STUDIES IN MODELLING AND IDENTIFICATION OF

DISTRIBUTED PARAMETER SYSTEMS⁺

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INTRODUCTION

The paper presents a summary of some recent studies conducted in the department of automatic control in the field of modelling and identification of complex distributed parameter systems corresponding to engineering applications. These are :

- compact cross-flow heat exchangers in aircraft air-conditioning systems
- fixed catalyst bed chemical reactor
- identification in microwave guides (microstrip lines)

I. DYNAMIC MODELLING AND IDENTIFICATION OF A CROSS-FLOW COMPACT HEAT EXCHANGER

The general subject is a detailed analysis of the dynamic behaviour of an on-board air conditioning system in aircraft industry. An example

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of such a system, a boot-strap system is given in Figure 1.

In this system, there is a regulation to be determined. The scaling of the air-conditioning system is made by guaranteeing the static performances in the flight domain. The regulation study is always built upon a simple transfer function between the cabin temperature and the position of the regulator valve. As for the dynamic behaviour of the system, the most interesting elements are the heat exchangers. These exchangers - precooling and main exchangers - are compact cross flow compact exchangers for which the performances are high for a given volume but for which the geometrical complexity is also high and unknown from the user. An example with two elements is given in Figure 2.

While the statics of such exchanges have been extensively studied for all kinds of configurations and internal geometry (see, for example, KAYS and LONDON, Ref. 2), the dynamics are not well documented, one of the reasons being that a cross flow set up leads to a characterization with three or four independant variables. The subject presented in this paper is then a detailed modelling in temperature of the exchanger.

I.1 - Characterization of an element and equations

Geometric and physical simplifying hypothesis are needed to obtain a tractable model of the object. The main assumptions are :

- the exchange surface is plane and rectangular
- the temperatures are uniform on the z-axis
- the physical parameters (velocities, of the fluids, specific heats,...) are constant.

Several models can be written according whether one considers outside walls or not, or whether one or two fluids are considered as mixed (uniformisation of the temperature on the direction perpendicular to the flow)

The practical case (no mixing, perfect insulation) is described by three temperatures (the two fluids and the separation wall) with the three independent variables x, y, t . Written in dimensional form, the equations are :

$$\left\{ \begin{array}{l} \frac{\partial \theta_1}{\partial x} + \frac{\partial \theta_1}{\partial t} = a_1 (\theta_p - \theta_1) \\ \frac{\partial \theta_2}{\partial y} + r \frac{\partial \theta_2}{\partial t} = a_2 (\theta_p - \theta_2) \\ \frac{\partial \theta_p}{\partial t} = b (\theta_1 - \theta_p) + r' b (\theta_2 - \theta_p) \end{array} \right.$$

with the following definitions :

$$x = \frac{x}{x_0}, \quad y = \frac{y}{y_0}, \quad t = \frac{t}{\tau_1} \quad \tau_1 = \text{transit time} \quad l = \frac{x_0}{v_1}$$

$$\tau_2 = \frac{y_0}{v_2}, \quad a_1 = \frac{\tau_1}{R_1 C_1}, \quad a_2 = \frac{\tau_2}{R_2 C_2}, \quad r = \frac{\tau_2}{\tau_1}, \quad r' = \frac{R_2}{R_1}, \quad b = \frac{\tau_1}{R_1 C_p}$$

("RC" = thermal time constants)

1.2 - Solution by Laplace transforms

The analysis of the dynamic behaviour leads to solve the above equations. This has been done after successive Laplace transformations giving the transfer functions of the system. The advantages of this approach are the possibility of obtaining independently the transforms of the temperatures in any point (x, y) , to calculate the mean temperatures at the outputs $x = 1$ and $y = 1$ which are the only temperatures usually measurable, and also to avoid the difficulties due to discontinuities and time consuming of difference methods. The transfer functions are however quite complex.

