseteq M_2 .$$

The relation Rl is called the forward reachability in one step.
Define $R = (Rl \cup Rl^{-1})^*$.
The relation R is called the forward and backward reachability of N (or simply reachability ralation of N). If N is not understood we shall write Rl_N, R_N . Note that R is an equivalence relation. For every $M \in 2^P$, let $[M]_R$ (or simply $[M]$) denote the equivalence class of R containing M (compare Petri(1978)).

By a marked simple net (abbr. ms-net) we mean any triple:

$$MN = (T,P,Mar),$$

where: $N=(T,P)$ is a s-net,

$Mar \subseteq 2^P$ is a set of markings of MN,

$$Mar = \bigcup_{M \in Mar} [M]_{R_N} .$$

A ms-net $MN=(T,P,Mar)$ is called compact iff:

$$(\forall M \in Mar) \quad Mar = [M]_{R_N} .$$

We shall prove in the sequel that compact marked nets have very regular properties. Petri (1978) has restricted his attention to compact nets, and has assumed that every Condition-Event-System is compact in the sense defined above.

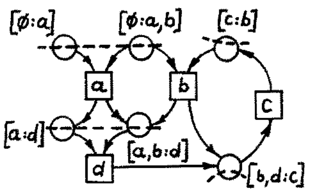
A transition a T is called fireable $\Longleftrightarrow$

$(\exists M_1, M_2 \in Mar) \quad {}^\bullet a \subseteq M_1, \; a^\bullet \subseteq M_2.$

A ms-net is called locally fireable if every transition is fireable, and every locally fireable and compact ms-net is called fireable.

Example 6.1.

Consider the following ms-net $MN=(T,P,Mar)$.



$$Mar = \{ \{[\emptyset:a],[\emptyset:a,b]\}, \{[a:d],[a,b:d]\}, \{[b,d:c]\}, \{[c:b]\} \}.$$

Observe that this net is compact, but the transition "b" is not fireable, so this net ia not locally fireable. □

A ms-net $MN=(T,P,Mar)$ is said to be safe iff

$(\forall C \in 2^P)(\forall a \in T)$

$$({}^\bullet a \cap C=\emptyset \;\&\; (\exists M \in Mar)\; {}^\bullet a \cup C \subseteq M) \Longleftrightarrow (a^\bullet \cap C=\emptyset \;\&\; (\exists M \in Mar)\; a^\bullet \cup C \subseteq M) .$$

The net from Example 6.1 is safe.

Let $MN=(T,P,Mar)$ be a ms-net and let (T,P) be an elementary net. Each ms-net of the above form will be called marked elementary net. It turns out that in the case of marked elementary nets, safeness describes very regular structure of markings.

Lemma 6.1.

A marked elementary net $MN=(T,P,Mar)$ is safe $\Leftrightarrow$ $Mar = \{\{p\} \mid p \in P\}$. ■

Lemma 6.2.

Every safe marked elementary net is compact and fireable. ■

7. Concurrency-like relations.

In this section we recall and modify some notions and results from Janicki (1979) and Petri (1977).

Let X be a set, and let $id \subseteq X \times X$ be the identity relation.

A relation $C \subseteq X \times X$ is called the sir-relation (from symmetric and irreflexive) iff:

1. $(\forall a,b \in X) \quad (a,b) \in C \Leftrightarrow (b,a) \in C$,
2. $(\forall a,b \in X) \quad (a,b) \in C \Rightarrow a \neq b$.

Let C be a sir-relation.

Define the families of subsets of X: kens(C), $\overline{\text{kens}}(C)$ in the following way (compare Petri(1977)),

$A \in \text{kens}(C) \Leftrightarrow$ 1. $(\forall a,b \in A) \quad (a,b) \in C \cup id$,
2. $(\forall c \notin A)(\exists a \in A) \quad (a,c) \notin C$,

$A \in \overline{\text{kens}}(C) \Leftrightarrow$ 1. $(\forall a,b \in A) \quad (a,b) \notin C$,
2. $(\forall c \notin A)(\exists a \in A) \quad (a,c) \in C$.

Corollary 7.1.

For every sir-relation $C \subseteq X \times X$, kens(C), $\overline{\text{kens}}(C)$ are covers of X. ■

We shall now consider some connections between covers and sir-relations. Let cov be a cover of X, and let $sir(cov) \subseteq X \times X$ be the relation defined as follows:

$(a,b) \in sir(cov) \Leftrightarrow \{A \mid A \in cov \ \& \ a \in A\} \cap \{A \mid A \in cov \ \& \ b \in A\} = \emptyset$.

