Monte Carlo Evaluation of the Hausdorff Distance for Shape Matching

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Abstract. In this work, we present a Monte Carlo approach to compute Hausdorff distance for locating objects in real images. Objects are considered to be only under translation motion. We use edge points as the features of the model. Using a different interpretation of the Hausdorff distance, we show how image similarity can be measured by using a randomly sub-sampled set of feature points. As a result of computing the Hausdorff distance on smaller sets of features, our approach is faster than the classical one. We have found that our method converges toward the actual Hausdorff distance by using less than 20 % of the feature points. We show the behavior of our method for several fractions of feature points used to compute Hausdorff distance. These tests let us conclude that performance is only critically degraded when the sub-sampled set has a cardinality under 15 % of the total feature points in real images.

1 Introduction

Shape matching is necessary for several robotic vision tasks such as visual object tracking and visual servoing [1]. Locating a model in an image consists in, given a template composed by a set of features that characterize an object, determining its position and pose in a target image. A pose is the set of parameters, e.g., size and orientation, that defines the object appearance in the image under test. Many methods have been developed for model based recognition (for example, Zitova [2] and Zhang [3] present recent surveys). A non exhaustive list includes correlation methods, template matching, frequential methods and scale-space methods. Each approach has its advantages and drawbacks in view of the specific application.

Hausdorff distance [4] is a similarity measure between two sets of points issued from a feature extraction step. Features like line segments [5], curves and skeletons[6], edge points and salient points [7] have been used for image matching. In our work, we use the edge points as features of the model obtained by applying a Canny-like edge operator. We can compare how similar is our model to a region in the image under test. If we use an exhaustive search approach, we need to look at all the possible positions and at each location, all the possible

poses of our model in the zone where the model could be present in the image under test. Recent applications of Hausdorff distance for shape matching include face detection and tracking [8]. Medical image registration also uses Hausdorff distance techniques for kidney images acquired using CT devices [9]. A generic object tracking framework is presented by Polat [10]. Another application addressed recently using Hausdorff distance-based methods is image retrieval [11].

Main drawback of Hausdorff distance is the high computational complexity involved in its calculation. When the object model presents some transformation in the test image, some works report search times up to 6s [4]. We need then to implement some heuristics to reduce necessary time for locating the object in an image. Monte Carlo techniques [12] enable us to compute static properties of models by using only a fraction of the points (chosen in a random manner) where these properties apply. Computing the static properties with randomly sampled points reduces the computational load. The only question consist in how to determine if the value converges toward the value computed using all the points where the property holds. Monte Carlo techniques have been used previously in Hausdorff-based shape matching for the generation of outlier shapes by Boykov in the testing of a Bayesian recognition method [13] and for the reduction of an affine transformation parameter space search by Jian-qi [14].

Our approach is to use Monte Carlo sampling to obtain a pattern with a lesser number of feature points and to apply Hausdorff-based shape matching on the sampled patterns. We have applied this approach and experimentally we have found that it converges by using only a small fraction $f = 0.12$ of the points composing the original model. Rest of this paper is organized as follows: In Section 2, we present the formal definition of Hausdorff distance and an alternative interpretation of it. This analysis leads in Section 3 to define the Monte Carlo version that we propose. Test and results for our experimental validation related to the convergence of the proposed approach are found in Section 4. Finally, our conclusions and perspectives are presented in Section 5.

2 Hausdorff Distance and an Alternative Interpretation

2.1 Hausdorff Distance Formulation

Hausdorff distance is computed as follows:

Given two set of points $A = \{a_1, \dots, a_p\}$ and $B = \{b_1, \dots, b_q\}$, Hausdorff distance $H(A, B)$ is:

$$H(A, B) = \max(h(A, B), h(B, A)) \quad (1)$$

where $h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\|$ and $\|\cdot\|$ is a two-dimensional Hilbert metric applied to points a and b .