With zero initial conditions and the boundary conditions :

$$\theta_1 (0, y, t) = u(t) = \text{step function}$$

$$\theta_2 (x, 0, t) = 0$$

We have the output transfer functions :

- direct response :

$$H_1(p, y) = \frac{\theta_1(1, y, p)}{\theta_1(0, p)}$$

$$= e^{-\lambda_1} \left[e^{-\lambda_2 y} I_0(2 \sqrt{\alpha_1 \alpha_2 y}) + \lambda_2 \int_0^y e^{-\lambda_2 u} I_0(2 \sqrt{\alpha_1 \alpha_2 u}) du \right]$$

- cross response

$$H_2(p, x) = \frac{\theta_2(x, 1, p)}{\theta_1(0, p)}$$

$$= \frac{\alpha_2 e^{-\lambda_1 x}}{\lambda_2} \left[e^{\frac{\alpha_1 \alpha_2}{\lambda_2} x} - e^{-\lambda_2} \left\{ I_0(2 \sqrt{\alpha_1 \alpha_2 x}) + \right. \right.$$

$$\left. \left. + \frac{\alpha_1 \alpha_2}{\lambda_2} e^{\frac{\alpha_1 \alpha_2 x}{\lambda_2}} \int_0^x I_0(2 \sqrt{\alpha_1 \alpha_2 u}) e^{-\frac{\alpha_1 \alpha_2 u}{\lambda_2}} du \right\} \right]$$

where :

$$\lambda_1 = p + a_1 - \frac{b a_1}{p + b(1 + r')}$$

$$\lambda_2 = r p + a_2 - \frac{r' b a_2}{p + b(1 + r')}$$

$$\alpha_1 = \frac{r' b a_1}{p + b(1 + r)}$$

$$\alpha_2 = \frac{b a_2}{p + b(1 + r')}$$

I_0 = Bessel function

Some interesting properties can be deduced from these transfer functions in the simplest cases, especially the separation of transport terms (time lag and discontinuity). However, the expressions are too complex to deal with analytically. Numerical inversion, through Dubner and Abate algorithm coupled with a Fast Fourier transform, has been implemented to obtain the transient responses. Typical responses are represented in Figure 3.

I.3. - Approximate characterization by rational transfer functions through identification

The transfer functions are too complex to deal with and to be a practical representation when inserting the element in a complete system or for synthesis purpose. Furthermore the temperatures are not homogeneous at the outputs (in case of no mixing) and what is needed is a global representation as simple as possible; we will then consider the output mean temperatures which are in the other hand, the only measurable variables. After integrating and expanding the Bessel functions, the exact expressions of the corresponding transfer functions are :

$$H_{1m}(p) = e^{-\lambda_1} \left(1 - \frac{\alpha_1 \alpha_2}{\lambda_2} \sum_{n=0}^{\infty} \frac{(\alpha_1 \alpha_2)^n}{n! \lambda_2^n} + e^{-(\lambda_1 + \lambda_2)} \sum_{n=1}^{\infty} \frac{(\alpha_1 \alpha_2)^n}{n! \lambda_2^{n+1}} \right)$$

$$\sum_{k=0}^{n-1} \sum_{j=0}^k \frac{\lambda_2^j}{j!}$$

$$H_{2m}(p) = \alpha_1 \left[\sum_{n=0}^{\infty} \frac{(\alpha_1 \alpha_2)^n}{(\lambda_1 \lambda_2)^{n+1}} \left(1 - e^{-\lambda_2} \sum_{j=0}^{\infty} \frac{\lambda_2^j}{j!} \right) \left(1 - e^{-\lambda_1} \sum_{k=0}^n \frac{\lambda_1^k}{k!} \right) \right]$$

These expressions have been approximated by rational transfer functions

after extracting the time lag and discontinuity due to transport phenomenon according to (other cases have been investigated) :

$$H_{1m}(p) = e^{-P} (e^{-a_1} + F(p)) \text{ with } F(p) = \frac{K_1 (1 + \frac{P}{Z_0})}{1 + 2z \frac{P}{\omega_1} + \frac{P^2}{\omega_1^2}}$$

$$H_{2m}(p) = F_2(p) = \frac{K_2 (1 + \frac{P}{Z'_0})}{(1 + 2z' \frac{P}{\omega_2} + \frac{P^2}{\omega_2^2})(1 + T_p)(1 + \frac{P}{P_0})}$$

These approximations are justified by the shape of the transient responses and the properties of the limiting case, - case without separation wall.

