NONSTATIONARY PROCESSES FOR MATHEMATICAL PROGRAMMING
PROBLEMS UNDER THE CONDITIONS OF POORLY FORMALIZED
CONSTRAINTS AND INCOMPLETE DEFINING INFORMATION

I.I. Eremin, Vl.D. Mazurov

Institute of Mathematics and Mechanics, Ural
Scientific Center of the Academy of Sciences
of the USSR

S.Kovalevskaja st. 16, Sverdlovsk, USSR

1. Introduction.

The methods of Mathematical Programming are an effective means for the analysis of complicated systems: technical, economical, some natural systems, etc. However, the following conditions are the reason of the necessity of adaptation of these methods:

- nonstationary and poorly defined processes;
- the relations between the components of the system are poorly formalized;
- the degree of determinancy of the available information can be arbitrary.

It seems, that this problem of adaptation can be solved by means of generalization and development of iterative methods, taking into consideration from step to step a more accurate definition of model and basic information. For example, the poorly formalized relations can be defined by means of methods of Pattern Recognition.

The characteristic properties of iterative procedures are the following:

- 1). The solution has a form of consequence.
- 2). Simplicity of iteration.
- 3). Stability in spite of random noise.
- 4). Non-sensibility of iterative operator to the change of basic information.
- 5). The slow convergence (it follows from 1).- 4).).

It is possible to assume that the iterative sequence $\{x_k\}$ is given in general by means of a multivalued transformation F of type

$$x_{k+1} \in F(x_k)$$

rather than by an iterative operator.

In model $\mathcal{M}$ (of plant $\mathcal{M}$) of Mathematical Programming we point out its ingredients:

- x is the state space vector;
- $f(x)$ is the criterion function;
- s is the defining information;
- ϕ is the system of restrictions.

Thus, if, e.g., $\mathcal{M}$ is the Linear Programming model of type

$$\max \{ \langle c, x \rangle : Ax \leq b, x \geq 0 \} \quad (1.1)$$

then $s = (c, A, b)$, $f(x) = \langle c, x \rangle$, ϕ is the set of relations in between s and x .

Let F be the transformation (iterative operator):

$$[x_{t+1} \in F(x_t) \quad (\forall t)] \Rightarrow [\{x_t\}' \subset \text{Arg } Z] \quad (1.2)$$

where Z is the Mathematical Programming Problem. In the transformation F we extract in explicit form the information ingredient s of model $\mathcal{M}$: $F = F[s]$.

If $s = s_t$ is the sequence then we get the process

$$x_{t+1} \in F[s_t](x_t); \quad (1.3)$$

s_t is the parameter of F .

We give a further generalization of process (1.3):

$$x_{t+1} \in \Phi[s_t, u_t](x_t), \quad (1.4)$$

where u_t is the controlling vector. This vector can be the formal (vector, algorithm, etc.) or non-formal (expert) object.

2. Iterative transformation including the block of pattern recognition for taking into account the poorly formalized restrictions.

We give a further generalization of process (1.4) which presupposes the possibility of a step by step innovation procedure for the poorly formalized constraints. Here we consider, for example, the Mathematical Programming Problem

$$\max \{ f(x) : x \in \mathcal{M}, g(x) \geq 0 \}. \quad (2.1)$$

Let the restriction $g(x) \leq 0$ be poorly formalized: the function $g(x)$ is not given. The expert $(\equiv [*])$ gives the information about $g(x)$ for each $x \in R^n$:

$$[*](x) = \begin{cases} +1 & \text{if } g(x) \leq 0, \\ -1 & \text{if } g(x) > 0. \end{cases}$$

If $g_t(x)$ is the approximation of $g(x)$, $g_t(x) \in G$, then s_t is the informational ingredient of the model

$$\max \{ f(x) : x \in M, g_t(x) \leq 0 \}. \quad (2.2)$$

Let the elements x_t , $g_t(x) \in G$ and sets M_t , N_t be given:

$$g_t(x) \leq 0 \quad (\forall x, x \in M_t), \quad g_t(x) > 0 \quad (\forall x, x \in N_t).$$

We find x_{t+1} , M_{t+1} , N_{t+1} , $g_{t+1}(x)$, s_{t+1} :

$$x_{t+1} \in \Phi[s_t, u_t](x_t),$$

$$M_{t+1} = \begin{cases} M_t \cup \{x_{t+1}\} & \text{if } [*](x_{t+1}) = +1; \\ M_t & \text{if } [*](x_{t+1}) = -1; \end{cases}$$

$$N_{t+1} = \begin{cases} N_t & \text{if } [*](x_{t+1}) = +1; \\ N_t \cup \{x_{t+1}\} & \text{if } [*](x_{t+1}) = -1; \end{cases}$$

$g_{t+1}(x) \in G$, $g_{t+1}(x)$ is a discriminant function for M_{t+1} and N_{t+1} ;

s_{t+1} is the informational ingredient of (2.2) by $t \rightarrow t+1$.

