DYNAMIC MODELS OF TECHNOLOGICAL CHANGES

L.V. Kantorovich, V.I. Zhjanov

Moscow

The report is devoted to one of the problems of mathematical modelling of economy - the problem of modelling technological change.

Alongside with multi-product models such as the input-output model, the Von Neumann model of expanding economy, models of linear programming, there are macroeconomic models of national economy on the basis of such aggregate indices as national income, aggregate demand, etc., which are widely applied for qualitative and quantitative analysis of economy.

Such models have been developed since the thirties: Feldman's Growth Model, research by the Keynes school, especially works by Harod, Domar, Samuelson, Hicks, Solow and other authors. It is well known that these models have been given practical application in government regulation of economy, in measures for reducing crises, unemployment, though with partial success.

The simplest one-product models of the development of economy can be used for global analysis of development of the economic system and consideration of the effect of technological change on the dynamics of the economic system. On the basis of these models it is possible to study the effect of technological change on the most important economic characteristics.

Let us consider an economic system in which a single product is produced. One part of this product is allocated to consumption and the other part to accumulation of fixed and working stocks, whereas in this model no distinction is made between these two stocks.

Under the assumption of instantaneous transformation of capital stocks and consideration of technological change this model may be described by the following equation:

$$\frac{dK(t)}{dt} = P(t) - V(t) = e^{\delta t}\, \mathcal{U}[K(t), T(t)] - V[t, K(t), T(t), P(t)] \tag{1}$$

where $P(t)$ is the net product or the national income;

$V(t)$ is the total consumption, given in unit time. The production function $\mathcal{U}[K(t), T(t)]$ characterizes the amount of net output which can be produced by capital stock $K(t)$ per unit time and labour resources $T(t)$, δ characterizes the rate of neutral technological change.

Within the framework of this model an important parameter of the economic system can be determined, i.e., norm of efficiency of investments $\eta_э$. For $\eta_э$ the following expression is derived

$$\eta_э = \frac{\frac{1}{P(t)}\frac{dP(t)}{dt} - \frac{1}{T(t)}\frac{dT(t)}{dt} - \delta}{1 - \frac{V(t)}{P(t)} - \frac{1}{T(t)}\frac{dT(t)}{dt}\frac{K(t)}{P(t)}} \tag{2}$$

All the variables in this formula have a precise economic content. Calculation of the norm of efficiency according to this formula without consideration of technological change gives a value of 22%, and with consideration of technological change, lag, obsolescence of stocks this value is equal to 18%.

As has been pointed out, it is assumed in the model the hypothesis about the instantaneous transformation of stocks, i.e., the assumption that the stocks can always be transformed from one form to another and, in particular, due to this it is possible to change from one production structure (labour-capital ratio) to another one without loss. More practicable is a similar model without the assumption about the instantaneous transformation of stocks.

Let us consider the structure of the one-product model.

$T(t)$ - labour resources at instant t is a fixed function. We introduce $\lambda(t)$ - type (or structure) of new stocks created at instant of time t, which are characterized by the value (expressed in the product) of single stocks (stocks per unit of labour). It is assumed that the stocks created at each instant of time t are single-type ($\lambda(t)$ - single-valued function of t). $\varphi(t)$ denotes the intensity of creating stocks, i.e., $\varphi(t)dt$ is the number of new work positions created during the time $[t, t+dt]$, then $\lambda(t)\varphi(t)dt$ is the volume of newly created stocks in the same interval. The functions $\lambda(t)$ and $\varphi(t)$ in the model are subject to calculation.

It is assumed that potential modes of production are characterized by the production function $\mathcal{U}(x,y)$, which indicates the amount of net product created by labour y when using the fixed stocks x per unit time (at the initial instant). It is assumed that the function $\mathcal{U}(x,y)$ is positively homogeneous of the first degree

$$\mathcal{U}(\lambda x, \lambda y) = \lambda\,\mathcal{U}[x,y] \qquad \text{when} \quad \lambda > 0$$

and based on optimal modes, which renders a natural assumption about convexity $\mathcal{U}[x,1]$.

Technological change is present in the model by the following method. It is assumed that the amount of net product, produced per unit at the given quantity of stocks and expenditures of labour, increases exponentially depending on the instant of creating stocks τ, i.e., it exceeds by $e^{\delta\tau}$ times (δ is fixed non-negative number) the amount of products produced by the stocks, created at the initial instant, under the same conditions.

It is also assumed that in the process of development of the economy the labour resources are removed from stocks of a lower structure, which have been created formerly. The labour resources, which have been freed from the removed stocks, are used on stocks created anew, the remaining stocks are not used subsequently. Under the assumption of continuous growth of capital organic composition (new stocks) the policy of removing stocks from production is characterized by the function $m(t)$, namely all stocks which have been created up to a certain instant of time $m(t)$ are freed to the instant of time t. The function $m(t)$ in the model is subject to determination.

The investments for increasing the fixed and working stocks are specified through their intensity so that $\partial\ell(t)\,dt$ is volume of investments at time interval $[t, t+dt]$. The function $\varkappa(t)$ is specified in the model, however it can be placed in dependence on the national income at instant t or other parameters of the model.