The identification has been made in the time domain by a quasi-Newton algorithm with a mean square error criterion. An example of results is given in figure 4. Comparison in the frequency domain has also been made in order to observe the influence of high frequency modes not contained in the domain of interest and, in fact, over all frequencies, the high modes being severely attenuated.

I.4 - Further investigations and conclusion

Different kinds of setting-up of elementary units have been theoretically and numerically investigated -parallel, counter current, mixed, two or three elements - . Each element is then represented as a linear two-input, two-output multi-variable system with its exact and approximate matrix transfer function. Comparisons of the two representations are very good and then justify the method of analysis taken for the elementary unit.

In order to reduce the order of characterization of the assembly which is probably too rich, we applied again the identification procedure. It is interesting to note then that all the transfer functions are satisfactorily represented by generalized second order functions

(plus a lag-time for the direct response). It is this representation which has been used for the study of an experimental heat exchanger.

Then the important problem of establishing relations between physical parameters (introduced in the internal representation by partial differential equations) and the dynamic parameters obtained by identification and the problem of reducing then the number of parameters has been investigated. This type of investigation has been made by KAYS and LONDON, in the static field but is here more difficult and awkward. A systematic approach by varying the physical parameters has given coherent results but the relations which are established are too approximate and concern too few data to conclude.

A simpler representation by gains and time constants has then also been performed. The validity of the representations by frequency domains has been established. Experimental work has been conducted at the Centre d'Essais Aéronautique de Toulouse on a heat exchanger whose diagram is represented below.

The study presents a method of approach for the detailed analysis and identification of the dynamic behaviour of cross flow compact heat exchangers used in aircraft industry and also in other industries (gas turbine applications.....) The approach has proved to be consistent on a concrete heat exchanger but more data, especially higher frequency components in the available inputs and other applications on real exchangers are necessary to conclude.

II. MODELLING - SIMULATION - IDENTIFICATION OF A FIXED BED CATALYST CHEMICAL REACTOR

II. 1 The bidimensional model

The model that we consider in this application concerns a cylindrical tubular reactor with diffusion terms in the stationary case. A real pilot reactor is available for the measurements.

The balance equations for mass and energy are :

1. fluid

$$\left\{ \begin{array}{l} \epsilon_R Da \frac{\partial^2 C}{\partial X^2} + \epsilon_R Dr \left\{ \frac{\partial^2 C}{\partial R^2} + \frac{1}{R} \frac{\partial C}{\partial R} \right\} - \epsilon_R u \frac{\partial C}{\partial X} = S_V K_g (C - C_s) \\ \epsilon_R K_a \frac{\partial^2 T}{\partial X^2} + \epsilon_R K_R \left\{ \frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right\} - \epsilon_R \rho_f Z_f \frac{\partial T}{\partial X} = S_V h_g (T - T_s) \end{array} \right.$$

where C and T are the concentration and temperature on the fluid, C_s and T_s the concentration and temperature at the surface of catalyst particles

2. catalyst

$$\left\{ \begin{array}{l} S_V K_g (C - C_s) = V_V f (C_s, T_s) \\ S_V h_g (T - T_s) = V_V \Delta H_f (C_s, T_s) \end{array} \right.$$

where f represents the kinetics law.

