IDENTIFICATION AND CONTROL FOR LINEAR
DYNAMIC SYSTEMS OF UNKNOWN ORDER

A.V.Medvedev

Institute of Automation, Academy of Sciences of
Kirghiz SSR, Frunze, USSR

The linear dynamic systems theory appears to be the most developed part of the control theory. Comparative simplicity of this theory due to the superposition principle dominating there, makes it widely accessible when developing control systems for different kind of processes. Monographs [1, 2, 3] are devoted to numerous analysis and synthesis problems concerning these systems. In the following an approach is developed to linear dynamic plants control algorithms aimed at the estimation of the plant reverse operator for the case when the order of the equation describing the process is unknown.

I. Processes described by ordinary differential
equations

Let a plant under study be described by the equation of the form

$$D^n q(t) = D^m U(t), \quad (1)$$

where $q(t)$ is plant output, $U(t)$ - control action, D - differential operator, n, m - order of operator D , $n > m$, the order of operators D^n , D^m being unknown. Considering that the operator connecting the output of plant $q(t)$ with the input of $U(t)$ and vice versa as well as $U(t)$ with $q(t)$ is described by convolution integral, it is necessary to estimate operator $U(t) = Aq(t)$. And now, by picking up $\dot{q}(t)$ as forward action, we may calculate control $U^*(t)$ [2] which corresponds to it. Transient characteristics taken at the plant serve as a basis for operator A estimation.

The estimation of operator A is subdivided into two stages.

Primarily, we solve the identification problem which consists in estimating operator B, $q(t) = BU(t)$ which is followed by estimating operator A.

It is known that at zero initial conditions B may be represented by the integral

$$q(t) = \int_0^t h(t-\tau)U(\tau)d\tau, \quad (2)$$

or in the discrete type

$$q_s = q(t_s) = \sum_{j=0}^{S-1} h(t_s-t_j)u(t_j)\Delta t_j \quad (3)$$

where $h(t-\tau)$, $h(t_s-t_j)$ is weight function of system (1) in the analogue and discrete variants correspondingly. Operator A, which is reverse to (2) alongside with B, is linear and bounded operating in continuous functions space and has the form [2]

$$U(t) = \int_0^t k(t-r)q(r)dr \quad (4)$$

or in the discrete form

$$U_s = U(t_s) = \sum_{j=0}^{S-1} k(t_s-t_j)q(t_j)\Delta t_j. \quad (5)$$

The method of estimation of operators (3), (5) (in connection with the all-round application of computers in control systems, we shall further apply a discrete registration only) is based on non-parametric estimation of unknown stochastic dependences.

Let the N-number of transient characteristics be taken on the plant described by linear differential equation (1). Consequently, we shall have observations $q_{\gamma k}$, mixed with disturbances at time moments t_{j1} , $i = \gamma k$, $k = 1, \dots, \lambda$, $\gamma = 1 \dots N$. For non-parametric estimate (3) we take the statistics class

$$q_{\gamma 2}(t_s) = \sum_{j=0}^{S-1} U(t_j) \frac{\sum_{i, \rho=1}^{\lambda} q_{i1} \left[F\left(\frac{t_s-t_j-t_i}{C(\tau)}\right) \phi\left(\frac{t_s-t_j-t_\rho}{C(\tau)}\right) - F\left(\frac{t_s-t_j-t_\rho}{C(\tau)}\right) \phi\left(\frac{t_s-t_j-t_i}{C(\tau)}\right) \right] \Delta t_j}{\sum_{i, \rho=1}^{\lambda} \phi\left(\frac{t_s-t_j-t_i}{C(\tau)}\right) \phi\left(\frac{t_s-t_j-t_\rho}{C(\tau)}\right)} \quad (6)$$

where $r = N\lambda$, $q_r(t_s)$ is the value of the system reaction to a disturbance $U(t_j)$, $j = 0, 1, \dots, S-1$.

Functions $F\left(\frac{t_s-t_j-t_i}{C(\tau)}\right)$, $\phi\left(\frac{t_s-t_j-t_i}{C(\tau)}\right)$ satisfy conditions

$$\int_{\Omega(t_s)} \phi\left(\frac{t_s - t_j - t_i}{c(\tau)}\right) d\Omega = 1, \quad \int_{\Omega(t_s)} (t_s - t_j - t_i) F\left(\frac{t_s - t_j - t_i}{c(\tau)}\right) d\Omega = -1. \quad (7)$$

If the type of disturbance $U(t)$ is known and $U(t)$ can be differentiated, then by estimating the known analogue of the integral convolution (2) we find (at zero initial conditions) that

$$q_z(t_s) = \sum_{j=0}^{S-1} U'(t_s - t_j) \sum_{i=1}^z q_i \phi\left(\frac{t_j - t_i}{c(\tau)}\right) \left[\sum_{i=1}^z \phi\left(\frac{t_j - t_i}{c(\tau)}\right) \right]^{-1} \Delta t_j. \quad (8)$$

For nonparametric estimate (6) the following theorem is appropriate: Theorem I. Let $h(t_s - t_j)$ be bounded for any $0 \leq t_j < t_s$ and have finite quantities of the second order, whereas random process

$$\xi_z = |q(t_s) - q_z(t_s)|, \quad \xi_z \in E, \quad M\{\xi_z\} < \infty, \quad z = 1, 2, \dots$$

which is set at probability space $\{\Omega, \mathcal{U}, P\}$ be measured relatively to the inclosed σ - algebras, U_r , $r = 1, 2, \dots$. In this case, with the probability one, the random process ξ_r , $r = 1, 2, \dots$ converges to zero with $c(r) = 0$, $rc(r) = \infty$ and $\nu \rightarrow \infty$, $N = \infty$. Also $\lim_{\nu \rightarrow \infty} \xi_z = 0$ at $\nu = \infty$, $N = \infty$.