The relation sir(cov) is called the sir-relation defined by the cover cov.

A sir-relation C is called K-dense (compare Petri(1977), Janicki(1979)) iff: $(\forall A \in \text{kens}(C))(\forall B \in \overline{\text{kens}}(C)) \quad A \cap B \neq \emptyset$.

A cover cov of X is called minimal iff

$(\forall A \in cov) \quad cov - \{A\}$ is not a cover of X.

Theorem 7.2.

Let $C \subseteq X \times X$ be a sir-relation.

If $\overline{\text{kens}}(C)$ is a minimal cover then C is K-dense. ■

8. Concurrency (coexistency) defined by the whole structure of net.

In this section we shall show in which way the static (unmarked) structure of nets describes the concurrent and dynamic structure of nets. We shall restrict our attention to proper nets.

Let $N=(T,P)$ be a proper simple net.
Assume that $\text{elem}(N)=\{N_1,\dots,N_m\}$, where $N_i=(T_i,P_i)$ for $i=1,\dots,n$.
Define $\text{cov}(P) = \{P_1,\dots,P_m\}$. Of course, $\text{cov}(P)$ is a cover of P.

Let $\text{coex}_N \subseteq P\times P$ be the following relation: $\text{coex}_N = \text{sir}(\text{cov}(P))$.
Other words: $(a,b)\in \text{coex}_N \iff \{P_i \mid a\in P_i\} \cap \{P_i \mid b\in P_i\} = \emptyset$.
The relation coex_N will be called the <u>coexistency defined by the whole structure of the net N</u>. This relation describes the concurrent structure defined by N. When N is the net of occurrences (see Petri (1977) and Petri(1978)) then coex_N is the concurrency relation from Petri (1977) restricted to places and minus identity.

<u>Theorem 8.1.</u>
For every proper net $N=(T,P)$, the triple $(T,P,\text{kens}(\text{coex}_N))$ is a marked simple net. ■

The above theorem enable us to introduce the following notions. For every proper net $N=(T,P)$, the marked net $(T,P,\text{kens}(\text{coex}_N))$ will be denoted by the symbol $\dot{N}$, and the marking $\text{kens}(\text{coex}_N)$ will be called <u>natural</u>. The marked net $\dot{N} = (T,P,\text{kens}(\text{coex}_N))$ will be called <u>naturally marked net</u>. Generally, a marked net $MN=(T,P,Mar)$, where $N=(T,P)$, is called <u>naturally marked</u> if N is a proper s-net and $Mar=\text{kens}(\text{coex}_N)$.

The basic properties of naturally marked nets are the following.

<u>Theorem 8.2.</u>
For every proper net $N=(T,P)$, the ms-net $\dot{N}=(T,P,\text{kens}(\text{coex}_N)$ is safe and locally fireable. ■

<u>Lemma 8.3.</u>
For every proper net $N=(T,P)$,

coex_N is K-dense $\Rightarrow$ $\overline{\text{kens}}(\text{coex}_N)=\text{cov}(P)$. ■

The property $\overline{\text{kens}}(\text{coex}_N)=\text{cov}(P)$ means that the decomposition of net into sequential components given by the "topological" structure and the decomposition given by the natural concurrency structure are identical. The K-density of coex_N is interpreted as the fact that every sequential component of the net N (i.e. the system which is represented by N) has one common element with any "case" of that net (i.e. system). For more details on the subject of K-density the reader is advised to refer to Petri (1977), Petri (1978), Best (1977), Janicki (1979).

Corollary 8.4.
For every proper net N=(T,P), if cov(P) is a minimal cover of P then coex_N is K-dense and $\overline{\text{kens}}(\text{coex}_N)=\text{cov}(P)$. ■

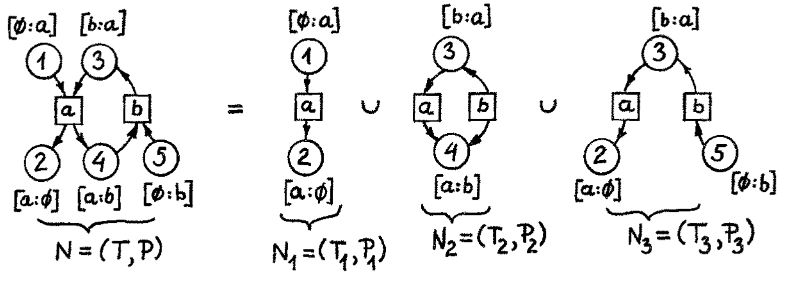
Theorem 8.5.
For every proper net N=(T,P), if $\dot{N}=(T,P,\text{kens}(\text{coex}_N))$ is compact then coex_N is K-dense. ■

From the above theorems it follows that naturally marked compact nets have very regular structure. This allows us to propose the following definition: every compact and naturally marked net will be called regular.