The boundary conditions are :

$$1. \quad 0 < X < L$$

$$R = 0 : \left\{ \frac{\partial C}{\partial R} = \frac{\partial T}{\partial R} = 0 \right.$$

$$R = R_0 : \begin{cases} \frac{\partial C}{\partial R} = 0 \\ -K_r \frac{\partial T}{\partial R} = h_w (T - T_w) \end{cases} \quad T_w : \text{wall temperature}$$

$$2. \quad 0 < R < R_0$$

$$X = X_0 : \begin{cases} P_f^{uz_f} (T_0 - T) = -K_a \frac{\partial T}{\partial X} \\ u (C_0 - C) = -D_a \frac{\partial C}{\partial X} \end{cases}$$

$$X = L : \begin{cases} \frac{\partial C}{\partial X} = \frac{\partial T}{\partial X} = 0 \end{cases}$$

II. 2 Equations in normalized form

With the definitions :

$$\xi = \frac{C_0 - C}{C_0}, \quad \theta = \frac{T - T_0}{T_0}, \quad X = \frac{X}{L}, \quad r = \frac{R}{R_0}$$

The equations are written :

$$\left\{ \begin{aligned} a_1 \frac{\partial^2 \xi}{\partial x^2} + a_2 \left[\frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r} \frac{\partial \xi}{\partial r} \right] - \frac{\partial \xi}{\partial x} &= a_3 (\xi - \xi_s) \\ a_3 (\xi - \xi_s) &= \alpha_1 f(\theta_s, \xi_s) \\ b_1 \frac{\partial^2 \theta}{\partial x^2} + b_2 \left[\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right] - \frac{\partial \theta}{\partial x} &= b_3 (\theta - \theta_s) \\ b_3 (\theta - \theta_s) &= \gamma_1 f(\theta_s, \xi_s) \end{aligned} \right.$$

with the boundary conditions :

$$\left. \frac{\partial \xi}{\partial r} \right|_{r=0} = \left. \frac{\partial \theta}{\partial r} \right|_{r=0} = 0$$

$$\left. \frac{\partial \xi}{\partial r} \right|_{r=1} = 0, \quad \left. \frac{\partial \theta}{\partial r} \right|_{r=1} = -Bi (\theta - \theta_w)$$

$$\xi - a_1 \left. \frac{\partial \xi}{\partial x} \right|_{x=0} = 0, \quad \theta - b_1 \left. \frac{\partial \theta}{\partial x} \right|_{x=0} = 1$$

$$\left. \frac{\partial \xi}{\partial x} \right|_{x=1} = \left. \frac{\partial \theta}{\partial x} \right|_{x=1} = 0$$

Kinetics laws :

The model can be used for a reaction with a kinetics law which is an arbitrary function of the temperature and concentrations of chemical species occurring in the reaction. Two main laws have been used for the oxydation of SO_2

1. A first law which has been determined on a laboratory reactor (Ref. 4)

$$f(\xi_s, \theta_s) = g(\xi_s, \theta_s) \times F_{eq} \times \phi$$

with :

$$g(\xi_s, \theta_s) = 72500 \exp^{-6.18 \cdot 10^3 / T_0 \theta_s} \cdot P_{SO_2}^{0.75} \cdot P_{O_2}^{1.1} \cdot P_{SO_3}^{-0.67}$$

$$F_{eq} = 1 - \frac{P_{SO_3}/K_p}{P_{SO_2} P_{O_2}^{1/2}}, \quad k_p = 10^{(5005/T_s \theta_s - 4.743)}, \quad \phi = 1$$

and where :

$$P_{SO_2} = \frac{a(1 - \xi_s)}{1 - a\xi_s/2}, \quad P_{SO_3} = \frac{a\xi_s + \epsilon}{1 - a\xi_s/2}, \quad P_{O_2} = \frac{1 - a - a\xi_s/2}{1 - a\xi_s/2}$$

2. Mars and Maesson law

$$f = g(\xi_s, \theta_s) \times F_{eq} \times \phi$$

with :

$$\left\{ \begin{aligned} g &= 6.56 \cdot 10^9 e^{-\frac{13853}{T_s \theta_s}} \times \frac{P_{O_2} \cdot P_{SO_2}}{P_{SO_3}} \times \left(1 + \sqrt{2.3 \cdot 10^{-8} e^{\left(\frac{13689}{T_s \theta_s} \frac{P_{SO_2}}{P_{SO_3}}\right)^{-2}} \right)} \\ \phi &= 72 \cdot 10^{-4} (11.72 \times 2.808 \cdot 10^{17} e^{-\frac{27542}{T_s \theta_s}} \times T_s \frac{1 + k_p}{K_p})^{1/2} \times \eta \\ \eta &= \left(\frac{3}{Th3\phi} - \frac{1}{\phi} \right) / 3 \phi \end{aligned} \right.$$