The function $h(A, B)$ is named the forward Hausdorff distance from A to B and defines the point $a \in A$ that is farther from any point in B according to the norm $\|\cdot\|$. Distance $h(B, A)$ is known as the reverse Hausdorff distance. Thus, Hausdorff distance $H(A, B)$ is the maximum between $h(A, B)$ and $h(B, A)$.

In computer vision tasks, sets A and B are usually named M and I , standing for the model set and the image set respectively. Rest of this paper will use this notation.

2.2 Alternative Interpretation of the Hausdorff Distance

Let us define distance d_m^f as follows:

$$d_m^f = \min_{i \in I} \|m - i\| \text{ for all } m \in M \quad (2)$$

Thus d_m^f is the minimal distance from the point $m \in M$ to any point $i \in I$. We have then for the computation of $h(M, I)$ a decreasing ordered set of distances, called D^f , such that:

$$D^f = \{d_0^f, d_1^f, \dots, d_p^f\} \quad (3)$$

where p is the cardinality of M ; (i.e., $p = \#(M)$) and in a similar way, we define distance d^r from the point $i \in I$ to any point $m \in M$. We have then for the computation of $h(I, M)$ a decreasing ordered set of distances, called D^r , such that:

$$D^r = \{d_0^r, d_1^r, \dots, d_q^r\} \quad (4)$$

$q = \#(I)$. Evaluation of Hausdorff distance can be interpreted according to these two sets D^f and D^r where, The Hausdorff distance (Equation 1) can be re-defined as:

$$H(M, I) = \max(d_0^f, d_0^r) \quad (5)$$

3 Monte Carlo Hausdorff Distance (MCHD)

3.1 Random Sampling for the Hausdorff Distance Evaluation

Hausdorff distance is a very time demanding method. Even if some works have proposed optimizations for computing the current pose of the model [15,16], computational load remains heavy specially when considered for dynamical vision applications as visual servoing, where 4 Hz operating frequencies are typical. The proposed method evaluates the Hausdorff distance by using a Monte Carlo approach. This method provide, in one hand, efficiency because we will compute the similarity property between a model M and an image I by using only a randomly sampled points subset for each model. In the other hand, our method is robust because we can overcome some of the problems associated with model based recognition methods as a result of random sampling.

Let us consider the sets $I_{MC} \subset I$ and $M_{MC} \subset M$. The sets I_{MC} and M_{MC} are random samples of cardinality n taken using an uniform probability sampling function from the sets I and M , respectively. Thus, we can also say:

$$\lim_{n \rightarrow p} I_{MC} = I \quad (6)$$

Fig. 1. (a) A synthetic target, (b) its edges, (c) the sampled edges using 75 %, (d) 50 %, (e) 25%, (f) 15 % and (g) 5 % of the feature points

$$\lim_{n \rightarrow q} M_{MC} = M \quad (7)$$

Probability of a point belonging to the random sample subset is defined by the ratio of points being kept to the total number of feature points in the I and M sets. For each feature point in both sets, we decide to include it in the sampled subset by using a random number ζ chosen from a uniform distribution in $[0, 1]$. The feature point will be included in the Monte Carlo sampled subset if $0 \leq \zeta < th$, th being a *a priori* chosen threshold.

Figure 1 shows the effect of random sampling for choosing feature points (in this case, edge points). In this example, we use a synthetic image (a) and the resulting edge points models when we use all the edge points (b). The subset M_{MC} is also shown when using (c) 75 %, (d) 50 %, (e) 25 %, (f) 15 % and (g) 5 % of the feature points.

3.2 MCHD Definition

We can define Monte Carlo variants for the forward and reverse Hausdorff distances by using the sub-sampled sets of I and M .

$$h_{MC}(M, I) = h(M_{MC}, I) = \max_{m \in M_{MC}} \min_{i \in I} \|m - i\| \quad (8)$$

$$h_{MC}(I, M) = h(I_{MC}, M) = \max_{i \in I_{MC}} \min_{m \in M} \|i - m\| \quad (9)$$

We define the Monte Carlo Hausdorff Distance (MCHD) as:

$$H_{MC}(M, I) = \max(h_{MC}(M, I), h_{MC}(I, M)) \quad (10)$$

If M and I present a perfect matching (as only arises in ideal images) then

$$H_{MC}(M, I) = H(M, I) \quad (11)$$

Fig. 2. (a) Image for model selection, (b) the selected model and its feature map, (c) near perfect matching zone, (d) partial matching zone and (e) non matching zone. Images (c), (d) and (e) were taken from a second image acquisition.

regardless of the cardinalities for I_{MC} and M_{MC} . The fraction of points considered for the computation will influence the MCHD estimation when we deal with real images (i.e., images that present different feature maps even in very similar environmental conditions).

