On Uncertainties of Laser Interferometric Absolute Ballistic Gravimeters Due to Magnetic Effects in the Free-Fall Gravity Measurements

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Abstract

At present, the uncertainty in measurements of free fall acceleration (FFA) by the designed absolute ballistic gravimeter (ABG), developed in VNIIM, attains microgal values ($1 \text{ Gal} = 1 \text{ cm/s}^2$). At such a high level of accuracy, the effects of the interaction of a test body (TB) falling in the vacuum chamber of the ABG in the geomagnetic field could comprise sources of systematic errors. Furthermore, individual units and systems of the ABG itself also can be sources of the magnetic field (MF). We report about more rigorous calculations of the possible effects than those performed in the past. A consecutive self-consistent method of calculation of the desired correction to the FFA has been developed. It results in the differential equation, including not only the field itself, but also its vertical first and second derivatives and variation of the velocity along the trajectory. The desired correction is obtained by its numerical solution on the basis of the parameters of the field generated by the magnet of the stepper motor. The derived correction proved to be at the level of a tenth part of microgal.

Keywords

Absolute ballistic gravimeters · Free fall acceleration · Geomagnetic field · Magnetic induction · Systematic error

1 Introduction

At present, the uncertainty in measurements of free fall acceleration (FFA) by absolute ballistic gravimeters (ABG) attains microgal values ($1 \text{ Gal} = 1 \text{ cm/s}^2$) (e.g., (Vitushkin et al. 2020)). The general operation principles of laser-interference ABG and the principles of absolute measurements of FFA with their help are presented, for example, in (Vitushkin 2014; Absolutnye gravimetry 2017; Niebauer et al. 1995). At such a high level of accuracy, it is necessary to take into account a number of effects that influence the result of FFA

measurements. Such effects, being the sources of systematic errors, include, in particular, the effects of the interaction of a test body (TB) falling in the vacuum chamber of the ABG in the geomagnetic field. Furthermore, individual units and systems of the ABG itself also can be sources of the magnetic field (MF). Those units are the ion (magnetic discharge) pump, electric motor used in some ABG designs, and others. Estimates of a possible effect of inhomogeneous MF on ABG readings have been obtained in the past, e.g. in (Niebauer et al. 1995; Murata 1978, 1980) and references therein.

In (Niebauer et al. 1995), the effect of the interaction of a conducting TB with an inhomogeneous MF has been estimated in the case of the FG5 type ABG. The effect is due to the emerging Foucault currents. The measured fields of the ion pump magnet, servo motor and Faraday optical isolators happened to be less than the geomagnetic field, estimated at $50 \mu\text{T}$, and do not create significant effects on FFA measurements with this type of ABG. It was proved experimentally that there was no any effect of this kind.

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To this end, MFs of the order of 100 μT were applied to the system with a set of Helmholtz coils. This experiment showed absence of effects at a level of 1 μGal . It should be noted that in FG5-type gravimeters, the free-fall path of the TB is about 21 cm and the fall velocity attains 2 m/s.

In (Murata 1978, 1980), the motion of a TB charged due to the supposed contact phenomena in a geomagnetic field under the action of the Lorentz force was considered. The estimates also show a negligible effect of the geomagnetic field on the motion of the TB. As a consequence, gravimeters are usually designed without adjusting for the magnetic effects of eddy currents. In the present work, we report about more rigorous calculations, we have been undertaken for the design of the ABG Grot. The correction is calculated on the basis of the parameters of the MF generated by the magnet of the stepper motor. The derived correction proved to be at the level of a tenth part of microgal. To a great extent, it can be taken into account by calibrating and comparing the gravimeters. However, in some cases, the spread can significantly exceed the indicated value. This may be the case if magnetized parts are used. Therefore, it is important to understand the nature of the correction. Without this understanding, the readings of gravimeters of different types can lead to incorrect results.

Bearing this in mind, in the first place the principles of the ABG operation are reminded in the next section. The method of assessment of the correction is described in Sects. 3 and 4. Numerical estimations of the integral correction to apply to the determined value of FFA are made in Sect. 5. In the last Sect. 6, the results are summarized, conclusions are drawn, and further prospects are considered.

2 Operation Principle of the ABG Grot

Let us evaluate the uncertainty caused by induction currents and their interaction with the geomagnetic field, as well as with the MF produced by individual parts of the gravimeter. The calculation procedure for all types of MF sources is the same. The ABG scheme is usual. The TB falls from a small height of 15 cm. Its motion is monitored by the laser-interference method. The system generates a set of instantaneous values of time $t_i(z_i)$, $i = 1, 2, \dots, n$, into which the TB passes the points z_i , fixed in height and spaced half-wave apart from each other. Up to the vertical gradient γ , the FFA at the point z can be written as follows:

$$g(z) = g_0 + \gamma(z - z_0), \quad (1)$$

where g_0 is the FFA value at the point z_0 . Choosing point z_0 as that where the TB is at the initial moment of time $t_0 = 0$, and designating the velocity of the TB at this point v_0 , we arrive at the following set of coupled equations, relating the

successive times t_i with the points z_i on the trajectory:

$$z_i = z_0 + v_0 t_i + \frac{g_0 t_i^2}{2} + \gamma \left(\frac{v_0}{6} t_i^3 + \frac{g_0}{24} t_i^4 \right). \quad (2)$$

The z_0 and v_0 values at the initial time $t_0 = 0$ form a set of initial conditions, which unambiguously defines the trajectory.

