Statistical Recursive Filtering Estimation of Detector Offset Nonuniformity in Infrared Imaging Systems^{*}

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Abstract. In this paper, a recursive filter to estimate the offset nonuniformity for infrared focal plane array imaging systems, using only the scene data, is presented. The proposed algorithm operates frame by frame in a pixel-by-pixel basis and there is not inter-related operations among the detectors. The method assumes that the input irradiance at each detector is a random and uniformly distributed variable in a range that is common to all detectors in the infrared focal plane array. The method is designed to operate in infrared imaging system, which exhibit important offset nonuniformities with slow temporal drift. The ability of the method to compensate for offset nonuniformity is demonstrated by employing several infrared video sequences obtained using an infrared camera.

Keywords: Image Sequence Processing, Infrared Focal Plane Arrays, Recursive Filtering.

1 Introduction

Infrared (IR) cameras use an IR sensor to digitize the information, and due to its high performance, the most employed integrated technology in IR sensors is the Focal Plane Array (FPA). An IR-FPA is a die composed of a group of photodetectors placed in a focal plane forming a matrix of $X \times Y$ pixels, which gives the sensor the ability to collect the IR radiation.

Nonuniformity noise in IR imaging sensors, which is due to pixel-to-pixel variation in the detectors' responses, can considerably degrade the quality of IR images since it results in a fixed-pattern-noise (FPN) that is superimposed on

^{*} This work was partially supported by Grants Milenio ICM P02-049 and by FONDECYT 1060894. The authors wish to thank Ernest E. Armstrong (OptiMetrics Inc., USA) and Pierre Potet (CEDIP Infrared Systems, France) for collecting the data, and the United States Air Force Research Laboratory, Ohio, USA.

the true image. Further, what makes matter worse is that the nonuniformity slowly varies over time, and depending on the FPA technology, this drift can take from minutes to hours. In order to solve this problem, several scene-based nonuniformity correction (NUC) techniques have been developed [1,2,3,4]. Scene-based techniques perform the NUC using only the video sequences that are being imaged, not requiring any kind of laboratory calibration technique. Our group has been active in the development of novel scene-based algorithms for NUC based on statistical estimation theory. In [5,6] we have developed a Gauss-Markov model to capture the slow variation in the FPN and have utilized the model to adaptively estimate the nonuniformity in blocks of infrared video sequences using a Kalman Filter.

In this paper, a recursive filter [7] to estimate the offset of each detector on the FPA from scene data is developed. The method is developed using two key assumptions: i) the input irradiance at each detector is a random and uniformly distributed variable in a range that is common to all detectors in FPA; ii) The FPA technology exhibits important offset nonuniformity with slow temporal drift. The proposed algorithm is developed to operate on one block, short enough to assume that the offset is constant within the block, of infrared images. In this block, the offset of each detector is optimally and recursively estimated.

This paper is organized as follows. In Sections 2 and 3 the IR-FPA model and the scene based method for NUC by means of a recursive filtering are presented respectively. In Section 4 the main parameters that the algorithm needs for a computer implementation are discussed. In Section 5 the recursive filtering technique is tested with video sequences of real raw IR data captured using an infrared camera. In Section 6 the conclusions of the paper are summarized.

2 Model

In this paper, we model the pixel-to-pixel variation in the detectors' responses (nonuniformity) using the commonly used linear model for each pixel on the IR FPA. For the $(ij)^{\text{th}}$ detector, the measured readout signal y_{ij} at a given time n can be expressed as:

$$y_{ij}(n) = A_{ij}(n)x_{ij}(n) + B_{ij}(n) + v_{ij}(n),$$

where $A_{ij}(n)$ and $B_{ij}(n)$ are the gain and the offset of the ij^{th} detector respectively and $x_{ij}(n)$ is the real incident infrared photon flux collected by the detector. It is assume that $x_{ij}(n)$ is a uniform distribute random variable in range common to all detectors in the IR FPA. The term $v_{ij}(n)$ is additive electronic noise represented by a zero mean Gaussian random variable that is statistically independent of noise in other detectors.