We get the tracking procedure

$$x_{t+1} \in \Phi[s_t, u_t](x_t).$$

3. Examples.

Here we consider the convex programming problem

$\max \{f[s](x) : f_j[s](x) \leq 0 \quad (j \in \overline{1, m})\} \equiv \tilde{f}[s](x); (3.1)$
 $-f, f_j$ are convex functions $(\forall s)$ and continuous functions concerning $x = (s; x)$.

3.1. Normal processes.

The nature of nonstationarity of s in (3.1) may be of various origin. Here we consider the case, when s is characterized by an uncertainty which may be gradually decreased and which hence allows an innovation of the estimate for vector s itself. Formally,

$s_t \in S_t$, $S_t \supset S_{t+1} \quad (\forall t)$. If $s_t = \arg \min \{\|s\| :$

$s \in S_t\}$ then we call $\{s_t\}$ a normal sequence.

Let S_t be convex and closed $(\forall t)$, $\bigcap S_t = S \neq \emptyset$, $\{s_t\}$ a normal sequence. Then $s_t \rightarrow \bar{s} \in S$. We define

$$\mathcal{M}(s) = \{x : f_j[s](x) \leq 0 \quad (j \in \overline{1, m})\}.$$

Theorem 3.1. Let $F[s](x)$ be a closed multivalued $\mathcal{M}(s)$ - Feyer - transformation of R^n into set of all subsets of R^n ; the system $f_j[\bar{s}](x) \leq 0 \quad (j \in \overline{1, m})$ satisfies the Slater constraint qualification; let us assume that $\{s_t\}$ is a normal sequence, $\bar{s} = \lim s_t$. Then for the sequence

$$x_{t+1} \in F[s_t](x_t) \quad (3.2)$$

exists the element $\bar{x} \in \mathcal{M}(s)$ such as $\lim x_t = \bar{x}$.

Theorem 3.2. We assume the conditions of theorem 3.1 and (3.1) is solvable for $s = \bar{s}$, $\bar{s} = \lim s_t$, s_t is a normal sequence, the function $f[\bar{s}](x)$ is strongly convex. Define

$$\varphi[s, u] = \operatorname{Arg} \max_x \left\{ f[s](x) - \sum_{j=1}^m u_j f_j^{+2}[s](x) \right\},$$

$$u = (u_1, \dots, u_m) \geq 0, \quad (3.3)$$

$$x_{t+1} \in \varphi[s_t, u_t], \quad u_j^{t+1} = u_j^t + f_j^+[s_t](x_t),$$

$$j = 1, \dots, m.$$

Then

$$x_t \rightarrow \bar{x} = \arg \max_x \{f[\bar{s}](x) : x \in \mathcal{M}(\bar{s})\}.$$

3.2. The use of Pattern Recognition methods.

We consider the process (2.3) for (2.1).

Theorem 3.3. Assume $\mathcal{M}$ is a limited closed convex set;
 $g(x)$ is an affine function; $\mathcal{M} \cap \{x: g(x) \leq 0\} \neq \emptyset$;
 G is the set of all affine functions; $g_{t+1}(x) \in G$;

$\{x: g_{t+1}(x) = 0\} = \mathcal{H}_{t+1}$; $\mathcal{H}_{t+1} = \arg \max_{\mathcal{H}} \{ \rho :$
 $\| \mathcal{H} - \mathcal{M}_{t+1} \| = \| \mathcal{H} - \mathcal{N}_{t+1} \| = \rho$, $\mathcal{H}$ is discri-
 minant hyperplane for $\mathcal{M}_{t+1}, \mathcal{N}_{t+1} \}$; $x_t = \arg (2.2)$;
 $x_t \in \mathcal{H}_t$ ($\forall t$) . Then $\lim \| \text{co } \mathcal{M}_t - \text{co } \mathcal{N}_t \| = 0$.

3.3. Processes under incomplete information.

Various interpretations of essence may be given for the processes under incomplete defining information. These, for example, are the following situations:

- the conduct of the modelled system is determined, but the information on the system is incomplete;
- the nonstationary properties of the defining information are in exact correspondence with those of modelled system;
- the information nonstationarity is an artificial technique due to the computational algorithm, when the informational ingredient acts as the control for the algorithm;
- the nonstationary informational properties are due to poor formalizability of the restrictions.

The Pattern Recognition methods may be used to construct the process models in the form of "input-output" transformations (with no assumptions of the availability of any interior descriptions of the system whatsoever) if the number of different output of the system is finite and is sufficiently small in relation to the capacity of the set of all different outputs. As an example one may demonstrate the problems of estimation by Pattern Recognition methods of the parameters for substance mixtures.

References

- I. И.И. Ерёмин, Вл.Д. Мазуров. О нестационарных процессах математического программирования. - In: "Нестационарные процессы математического программирования", Works of Institute of Mathematics and Mechanics, N 14, Sverdlovsk, 1974.