3 Physical Premises for the Uncertainties Arising in the Presence of MF

Let us consider a simple model. A conductive loop drops down city of $v(z)$ along the z axis in an inhomogeneous MF with induction $\mathbf{B}$, remaining in the horizontal plane. Since the MF is inhomogeneous, the magnetic flux through the contour changes during the motion (Fig. 1). As a result, an electromotive force appears in the contour, which in turn causes circular electric current. As is known, the closed frame with current has the magnetic moment $\mathbf{m}$:

$$\mathbf{m} = \frac{S^2}{R} \left(\mathbf{v} \frac{\partial B_z}{\partial z} \right) \equiv A \left(\mathbf{v} \frac{\partial B_z}{\partial z} \right). \quad (3)$$

Fig. 1 Scheme of the fall of the conductive loop in an inhomogeneous MF with induction $\mathbf{B}$. The magnetic flux passing through the loop is conventionally depicted by the number of lines of force. When the loop falls, the number of field lines changes. As a consequence, this induces an electric current along the loop and the conjugate magnetic moment of the circuit $\mathbf{m}$

In Eq. (3), S is the area of the loop, R – its electrical resistance. Parameter A characterizes the physical properties of the TB: its area, electrical conductivity, and the shape in general case. In turn, the produced magnetic moment interacts with the external MF. If the latter is inhomogeneous, then the loop is drawn into the region with the maximum gradient of the MF. Therefore, the force acting on it arises, which can be expressed as follows (Landau and Lifshitz 1960):

$$\mathbf{F} = -\nabla(\mathbf{mB}) \equiv -\nabla U_{\text{pot}}, \quad (4)$$

where U_{pot} is the potential energy of the interaction of the induced magnetic moment with the MF. Dividing the force Eq. (4) by the mass of the loop m_0 , one obtains the desired correction to FFA:

$$\Delta g = -\frac{A}{m_0} \frac{\partial}{\partial z} \left(v B_z \frac{\partial B_z}{\partial z} \right). \quad (5)$$

4 Equation for the Motion of the TB

The fall of the TB is described by the second Newton's law:

$$\frac{d^2 z}{dt^2} = g(z) + a_{e.m.}(z) \quad (6)$$

In Eq. (6), $g(z)$ is the desired FFA (1). $a_{e.m.}(z) \equiv \Delta g(z)$ is the correction Eq. (5) taking into account the geomagnetic and other electromagnetic fields. The correction for the geomagnetic field is small. The correction for the MF of the ionic pump can be leveled by the optimal orientation of its magnet. Let us examine the correction for the MF created by the permanent magnet of the stepper motor.

Performing differentiation in Eq. (5), one can expand Eq. (4) to read:

$$\begin{aligned} \Delta g &= -\frac{A}{m_0} \left(B_z \frac{\partial B_z}{\partial z} \frac{\partial v}{\partial z} + v B_z \frac{\partial^2 B_z}{\partial z^2} + v \left(\frac{\partial B_z}{\partial z} \right)^2 \right) \\ &\equiv F_1 + F_2 + F_3. \end{aligned} \quad (7)$$

As one can see from Eq. (7), the desired correction depends not only on the height z , but also on the velocity of the TB at a given point, and moreover, on its own acceleration in the point, as

$$dv/dz = \frac{1}{v(z)} \frac{d^2 z}{dt^2}.$$

Therefore, $\frac{d^2 z}{dt^2}$ is included in Eq. (6) twice: both on the left and on the right. Thus, the force induced by the MF depends on the acceleration of the TB at the given moment and at the given point in the trajectory, which is itself determined

by this force. For this reason, Eq. (6) must be solved in a self-consistent way, taking into account this interaction of acceleration with the force of electromagnetic induction. To this end, we collect both terms containing $\frac{dv}{dt} = \frac{d^2 z}{dt^2}$ on the left side of the equality. As a result, after simple transformations, one obtains the motion equation for the TB in the presence of the MF as follows:

$$\begin{aligned} \frac{d^2 z}{dt^2} &= \frac{1}{1 + \frac{A}{m_0} \frac{d^2 z}{dt^2} B_z \frac{\partial B_z}{\partial z}} \\ &\times \left\{ g(z) - \frac{A}{m_0} \frac{d^2 z}{dt^2} \left[\left(\frac{\partial B_z}{\partial z} \right)^2 + B_z \frac{\partial^2 B_z}{\partial z^2} \right] \right\}. \end{aligned} \quad (8)$$