For example, in Figure 2, we consider two images where the time interval between acquisitions was approximately 100 ms. Camera position was fixed and the environment had not any mobile objects. This setup let us know ground truth about the object of interest in the scene. We define the model M (shown in Figure 2b) from the first image and we try to locate it in the second one. If we compute $h(M, I)$ in the best matching point (the image zone shown in 2(c)), the value is typically not zero. This behavior is caused mainly by acquisition noise and varying illumination conditions.

We analyze the matching behavior for three cases: i) A near perfect matching, where the model is compared against the image subset located in the same position than the previous model location. These sets of points differ only in the outlier points caused by the imaging artifacts cited above. ii) A partial matching, where we compute the distance between the model zone and the image subset

Fig. 3. Graph of the set $\|m - i\|$ for a (a) Correct, (b) Partial and (c) Null matching

Fig. 4. Selected points belonging to M_{MC} for a (a) near perfect, (b) partial and (c) null matching of the model in the image under test

located on a zone that has a similar feature map than the model zone and iii) A null matching, where the distance between the image zone and the image subset located on a non matching nor similar zone in the image is computed. In each of these cases we have plotted the sorted values of $\|m - i\|$ in decreasing order. Figure 3 shows these plots for: (a) a near perfect matching, (b) a partial matching and (c) a null matching when the original Hausdorff distance definition is used.

Our proposed approach uses randomly sampled subsets of M and I . In Figure 4 we show the randomly selected points to evaluate the Hausdorff distance in a run where we have used only 20 % of the points. In this graph, randomly selected points are marked with a small square and with a vertical dotted line.

Let us consider how the random sampling influences the computation of the Hausdorff distance and the Monte Carlo variants proposed in this paper. Consider again the sets D^f and D^r (defined in Section 2.2).

For the computation of $h_{MC}(M, I)$, we have a set D_{MC}^f such that:

$$D_{MC}^f = \{d_{MC_0}^f, d_{MC_1}^f, \dots, d_{MC_n}^f\} \quad (12)$$

and for the computation of $h_{MC}(I, M)$, we have a set D_{MC}^r such that:

$$D_{MC}^r = \{d_{MC_0}^r, d_{MC_1}^r, \dots, d_{MC_n}^r\} \quad (13)$$

where n is the cardinality of the sets M_{MC} and I_{MC} .

We can then apply the alternative interpretation of the Hausdorff distance and obtain the following definition for the Monte Carlo variant of the Hausdorff distance:

Monte Carlo Hausdorff distance (MCHD) is defined as:

$$MCHD(M, I) = \max(d_{MC_0}^f, d_{MC_0}^r) \quad (14)$$

Next section presents how the proposed approach converges toward a useful estimate of Hausdorff distance between a model and a target region in the image under test.

4 Experimental Work

4.1 Convergence Validation of the MCHD

The fraction of points f taken into account for the computation of $H_{MC}(M, I)$ impacts on the precision of the estimation. In this section, we use the cumulative histogram of the D_{MC} and D vectors to show the effect of random sampling in the performance of the computation of Hausdorff distance and the determination of what is a good choice for f .

Each point hic_i in the cumulative histogram $HISC$ is defined as

$$hic_i = \sum_{j=1}^i \frac{f_j}{n} \quad (15)$$

Let us consider the cumulative histogram $HISC$ of those points in the D set from $h(M, I)$ and $D_{MC}(M, I)$ from $h_{MC}(M, I)$, from the three cases of matching presented in the previous section.