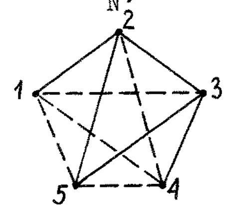
Corollary 8.6.
For every elementary marked net N=(T,P): $\text{coex}_N=\emptyset$, $\overline{\text{kens}}(\text{coex}_N)=P$, $\text{kens}(\text{coex}_N)=\{\{p\} \mid p\in P\}$, and $\dot{N}=(T,P,\text{kens}(\text{coex}_N))$ is regular. ■

Now we consider two examples which represent different aspects of connections between the relation coex_N and the net N. We shall define the relation coex_N by means of graphs.

Example 8.1.



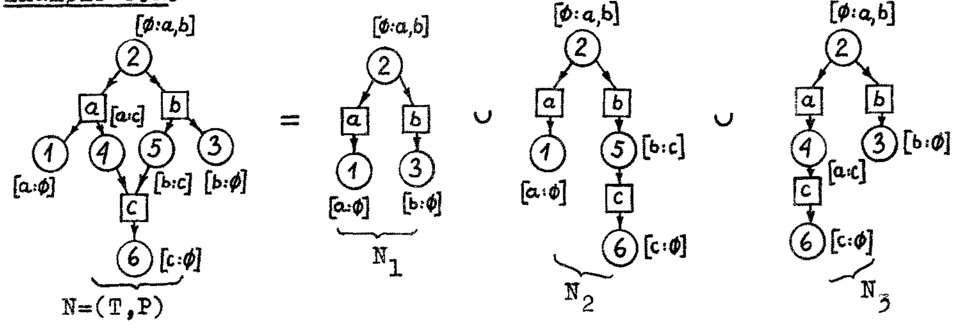
In this case : elem(N)=atoms(N)=$\{N_1,N_2,N_3\}$. The graph representing the relation $coex_N$ is of the following form (the line - - - - denotes $coex_N$, the line ——— denotes the relation $\overline{coex}_N$-id).



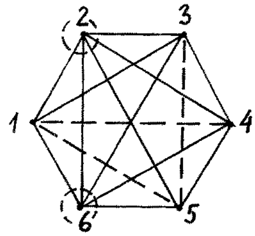
Properties of the net N are the following:
$kens(coex_N)= \{\{1,4,5\},\{1,3\},\{2,4\}\}$,
$\overline{kens}(coex_N)= \{\{1,2\},\{3,4\},\{2,3,5\}\}$,
$coex_N$ is K-dense, $\overline{kens}(coex_N)$=cov(P) ,
$\overline{kens}(coex_N)$ is a minimal cover of P.

The ms-net $\dot{N}$=(T,P,kens($coex_N$)) is compact, safe and fireable; other words the ms-net $\dot{N}$ is regular. □

Example 8.2.



Here elem(N)=$\{N_1,N_2,N_2\}$≠atoms(N), and the graph of $coex_N$ is the following.



Remaining properties of N are the following:
$kens(coex_N)=\{\{1,4\},\{3,5\},\{4,5\},\{2\},\{6\}\}$,
$\overline{kens}(coex_N)=\{\{1,2,3,6\},\{2,3,4,6\},\{1,2,5,6\}\}$,
$cov(P)= \{\{1,2,3\},\{2,3,4,6\},\{1,2,5,6\}\}$,
$\overline{kens}(coex_N) \neq cov(P)$,
$coex_N$ is not K-dense, because
$\{4,5\} \cap \{1,2,4,6\} = \emptyset$,

and $\overline{kens}(coex_N)$ is not a minimal cover of P. In this case the ms-net $\dot{N}$=(T,P,kens($coex_N$)) is safe and locally fireable, but not compact, therefore it is not regular. □

9. Concurrency (coexistency) defined by markings.

In this section we shall show in which way markings do influence on the concurrent structure of nets.

Let MN=(T,P,Mar) be a marked simple net, where N=(T,P) is not necessarily proper.

Let $coex_{Mar} \subseteq P \times P$ be the relation defined in the following way:

$$(\forall p,q \in P) \quad (p,q) \in coex_{Mar} \iff p \neq q \ \& \ (\exists M \in Mar) \quad \{p,q\} \subseteq M.$$

The relation $coex_{Mar}$ is called the coexistency defined by markings.