In this work, we are focused in IR FPA in which the nonuniformity is mainly generated by spatial and temporal variations in the detectors offset. Then, we suppose that A_{ij} is a known constant given by the IR camera manufacturer or it can be initially calculated by using known information of the particular IR camera. Using the foregoing, we re-write the model of each detector as

$$y(n) = Ax(n) + B(n) + v(n), \quad (1)$$

the subscript ij is omitted with the understanding that all operations are performed on a pixel by pixel basis. In the next section, we present the deduction of the proposed method: a recursive filtering for estimate the offset $B(n)$ in a block of frames of read-out IR data.

3 Recursive Estimation of the Offset

The goal of this paper is to develop a scene-based nonuniformity correction method for estimating the offset of each detector on the IR FPA using a block of frames short enough to assume that the offset is a constant in noise within the block. To do so, we first propose to estimate the offset frame by frame recursively in the form

$$\hat{B}(n) = C_n \hat{B}(n-1) + K_n y(n), \quad (2)$$

where $\hat{B}(n)$ and $\hat{B}(n-1)$ are the estimate of $B(n)$ and $B(n-1)$ respectively, C_n and K_n are the coefficients of the recursive filter. To find the C_n and K_n coefficients, we apply the orthogonality principle

$$E[(B(n) - \hat{B}(n))y(n-l)] = E[\varepsilon(n)y(n-l)] = 0, \quad (3)$$

where $\varepsilon(n)$ is the estimation error and $l = 0, 1, \dots, n$. In order to find an expression for the error variance, $\sigma_{\varepsilon(n)}^2$, we use

$$E[\varepsilon(n)B(n)] = \sigma_{\varepsilon(n)}^2, \quad (4)$$

where $E[\cdot]$ denotes the expectation operation.

Now, the main assumptions used in the develop of the filter are that the IR radiance $x(n)$ is random and uniformly distributed [5] within a known range $[X_{min}, X_{max}]$. We also consider that $x(n)$ is independent of $B(n)$ and $v(n)$, and that $B(n)$ is a zero mean random variable independent of $v(n)$. Before solving the equation (2), we introduce the necessary expressions for the development of the method.

The IR radiance $x(n)$ has a mean value given by $E[x(n)] = \mu_x = \frac{(X_{max} + X_{min})}{2}$ and variance $\sigma_x^2 = \frac{(X_{max} - X_{min})^2}{12}$, and the correlation function is denoted by $E[x^2(n)] = R_x(0) = \sigma_x^2 + \mu_x^2$. How $x(n)$ is independent of $B(n)$ and $v(n)$ then $E[x(n)B(n)] = E[x(n)]E[B(n)]$ and $E[x(n)v(n)] = E[x(n)]E[v(n)]$, are both equal to zero.

Now, based on the latter deduction, we want to obtain an expression for $E[y(n)y(n-l)]$ for $l = 0, 1, \dots, n$ using equation (1), that is

$$E[y(n)y(n-l)] = AE[x(n)y(n-l)] + E[B(n)y(n-l)] + E[v(n)y(n-l)]. \quad (5)$$

It can be easily shown that this expectation is reduced to

$$E[y(n)y(n-l)] = A^2 R_x(l) + R_B(l) + \sigma_v^2 \delta(l), \quad (6)$$

where $R_x(l)$ is the correlation function of the IR radiance $x(n)$, $R_B(l)$ is the correlation function of the offset $B(n)$ and $\sigma_v^2 \delta(l)$ is the correlation function of the electronic noise.