Thus, algorithm (6) enables us to calculate the plant reaction to arbitrary disturbance $U(t_j)$, $j = 0, 1, \dots, S-1$ provided there are transient characteristics, taken at a particular plant.

As to the transient characteristics needed for the estimation of (5) they will be "taken" now from plant (6) simulator. Supposing $q_r(t_s) = 1$, $S=1, \dots, \hat{z}$. Then U_s , $S=1, 2, \dots, \hat{z}$ will be determined from the matrix equation

$$U = H^{-1}Q,$$

where

$$U = \begin{Bmatrix} u(t_1) \\ u(t_2) \\ \vdots \\ u(t_{\hat{z}}) \end{Bmatrix}, \quad Q = \begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{Bmatrix}, \quad H = \begin{Bmatrix} h(t_1) & 0 & \dots & 0 \\ h(t_2) & h(t_1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(t_{\hat{z}}) & h(t_{\hat{z}-1}) & \dots & h(t_1) \end{Bmatrix}.$$

And now, when we have a set of values $U(t_i)$, $i=1, 2, \dots, \hat{z}$ we find a non-parametric estimate (5). At zero initial conditions it will be of the form

$$u_v(t_s) = \sum_{j=1}^{S-1} q(t_j) \frac{\sum_{i,p=1}^{\hat{z}} u_i \left[F\left(\frac{t_s - t_j - t_i}{c(\nu)}\right) \phi\left(\frac{t_s - t_j - t_p}{c(\nu)}\right) - F\left(\frac{t_s - t_j - t_p}{c(\nu)}\right) \phi\left(\frac{t_s - t_j - t_i}{c(\nu)}\right) \right] \Delta t_j}{\sum_{i,p=1}^{\hat{z}} \phi\left(\frac{t_s - t_j - t_i}{c(\nu)}\right) \phi\left(\frac{t_s - t_j - t_p}{c(\nu)}\right)}, \quad (9)$$

where $U=R\zeta$, R is number of "taken" transient characteristics. The following proposal holds good for (9):

Theorem 2. Let theorem 1 take place and $k(t_s - t_j)$, $0 \leq t_j < t_s$ have the second bounded derivatives. Then at $c(v) \rightarrow 0$, $vc(v) \rightarrow \infty$ with $R \rightarrow \infty$, $\zeta \rightarrow \infty$

$$\lim_{\substack{R \rightarrow \infty \\ \zeta \rightarrow \infty}} \frac{1}{R} \sum_{j=1}^R |U_v(t_s) - U(t_s)| = 0, \quad \lim_{\substack{R \rightarrow \infty \\ \zeta \rightarrow \infty}} P(\sup |U_v(t_s) - U(t_s)| < \varepsilon) = 1,$$

where ε is a certain positive constant.

Formula (9) just represents the control algorithm for linear dynamic system.

Thus, the expounded approach to the dynamic plants control algorithms synthesis as apriori information requires only an assumption of its being linear and does not require knowledge of the equation order describing the plant. In this case in order to find the control algorithm (9), it is necessary to take several transient characteristics from the plant. Though convergence theorems are of asymptotic character, estimate (5) can be obtained in practice with a sufficient accuracy rate by having a limited number of transient characteristics. The advantage of the method proposed lies in the fact that the identification and control algorithms can be applied in the systems described by the linear differential equations of the general type (1).

2. Processes described by differential equations in partial derivatives

Let the condition of linear controllable system with distributed parameters be described by the function $q(x, t)$, where x is space variable belonging to interval $[0, 1]$, while time $t \in [0, T]$, T is termination time for transient process. Disturbing action $U(t)$ is applied to the input of the system. Let function $U(t) \in L_2$ be measurable. At zero initial condition, i.e. $q(x, 0) = 0$, $0 \leq x \leq 1$ the plant condition function $q(x, t)$ can be expressed by the formula [3]

$$q(x, t) = \int_0^t G(x, t, r) U(r) dr, \quad (10)$$

where $G(x, t, r)$ is pulse transient function of the system.

