SOME STATEMENTS AND WAYS OF SOLVING DYNAMIC OPTIMIZATION PROBLEMS UNDER UNCERTAINTY

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Very many optimization problems, as related with system development, have a dynamic character and they are to be solved under uncertainty when for the part of initial information the probable description is neither known exactly nor available at all.

As is known [1], under uncertainty by a formalized solution of the problem it is generally impossible to find a single optimal variant. One may get but several rational variants that are the best on diverse criteria (Wald, Laplace, Savage et al.). But the final choice among them is made by experts based on their experience and intuition. Methods for solving such problems are insufficiently elaborated and they are investigated at the Siberian Power Institute, Ac.Sc. [2, 3 et al.].

Under uncertainty it is expedient to make final decisions only for those actions (variants, objects, etc.) which are urgent and should be realized immediately. These actions concern usually the nearest time period called further "the first step". The other part of the studied period (period of "afteraction") is to be taken for the account of the consequences of these immediate actions, but the final decisions of the system development for this period can be made later.

At such a statement the question of the variant formation way (trajectory) of system development for the period of "afteraction" is complex enough. Apparently, such variants have to correspond on the one hand to actions of "the first step" (the following system development depends upon the choice of immediate objects) and on the other hand to impending conditions of system development (later decisions are made upon concrete situations). However taking into account that at making subsequent decisions uncertainty of information is also available and so the final choice is made by men (intuitively, informalized), it is impossible to foresee these decisions and definitely select the variants of system development for the period of "afteraction". Therefore here can be proposed diverse approximate ways [3]. Some possible statements will be considered below. The economical effect in the mentioned problems can be estimated by the functional:

$$J = \sum_{t=1}^T J_t(x_{t-1}, x_t, y_t), \quad (1)$$

where: T = total number of time intervals ("steps");
 y_t = vector of indefinite variables that characterizes "nature states" at time interval t ;
 x_{t-1} and x_t = vectors of parameters to optimize at the beginning and at the end of time interval t .

The first statement, the most simple (and the most rough) is to make the dynamic problem a statical one. This is simply enough when the nature state y_s (s = number of nature state) is taken as a concrete realization of vector of random (indefinite) variables for the whole studied period T :

$$y_s = (y_{1s}, \dots, y_{ts}, \dots, y_{Ts}) \quad (2)$$

and the possible action of man x_i (i = number of action or variant) is the choice of a single-valued trajectory of the system development for the whole period T :

$$x_i = (x_{i1}, \dots, x_{ti}, \dots, x_{Ti}) . \quad (3)$$

At such a statement of the problem the general sequence of its solution can be the same as for the statical problems. Some realisations (2) and several possible actions (3) are chosen. This is done by the formal methods [3] or heuristic means.

For each action x_i and nature state we estimate the expenditures β_{is} by functional (I). This results in a "pay matrix" $\| \beta_{is} \|$ on the basis of which a variant for realisation is chosen.

The shortcomings of this statement are obvious.

The second statement (and all following ones) supposes that the aim of the dynamic problem solution is the choice of an expedient action for the nearest time period ("first step") only. One of the possible ways of solving such a problem is [2] as follows:

(a) several possible actions are planned at the first step

$$x_i \equiv x_{i1} ;$$

(b) a series of realisations (2) for the whole studied period are chosen as before;

(c) for each planned action at the first step and the chosen nature state determinate optimization calculations are carried out for all steps beginning with the second one; this gives a pay matrix $\| \beta_{is} \|$, where β_{is} = value of objective functional of the i -th action and the s -th state of nature is determined as

$$\beta_{is} = \beta_i(x_i, y_{is}) + \min_{x_t} \sum_{t=2}^T \beta_t(x_{t-1}, x_t, y_{ts}) . \quad (4)$$

The first summand in (4) is equal to the functional value in the first interval at fixed x_i and y_{is} ; the second summand is an extreme value of the functional at the period of afteraction;

(d) rational actions at the "first step" are chosen on the basis of this matrix using special methods and criteria of solutions theory.

The merit of such a statement is the flexible and rather logical adaptation of various actions to different states of nature.