The system of equations that describe the model takes the form:

$$\varphi(t) = \frac{dT(t)}{dt} - \varphi[m(t)]\frac{dm(t)}{dt} \tag{3}$$

$$\varphi(t)\lambda(t) = \varkappa(t) \tag{4}$$

$$\mathcal{U}[\lambda(t),1]\varphi(t) - \frac{\partial\mathcal{U}[\lambda(t),1]}{\partial x}\varkappa(t) - e^{\delta[m(t)-t]}\varphi(t)\mathcal{U}[\lambda[m(t)],1] = 0 \tag{5}$$

The system is resolved for $t > t_0$ (t_0 is a fixed number). The initial conditions are specified as $m(t_0) = m_0$ $(m_0 < t_0)$,

$\lambda(t) = \lambda_0(t)$, $\varphi(t) = \varphi_0(t)$ when $t \in [m_0, t_0]$

where m_0 is a fixed number, and $\lambda_0(t)$ and $\varphi_0(t)$ are functions determining the initial distribution (with $t \in [m_0, t_0]$) of stocks and labour.

Equation (3) reflects the labour balance. Equation (4) reflects the stocks balance. Equation (5) is a condition of differential optimization. This condition denotes that the increase of net product at each instant of time should be maximal, in other words the functions $m(t)$, $\lambda(t)$, $\varphi(t)$ should be determined so that the function $dP(t)/dt$ is maximal at each instant of time t. Here $P(t)$ is the amount of net product (national income) produced at instant t per unit time. For $P(t)$ the following formula holds true:

$$P(t) = \int_{m(t)}^{t} e^{\delta\tau}\,\mathcal{U}[\lambda(\tau),1]\,\varphi(\tau)\,d\tau \tag{6}$$

For the norm of efficiency of investments the following formula has been derived

$$n(t) = \frac{1}{\varkappa(t)} \left\{ \frac{dP(t)}{dt} - e^{\delta m(t)} \mathcal{U}[\lambda[m(t)],1] \frac{dT(t)}{dt} \right\} \qquad (7)$$

Let us specify the next form of the functions included in equations (3)-(5).

$\mathcal{U}[x,y] = x^{\alpha} y^{1-\alpha}$; $T(t) = T_0 e^{\rho t}$; $\varkappa(t) = \varkappa_0$ is a fixed positive number. Under the assumption of a possibility of expansion of equations (3)-(5) into infinite series and confining ourselves to linear terms of values δ and ρ we have

$$m(t) = \beta t \left[1 + \delta \frac{1-\beta}{2\alpha} t\right]$$

$$\varphi(t) = \frac{1}{t} \left[1 + \delta \frac{t+1}{2\alpha} + \rho \frac{T_0 (t+1)}{1-\beta}\right]$$

$$\lambda(t) = \varkappa_0 t \left[1 - \delta \frac{t+1}{2\alpha} - \rho \frac{T_0 (t+1)}{1-\beta}\right], \quad \beta = (1-\alpha)^{1/\alpha}.$$

By means of this solution it is possible to obtain a parametric representation of the norm of efficiency by coefficient δ which characterizes technological change and growth rate of labour resources ρ :

$$n(t) = \alpha \varkappa_0^{\alpha-1} t^{\alpha-1} \left[1 + \frac{\delta(1+\alpha)t + (1-\alpha)}{2\alpha} + \rho T_0 \frac{t+1}{1-\beta}\right].$$

In conclusion we shall note the possibility to introduce within the framework of the model the notion of variable transformability of stocks, that is incomplete transformability, whereby the degree of incompleteness is characterized by a variable coefficient.

Let us assume that the stocks removed from production (relative to its cost of reproduction) can be partly realized and the obtained capital directed into investments. Let us denote part of the realized value by π $(0 < \pi < 1)$. The equations of the model will take the following form:

$$\varphi(t) = \frac{dT(t)}{dt} + \rho[m(t)]\frac{dm(t)}{dt} \tag{8}$$

$$\varphi(t)\lambda(t) = \partial\ell(t) + \pi\,\partial\ell[m(t)]\frac{dm(t)}{dt} \tag{9}$$

$$\mathcal{U}[\lambda(t),1]\,\varphi(t) - \frac{\partial\mathcal{U}[\lambda(t),1]}{\partial x}\partial\ell(t) - $$

$$- e^{\delta[m(t)-t]}\varphi(t)\,\mathcal{U}[\lambda(m(t)),1] + \pi\frac{\partial\mathcal{U}[\lambda(t),1]}{\partial t}\partial\ell[m(t)]\frac{dm(t)}{dt} = 0. \tag{10}$$

The formula for the norm of efficiency of investments will be changed accordingly. Under the assumption of a small value of coefficient π we obtain for $n(t)$ the following formula

$$n(t) = n_0(t)\left[1 + \pi\frac{\partial\ell[m(t)]}{\partial\ell(t)}\;\frac{d^2T(t)}{dt^2}\right] \tag{11}$$

References

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