II.3 Problems on study

The problems which have been studied or are still on study are :

- simulation of the model
- sensitivity studies
- simulation of different kinetics laws
- comparison with experimental results
- identification of the Peclet and Stanton numbers (mass and heat) a_i , b_i , and the Biot number (B_i)

The measurements available are :

- the temperatures θ_{ij} at a limited number of discrete points

$$\theta_{ij} = \theta (x_i, r_j)$$

- the mean concentration of SO_2 at the output $x = 1$

$$\xi_m = 2 \int_0^1 r \xi (1, r) dr$$

II. 4 Discretization - Simulation

After finite difference discretization, we obtain :

$$\left\{ \begin{array}{l} L_{\xi} \xi = -\xi_S + Z_{\xi} \\ L_{\theta} \theta = -\theta_S + Z_{\theta} \\ \xi = g_{\xi} (\xi_S, \theta_S) \quad g_{\xi} = \xi_S + k_1 f (\xi_S, \theta_S) \\ \theta = g_{\theta} (\xi_S, \theta_S) \quad g_{\theta} = \text{-----} \end{array} \right.$$

where L_{ξ} , L_{θ} are block tridiagonal matrix and Z represent the boundary conditions

The system can be written :

$$A \underline{V} = \underline{B} (\underline{V}) + \underline{Z} \text{ with}$$

$$A = \begin{array}{|c|c|c|c|} \hline L_{\xi} & 0 & 0 & 0 \\ \hline I & -I & 0 & 0 \\ \hline 0 & 0 & L_{\theta} & 0 \\ \hline 0 & 0 & I & -I \\ \hline \end{array} \quad B = \begin{array}{|c|c|c|c|} \hline 0 & -I & 0 & 0 \\ \hline 0 & NL(\xi_S) & 0 & 0 \\ \hline 0 & 0 & 0 & -I \\ \hline 0 & 0 & 0 & NL(\theta_S) \\ \hline \end{array} \quad \underline{V} = \begin{array}{|c|} \hline \xi \\ \hline \xi_S \\ \hline \theta \\ \hline \theta_S \\ \hline \end{array}$$

where $NL(\xi_S)$ and $NL(\theta_S)$ represent the non-linear kinetics law.

The method of resolution associates two imbricate iterative schemes, one for the linear equation (Gauss - Seidel - relaxation) one explicit type scheme for the non linearities but imbricate with the first scheme and with relaxation. We have, for example, for ξ

$$\underline{\xi} |^k = L_{\xi}^{-1} (\underline{Z}_{\xi} - \xi_S |^{k-1})$$

$$\begin{aligned} \underline{\xi} |^k_i = \underline{\xi} |^{k,1}_i &+ \gamma D_i^{-1} \left[-E_i \underline{\xi} |^{k,1-1}_{i-1} - F_i \underline{\xi} |^{k,1-1}_{i+1} - D_i \underline{\xi} |^{k,1-1}_i \right. \\ &\left. + \underline{Z}_{\xi} |^{k,1-1}_i - \xi_S |^{k,1-1}_i \right] \end{aligned}$$

and $\underline{\xi}_S$ by :

$$\left\{ \begin{aligned} \underline{\xi}_S |^k &= \underline{\xi} |^{k-1} - k_1 f \left(\underline{\xi}_S |^{k-1}, \theta_S |^{k-1} \right) \\ \underline{\xi}_S |^{k+1,1} &= \underline{\xi}_S |^{k,1}_i + \omega_2 \left(\widetilde{\underline{\xi}_S} |^{k,1=\max}_i - \underline{\xi}_S |^{k,1}_i \right) \end{aligned} \right.$$