The cumulative histogram for the set D for the near perfect matching is shown in Figure 5a. In Figure 5d, we have the D $HISC$ for the partial correspondence. The plots in Figure 5b and 5e show $HISC$ for the D_{MC} for the correct and partial matching cases, respectively, but using only 20% of the points. In the histogram in Figure 5a, we can see that the point distribution has a significant upward slope, indicating that the 100% of the points in D has a very small distance value. From this plot we can see that the distance for the 80% of the points is equal to 0. The same tendency is observed in Figure 5b. Plots in Figures 5d and 5e have similarities each other. Because of the cumulative histograms show the tendencies in function of the point percentage, we can over plot the graphs for D and D_{MC} . In Figure 5c, we can observe the histograms in 5a and 5b in the same axes. Plots in Figures 5d and 5e are compared in Figure 5f. From these graphs, we can say that D and D_{MC} follow the same trend in their cumulative histogram. A certain difference between the D_{MC} histogram with respect to the D one is observed in Figures 5c and 5f. Such a deviation depends on the amount of points taken into account.

Fig. 5. Cumulative histograms for D and D_{MC} , respectively (a) and (b) for a near perfect matching (d) and (e) for a partial matching. (c) Comparison between histograms (a) and (b). (f) Comparison between histograms (d) and (e).

Figure 6 shows the $HISC$ results using (a) 75%, (b) 25%, (c) 15%, and (d) 5% of the points. We see that the number of points taken for M_{MC} determines the deviation of the D_{MC} $HISC$ from the one for D . Taking the 75% of points, vector D_{MC} has the same tendency than D , but with less points the variation increases. Using the 5% of the points produces a great variation. Such a variation implies a variation in the $H_{MC}(M, I)$ with respect to the $H(M, I)$ value. From Figure 6, we can see that the deviation between (a) and (b) is lower than that between graphs (c) and (d). All the results were estimated as the mean value for 100 experiments in each test. Main conclusion we can infer from these results is that variance of the cumulative histograms augments as we reduce the fraction of points we use to estimate MCHD. Nevertheless, accuracy is only degraded when we use a fraction of points lower than 15 % (Figure 6d). So if we use some $f = 0.15$ we will have an enough accurate estimate of Hausdorff distance but a reduced computation time (only about 15 % of the original computational time).

Applying an evaluation method for determining the Hausdorff distance to the D_{MC} vector, the result can be a lower or an equal distance than the obtained if we would apply such an evaluation to vector D .

$$|h(M, I) - h_{MC}(M, I)| \leq \varepsilon \quad \varepsilon \geq 0 \quad (16)$$

where ε depends on the amount of points taken into account in the M_{MC} set. The higher the number of points is, the lower the maximum ε will become, but computation time will not be reduced significantly. A design choice has to be made to select f . For example, in an application developed for a robotic object

Fig. 6. Variation of cumulative histograms of D_{MC} with respect to D using (a) 75%, (b) 25%, (c) 15%, and (d) 5% of the points. All data were computed for 100 runs of each case.

tracking and visual servoing system, $f = 0.15$ has shown to be a good choice for successful results.

5 Conclusions and Perspectives

We have presented a method for computing image similarity using a novel approach to compute Hausdorff distance. Our method computes Hausdorff distance on a random subsample of the feature point set and consequently it reduces the computational time needed to compute Hausdorff distance on real images that are linked by a translational transformation. We have shown the trend that follows the cumulative histograms for different fractions of randomly subsampled sets of feature points. We have found that our method's performance critically degrades when use under 15 % of the total of feature points of real images. Over this threshold, our method converges toward the classical Hausdorff distance value but as mentioned before, reducing the computational time needed to compute image similarity.

In the near future, we will also apply the alternative interpretations for the partial (PHD) and modified (MHD) Hausdorff distance in methods similar to the one presented here. That will result in even more substantial time reduction for complexity management in robotic vision tasks using shape matching. Specially, we aim to apply this approaches to dynamic vision problem as the visual tracking and visual servoing problems.

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