Lemma 9.1.
For every naturally marked net $\dot{N}=(T,P,Mar)$: $coex_{Mar} = coex_N$.■

We aim to formulate when a marked net is regular. To this end we must introduce some new notions.

A ms-net $MN=(T,P,Mar)$ is called K-dense iff $coex_{Mar}$ is K-dense.

A ms-net $MN=(T,P,Mar)$ is called c-compatible iff $kens(coex_{Mar}) = Mar$.

A ms-net $MN=(T,P,Mar)$ is said to be e-consistent iff $(\forall N_i=(T_i,P_i) \in elem(N))(\forall M \in Mar) \quad |M \cap P_i| \leq 1$ (where $N=(T,P)$).

The first property says that every sequential component described by the markings class of the ms-net and every "case" described also by that markings class have one element in common. The second property means that the class of all "cases" described by the concurrency relation $coex_{Mar}$ is identical with the class of all markings of that net, that is to say the concurrrncy relation $coex_{Mar}$ is compatible with the family Mar. And,the third property says that every elementary net contained in N and every marking have at most one element in common, then the class Mar is consistent with the family of all elementary subnets of the net N.

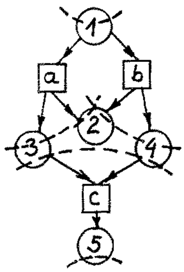
Theorem 9.2.
If $MN=(T,P,Mar)$ is compact, safe, fireable, K-dense and c-compatible then the s-net $N=(T,P)$ is proper.■

Lemma 9.3.
If $MN=(T,P,Mar)$ is compact, safe, fireable, K-dense and c-compatible, then for every $A \in \overline{kens}(coex_{Mar})$ the pair $(left(P),P)$ is an elementary simple net.■

Theorem 9.4.
A marked net is regular $\iff$ it is compact, safe, fireable, K-dense, c-compatible and e-consistent.■

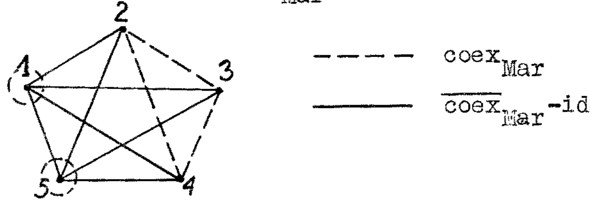
Example 9.1.

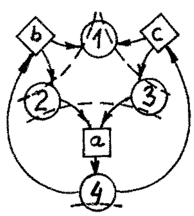
Consider the following four ms-nets (all of them will be denoted by (T,P,Mar)). The first of them is safe, locally fireable, K-dense, but it is not compact, c-compatible and the s-net (T,P) is not proper. The second net is compact, safe, fireable and K-dense, but it is not c-compatible, and, of course, the net (T,P) is not proper. The third net is compact, safe, fireable and c-compatible, but it is not K-dense, and (T,P) is not proper. The fourth ms-net is compact, safe, fireable, K-dense and c-compatible, the net (T,P) is proper, but it is not e-consistent, then it is not regular.



Mar = {{1}, {2,3}, {2,4}, {3,4}, {5}}

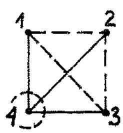
the graph of $coex_{Mar}$

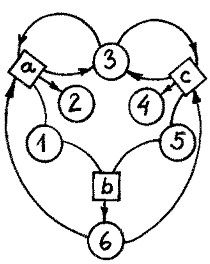




Mar = {{1,2}, {1,3}, {2,3}, {4}}

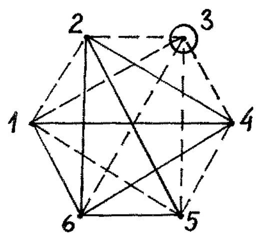
the graph of $coex_{Mar}$

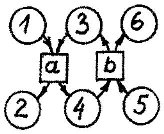




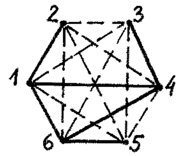
Mar = {{1,2,3}, {3,4,5}, {1,3,5}, {3,6}}

the graph of $coex_{Mar}$





$\text{Mar} = \{\{1,3,5\},\{2,4,5\},\{2,3,6\}\}$



□

10. Final comment.

Treating concurrent systems as the superposition of sequential subsystems or primitive concurrent subsystems is, to author's mind, the natural way of analysis and synthesis of those systems. This paper is an attempt to formal approach to this problem. Similar problems, but from a different point of view are considered in Lauer et al. (1978), Janicki (1978), Knuth (1979).

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