II.5 Results and discussion

A typical result is presented in Fig. 5 (fluid temperature and SO₂ concentration). The temperature evolution in the fluid and at the catalyst surface characterizes a quasi-adiabatic reactor: fast increase of the temperature on the x-axis, up to a top corresponding to the end of the reaction, much slower increase near the reactor wall due to thermal losses. As for SO₂ concentration, it decreases in the first part of the reactor due to the temperature increase; but rapidly it approaches its equilibrium value and the reaction speed is negligible; the reaction then takes place in a domain near

the wall where the equilibrium concentration is slower due to a much lower temperature. There is then a radial displacement of the maximum reaction speed in spite of the homogeneization effect of diffusion terms.

Comparison of the two first kinetics laws have been made for several experimental results. Figure 6 shows an example for the axial temperature (at $r = 0$). The second law is much better, the first law giving a too high reaction speed at the input. Other experiences are on study.

An identification algorithm has been implemented; it is based on the minimization by a Gauss-Newton algorithm of the following index :

$$J(\underline{p}) = \frac{1}{2} (\xi_m(\underline{p}) - \xi_m \text{ meas.})^2 + \frac{\lambda}{2} \sum_{i=1}^I \sum_{j=1}^J (\theta(x_i, r_j) - \theta_{ij \text{ meas}})^2$$

where $\underline{p}$ represents the vector of parameters a_i , b_i , and B_i . This identification procedure is now under study but first variations of parameter and a sensitivity study has been made together with the kinetics study in order to approach the experimental results on the pilot reactor.

III. IDENTIFICATION IN MICROWAVE GUIDES (MICROSTRIP LINES)

III. 1 - INTRODUCTION

This research is motivated by the increasing use of microstrip lines at higher microwave frequencies. The classical analysis is based on the T E M approximation from which is derived the characteristic impedance and the propagation wave number . However, this analysis is inadequate at higher frequencies where one has to consider hybrid mode configurations.

In this paper we present briefly the problem of identifying the propagation wave number in microstrip lines. A direct method (but iterative in resolution) by a non linear eigen value problem and an approach by identification from measurements of the electric field (or magnetic field when the measurements are available). A third method, Mittra's method, has also been implemented and has given excellent results.

III.2 - Propagation equations

The electromagnetic wave is defined by its components $\vec{E}_T$, $\vec{H}_T$ in Oxy and $\vec{E}_Z$, $\vec{H}_Z$ on Z axis. Maxwell equations give (in homogeneous field) :

$$\begin{cases} \vec{E}_T = \frac{1}{k^2} [\omega\mu (\vec{u} \wedge \text{grad } H_Z) - \beta \text{ grad } E_Z] \\ \vec{H}_T = \frac{1}{k^2} [\omega\epsilon (\vec{u} \wedge \text{grad } E_Z) + \beta \text{ grad } H_Z] \end{cases}$$

$$\begin{cases} \Delta E_Z + k^2 E_Z = 0 \\ \Delta H_Z + k^2 H_Z = 0 \end{cases}$$

$$\text{where } k^2 = \epsilon\mu\omega^2 - \beta^2$$

In the non-homogeneous case of microstrip lines k^2 depends (through ϵ) on the regions. A case with two regions is represented below.

The measurements will be :

E_y on OA

E_x on AB

III.3 - Identification of β by an eigenvalue problem

In the general case of hybrid modes, the Helmotz equations

$$\Delta E_Z + k^2 E_Z = 0$$

$$\Delta H_Z + k^2 H_Z = 0$$

where k^2 is unknown have to be solved. Then k^2 gives β and E_Z , H_Z gives $\vec{E}_T$ which is compared to measurements.