But it is laborious and doesn't quite correspond to the real situations. The "suboptimisation" of system development at the second and following steps at these or those conditions agrees with the assumption that further on (after the first step) we shall precisely know the impending conditions and shall act therefore optimally. In reality, making decisions at the next steps we shall be again under uncertainty and so cannot act optimally.

Taking into account the second circumstance we may not sometimes demand such a strict "suboptimisation" of system development at the period of "afteraction" and may take a simplified third statement. Here some possible actions at the first step and several nature states (2) for the whole studied period are also planned.

The difference is in the account method of the afteraction period and pay matrix calculation. For each action at the first step several (two-five) variants of system development at the period of afteraction are planned (numbers of these variants are designated by j):

$$(x_{2ij}, \dots, x_{tij}, \dots, x_{rij}) \quad j = 1, 2, \dots \quad (5)$$

Further, for all variants of j functional (I) is calculated at each chosen nature state. These calculations become not optimization but only "value" (at fixed values x_{tj}) calculations not as in the above-mentioned second statement. Such a calculation determines the expenditure value:

$$j_{isj} = j_i(x_i, y_{is}) + \sum_{t=2}^T j_t(x_{t-1ij}, x_{tij}, y_{ts}). \quad (6)$$

Now we suggest taking into account the adaptation of system development at the period of afteraction at diverse nature states by the choice of such a variant (from the mentioned number) for which the expenditures at the given nature state will be minimal:

$$j_{is} = \min_j j_{isj}. \quad (7)$$

This agrees with the assumption that at the second and following steps we shall choose this or that variant of the further system development dependent upon the factual conditions, but this circumstance is accounted here in the simplified form. The values j_{is} obtained by relation (7) are used for filling the pay matrix $\|j_{is}\|$. Its following analysis and choice of rational actions is going on in the usual order.

The fourth statement differs from the above ones as follows: each rational action at the first step (optimal on the corresponding criterion) is determined at the assumption that at all steps during the afteraction period the choice is implemented by this criterion. In other words, we choose a certain criterion, for instance that of Wald, and optimize the system development for the whole studied period. Optimal action obtained for the first step belongs to the rational actions. Then we optimize the system development (also for the whole studied period) by another criterion (for example the Savage criterion) and we get one more rational action at the first step, etc.

Having fixed the criterion of optimality K we come to the problem similar to stochastic dynamic problems (at the known probable description) when the extremum of the mathematical expectation of the functional is searched for. It can be solved using ideas and methods of dynamic programming. For instance, if we take the Wald criterion (minimax expenditures) then for each t -th time interval the following functional equation must be solved (starting from the end):

$$\overset{\circ}{K}_t^r(x_{t-1}) = \min_{x_t} \max_{y_t} [j_t(x_{t-1}, x_t, y_t) + \overset{\circ}{K}_{t+1}^r(x_t)], \quad (8)$$

where:

$\overset{\circ}{K}_t^r$ = minimal possible criterion value for the period from the beginning of the t -th interval to the end of the studied period (it depends upon the vector value x_{t-1}).

The solution of such a problem at the continuous values of vectors x and y , especially at their great dimension goes with great (possibly even unsolvable) computational difficulties. But, if a finite and not too large amount of vector values x_t is taken, characterising diverse system states and also some limited number of nature states (making the problem discrete), then the solution becomes practically possible. In [3] there is the algorithm of problem solution in such a statement on the base of which the computer program is worked out.

The expedient usage of this or that mentioned statements depends

upon the peculiarities of the problem: its general laboriousness (accounting for the computers used), the possible dates and the solution periodicity, etc. For each problem this question has to be specified. The second statement is the most widely used now.

In [3] there are examples of problem solutions for power systems accounting for the uncertainty of initial information.

References.

1. Льюс Р.Д., Райфа Х. Игры и решения. М., изд-во иностр. лит., 1961.
2. Беляев Л.С. Вопросы оптимизации больших систем в вероятностных ситуациях. "Экономика и математические методы", т.3, вып.6, 1967.
3. Сборник "Вопросы построения автоматизированных информационных систем управления развитием электроэнергетических систем", вып.1. Учет неопределенности исходной информации, СЭИ СО АН СССР, Иркутск, 1973.