Let unique disturbing action $l(t)=1$, $t \geq 0$ and $l(t)=0$, $t < 0$, $0 \leq t \leq T$ be applied to the distributed system input. Let us carry on a series of observations $q_v(x, t_k)$ in the point $0 \leq x \leq 1$ at moments t_i , $i=k, \nu$, $K=1, \dots, \lambda$, $\nu=1, \dots, N$, where N is the number of taken transient characteristics. Based on these observations we build a statistic es-

timate (10) which being within the class of non-parametric type approximations, has the form [4,5]

$$q_z(x,t) = \int_0^t u(\tau) \sum_{i,p,i'} q_i(x) [F(\tilde{c}(\tau)(t-\tau-t_i))\phi(\tilde{c}(\tau)(t-\tau-t_p)) - F(\tilde{c}(\tau)(t-\tau-t_i)) \times (11) \\ \phi(\tilde{c}(\tau)(t-\tau-t_p))] \phi(\tilde{c}(\tau)(x-x_i)) \left[\sum_{i,p,i'} \phi(\tilde{c}(\tau)(t-\tau-t_i)) \phi(\tilde{c}(\tau)(t-\tau-t_p)) \phi(\tilde{c}(\tau)(x-x_i)) \right]^{-1} d\tau,$$

where functions $F(\tilde{c}^{-1}(r)(t-r-t_i))$ and $\phi(\tilde{c}^{-1}(r)(t-r-t_i))$ satisfy conditions (7).

For the statistics of the class under consideration holds good the following

Theorem 3. Let $Q(x,t,\tau)$ be twice differentiable by $\tau \in [0,t]$ in point x , $0 \leq x \leq 1$. Then the random process $\hat{Q}_r = |q(x,t) - q_r(x,t)|$, $\hat{Q}_r \in E_1$, $r=1,2,\dots$ given on the probability space $\{\Omega, \mathcal{U}, P\}$ and measurable in relation to the inclosed σ -algebras, U_r , $r=1,2,\dots$ is convergeable in the mean and with the probability zero $\hat{Q}_r$, $r=1,2,\dots$ strives to zero at $\lambda \rightarrow \infty$, $N \rightarrow \infty$.

Now considering the formula (11) as the distributed plant simulator, we take transient characteristics analogically to the above mentioned way, picking up for $q(x,t)$ some given value of the plant output $q^*(x,t)$, $0 \leq t \leq T$. And only now on the basis of the transient characteristics obtained on the simulator (11) we may build up a non-parametric algorithm for distributed plant control.

In conclusion it should be observed that here too the method of building up identification and control algorithms is just the same as that for the processes described by usual differential equations. Primarily transient characteristics are taken from the plant under study and its simulator is made and only then on the simulator "reverse" transient characteristics are taken and the plant reverse operator is estimated which just represents the plant control algorithm. The class of control and identification algorithms can be realized only on digital computers and therefore can be applied for control of complicated technological processes in computer controls and digital computers.

3. Digital computer simulation. Discussion. Prospects.

Above it has been pointed out that dynamic system non-parametric identification and control algorithms prove to be fit for unknown order plants. This fact we illustrate by simulation results. The plant is described by the equation of the 1,2 and 3rd order correspondingly. Three transient plant characteristics have been "taken" from a digital computer. When "taking" a transient characteristic we have

carried out 50 measurements of the plant output subjected to the action of random disturbances belonging to the interval $[-\alpha, \alpha]$ at different intervals of Δt discretization.

As the result we have a set of points $\{q_i, t_i\}$, $i=1, \dots, 150$, $\lambda=50$, $N=3$. Based on these observations the plant operator was estimated in accordance with the formula (8). Further, sinusoidal disturbing action was applied to the system input (in Fig.1 - curve 1) and the plant response was calculated in accordance with the ordinary differential equations theory (curve-2) as well as with the non-parametric estimate of the system response (curve-3).

Fig. 1

Cases a, b, c in Fig.1 correspond to the plant simulators of 1, 2 and 3rd order, S is discrete time. Fig.2 shows the changes of quadratic mean error $\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (q(t_i) - \hat{q}_2(t_i))^2}$, $N=50$ depending on Δt . The curve in Fig.2a corresponds to the case when linearly increasing disturbance was applied to the plant input, while 2b corresponds

to sinusoidal disturbance. Owing to the all-round application of computing machines in up-to-date information control and processing systems it seems impedient to use a non-parametric algorithm proceeding from the two reasons: first, owing to little apriori information (only the correctness of assumption about the plant linearity is required); and second, a certain universality and the unique type of these procedures makes it possible to develop a program for control computers aimed at controlling different technological plants of the class under consideration. In this case it is sufficient to keep in the control computer's memory a set of transient characteristics of each technological plant.

The results, expounded above, appear to be the first step on the way of developing non-parametric algorithms for controlling linear systems of an unknown order. And the most serious attention should be given to the problems associated with incorrect setting of identification and control problems, adequacy of the linear assumption of the real structure of a plant, optimization of algorithms, estimation of $q_r(t)$ accuracy depending on Δt ; developing the algorithms covering more complicated cases and so on.

Consequently, a distinctive feature of the algorithms of the class under consideration is the absence of an apriori assumption about the order of the equation describing a process, which makes it possible to use the same algorithm (program) for controlling different production processes. The other important feature of these algorithms is their orientation to digital technique which will ever find a wider application in different branches of industry.

a

b

Fig. 2

R e f e r e n c e s

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