A first difficulty is due to the fact that k^2 is not the same in the air (k_A^2) and the dielectric (k_D^2)

Writing the boundary conditions :

$$E_Z = \frac{\partial H_Z}{\partial n} = 0 \text{ on the conducting surfaces}$$

$$H_Z = 0, E_Z \text{ symetric on symetry y-axis}$$

and the continuity relations at the separation surface (1)/(2)

$$(E_X)_{\text{air}} = (E_X)_{\text{diel.}} ; (\epsilon_0 E_Y)_{\text{air}} = (\epsilon E_Y)_{\text{diel.}}$$

$$H_{X \text{ air}} = H_{X \text{ diel}}$$

$$H_{Y \text{ air}} = H_{Y \text{ diel}}$$

We obtain the following eigenvalue problem, in terms of $\phi = H_Z$,

$$\eta = \frac{\omega \epsilon_0}{\beta} E_Z$$

$$\begin{vmatrix} A & B(\tau) \\ C(\tau) & D \end{vmatrix} \begin{vmatrix} \phi \\ \eta \end{vmatrix} = k_A^2 \begin{vmatrix} \phi \\ \eta \end{vmatrix}$$

where $\tau = \frac{k_A^2}{k_D^2}$

This is a non linear eigenvalue problem. B and C depend on τ ,
The following procedure has been used :

- 1) τ is chosen
- 2) the linear eigenvalue problem is solved. The minimum modulus eigenvalue k_A^2 and ϕ , η are obtained.
- 3) ω and β are obtained by

$$\omega^2 = \frac{k_A^2}{\epsilon_0 \mu} \frac{1/\tau - 1}{\epsilon_r - 1} \beta^2 = - \frac{k_A^2}{\epsilon_r - 1} \frac{\epsilon_r - 1/\tau}{\epsilon_r - 1}$$

This gives ω, k_A^2, k_D^2 and β as functions of τ

The experimental conditions fixing ω , the procedure is repeated for several τ

The procedure has been implemented and has given some results but does not seem to be a valuable general approach due to difficulties coming from the apparition of spurious modes, the spectral radii constraints and the high dimensionality of the problem.

III.4 Identification from measurements

The identification procedure is as follows :

1. initialization of β
2. calculate E_Z and H_Z
3. from E_Z, H_Z , calculate $\vec{E}_T$ then E_Y on OA and E_X on AB
4. compare to the experimental values. Define an error performance index.
5. minimization of the index with respect to β

A modal representation has been used here for the model

$$E_Z = \sum_{n=1}^N A_n f_n(x) g_n(y)$$

$$H_Z = \sum_{n=1}^N B_n p_n(x) q_n(y)$$

with the boundary conditions, we have :

$$f_n = \cos k_n x, \quad g_n = \text{Sh} \alpha_n y$$

$$p_n = \sin \hat{k}_n x, \quad q_n = \text{Ch} \alpha_n y$$

where :

$$\hat{k}_n = (n - 1/2) \frac{\pi}{L}, \quad \alpha_n = (\hat{k}_n^2 - k^2)^{1/2}$$

then :

$$E_x(y) = \sum_{n=1}^N X_n \text{Sh} \alpha_n y$$

$$E_y(x) = \sum_{n=1}^N Y_n \cos \hat{k}_n x$$

where X_n and Y_n are known expressions of β

The identification of β is made from the measurements on AB :

$$\underline{E}_x = \begin{bmatrix} \text{Sh} \alpha_1 y_1 & \dots & \dots & \text{Sh} \alpha_N y_1 \\ \vdots & & & \vdots \\ \text{Sh} \alpha_1 y_S & \dots & \dots & \text{Sh} \alpha_N y_S \end{bmatrix} \quad \underline{X} = M \underline{X}$$

where S is the number of measurements

Then a least square index gives :

$$\underline{X} = (M^T M)^{-1} M^T \underline{E}_x$$

and β is obtained by the minimization of the norm of the residue :

$$\|R\| = \|\underline{E}_x - M \underline{X}\|$$

Then the coefficients A_n and B_n of the model are obtained from the measurements $\underline{E}_x$ and $\underline{E}_y$. The results are satisfactory.

A special feature is that near T E M frequencies the minimum is flat (independency from β) while at higher frequencies we obtain a peak.

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FIG. 1.

FIG. 2.

$$a_1 = a_2 = 2 \quad b = r = r' = 1$$

Fig. 3

— OBJECT --- APPROXIMATION

FIG. 4

Fig. 5