By comparing Eq. (8) and Eq. (6), we obtain the desired correction to FFA:

$$\begin{aligned} \Delta g(z) &\equiv a_{e.m.}(z) = \frac{1}{1 + \frac{A}{m_0} \frac{d^2 z}{dt^2} B_z \frac{\partial B_z}{\partial z}} \\ &\times \left\{ g(z) - \frac{A}{m_0} \frac{d^2 z}{dt^2} \left[\left(\frac{\partial B_z}{\partial z} \right)^2 + B_z \frac{\partial^2 B_z}{\partial z^2} \right] \right\} - g(z). \end{aligned} \quad (9)$$

5 Results

The total weight of the TB, including the triple prism, is $P = 58.8$ g. The calculation shows that the combined coefficient $A_t = 0.022 \text{ m}^4 / \text{Ohm}$. Let the stepper motor be in the x, y plane. It is also necessary to take into account the design features of the device, in particular, that its geometric center is offset from the origin by approximately 1.5 cm in the horizontal plane in both x and y directions. The origin is specified by the z axis, which is directed down along the trajectory of the stepper motor in the point. The end point of the trajectory is 17.2 cm below the stepper motor. In the last section of approximately 8 cm length, the trajectory is measured using a laser interferometer, and several hundred points t_i (z_i) are used for processing. The measurements have shown that the MF of the stepper motor is approximately a dipole having components of the magnetic moment:

$$M_x = 0.16 \text{ Am}^2, \quad M_y = 0.017 \text{ Am}^2, \quad M_z = 0.014 \text{ Am}^2.$$

A numerical solution of Eq. (8) with typical initial conditions was obtained by the Runge-Kutta method. A representative trajectory $z(t)$ is plotted in Fig. 2a. It is close to the conventional parabola. The numerical solution also gives us values of instant velocity $v(t)$ along with acceleration $\frac{dv(t)}{dt}$. Once these values are obtained, one can find instant values of the electromagnetic induction correction $a_{e.m.}(z)$, which is plotted in Fig. 2a. By fitting the shape of the obtained trajectory to the experimental points of the trajectory t_i (z_i), one can find parameters v_0, g_0 . The implementation of this algorithm depends on a particular device, and also on external factors such as seismic vibrations of the base, frequency modulation of the laser beam of the gravimeter,

Fig. 2 A plot of a part of the calculated trajectory of the falling TB $z(t)$ (a), and the related correction to FFA (b), solid curve, the dashed curve being a linear fit

etc. It can be implemented in a number of ways. However, in order to evaluate the integral correction to the gravity acceleration $a_{e.m.}(z)$, one can proceed as follows.

Let us use the linear approximation of the curve for $a_{e.m.}(z)$, as shown in Fig. 2b:

$$\Delta g(z) = k + pz \equiv a + b(z - z_m). \quad (10)$$

In Eq. (10), z_m is the point of measuring the g_0 value. A χ^2 fit has resulted in the following values of the parameters: $k = -1.48666 \mu\text{Gal}$, and $p = 0.194504 \mu\text{Gal/cm}$. First, the latter values provide the following value of the desired correction in the lowest point of the trajectory: $a = -0.011 \mu\text{Gal}$. Bearing in mind that typical values of $a_{e.m.}(z)$ in the trajectory of Fig. 2 amount to tenth parts of microgal, one can assume that such a small final value

of the correction might be due to a random compensation of individual terms. In any case, one can conclude that the desired correction is at the level of $0.1 \mu\text{Gal}$. Second, using the p and k values obtained above, one can derive the value of $b = 0.1945 \mu\text{Gal/cm}$ from Eq. (10) in a similar way. The value of b provides a correction to the gradient of FFA γ . At the same time, the value of γ can be measured independently by a relative gravimeter. In principle, this makes it possible to compare the present theory with the experiment.

6 Discussion of the Results

A consecutive self-consistent method of calculation of the desired correction to the FFA has been developed. The resulting Eq. (8) explicitly takes into account the fact that the correction is determined not only by the field itself, but also by its vertical gradient, as well as by the variation of the velocity along the trajectory. For this purpose, a consecutive consideration of all the three terms F_1 , F_2 , and F_3 in Eq. (7) is performed. The correction is obtained by solving Eq. (8) numerically for a representative trajectory of the TB. The need for self-consistency is due to the fact that the correction depends on the acceleration, on which it has a direct effect. It should be noted that in the previous papers (Absolutnye gravimetry 2017; Niebauer et al. 1995; Murata 1978, 1980) only the term F_3 was taken into account, and self-consistency was not considered. The obtained estimates indicate that the desired value of correction is within the tenth part of microgal.

The developed method also makes it possible to calculate the correction to the gravity gradient at the geographical point of measuring. In principle, this can serve as a test for comparing the theory with the experiment, after the gradient has been measured using a relative gravimeter.

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