3.1 Solution for K_n : Case Where $l = 0$

In this case, substituting $y(n)$ by (1) in (3), and using (4) yields

$$E[\varepsilon(n)y(n)] = E[\varepsilon(n)Ax(n)] + \sigma_{\varepsilon(n)}^2 + E[\varepsilon(n)v(n)] = 0, \quad (7)$$

the first and third terms are calculated individually. In the first term, equations (1), (2) and (6) are used, since the irradiance $x(n)$ is independent of the offset, we get

$$E[\varepsilon(n)x(n)] = E[B(n)x(n)] - E[C_n \hat{B}(n-1)x(n)] - E[K_n y(n)x(n)] = -AK_n R_x(0), \quad (8)$$

and in the third term, (1), (2) and (6) are used, and that since the noise is white and orthogonal to the signal, we can write

$$E[\varepsilon(n)v(n)] = E[B(n)v(n)] - E[C_n \hat{B}(n-1)v(n)] - E[K_n y(n)v(n)] = -K_n \sigma_v^2. \quad (9)$$

Now, substituting these results in (7) yields

$$K_n = \frac{\sigma_{\varepsilon(n)}^2}{\sigma_v^2 + A^2 R_x(0)}. \quad (10)$$

3.2 Solution for C_n : Case Where $l > 0$

In this case,

$$E[\varepsilon(n)y(n-l)] = E[(B(n)y(n-l)] - C_n E[\hat{B}(n-1)y(n-l)] - K_n E[y(n)y(n-l)] = 0, \quad (11)$$

and using (1), (5) and (6) we obtain

$$E[\varepsilon(n)y(n-l)] = (1-K_n)R_B(l) - A^2 R_x(l)K_n - C_n E[\hat{B}(n-1)y(n-l)] = 0, \quad (12)$$

and for the term $E[\hat{B}(n-1)y(n-l)]$ we use $\varepsilon(n-1) = B(n-1) - \hat{B}(n-1)$ and (4), getting

$$E[\hat{B}(n-1)y(n-l)] = E[B(n-1)y(n-l)] - E[\varepsilon(n-1)y(n-l)] = R_B(l-1), \quad (13)$$

now, rewriting (12) for C_n we obtain

$$C_n = \frac{R_B(l) - K_n(R_B(l) + A^2 R_x(l))}{R_B(l-1)}. \quad (14)$$

Assuming that $R_B(l) = \alpha^l R_B(0)$, i.e., $B(n)$ is a signal that has an exponential correlation function, we obtain

$$C_n = \alpha \{1 - K_n(1 + A^2 \frac{R_x(l)}{\alpha^l R_B(0)})\}. \quad (15)$$

Using an auxiliary variable $\beta = 1 + A^2 \frac{R_x(l)}{\alpha^l R_B(0)}$ we finally get

$$\hat{B}(n) = \alpha \hat{B}(n-1) + K_n(y(n) - \alpha \beta \hat{B}(n-1)). \quad (16)$$

3.3 Solution for $\sigma_{\varepsilon(n)}^2$

We start using (2) and the previous results. Then, we can write

$$\sigma_{\varepsilon(n)}^2 = E[(B(n) - \hat{B}(n))B(n)] = (1 - K_n)R_B(0) - \alpha(1 - \beta K_n)E[B(n)\hat{B}(n-1)]. \quad (17)$$

Using the fact that $B(n)$ is a signal with an exponential correlation function, i.e., $B(n) = \alpha B(n-1) + \omega(n)$, where $\omega(n)$ is a zero mean gaussian process independent of $B(n)$, we obtain that

$$E[B(n)\hat{B}(n-1)] = \alpha E[B(n-1)\hat{B}(n-1)]. \quad (18)$$

Note that $\sigma_{\varepsilon(n-1)}^2 = E[\varepsilon(n-1)B(n-1)] = R_B(0) - E[B(n-1)\hat{B}(n-1)]$. Then, solving for $E[B(n-1)\hat{B}(n-1)]$ result in

$$\sigma_{\varepsilon(n)}^2 = (1 - K_n)R_B(0) - \alpha^2(1 - \beta K_n)(R_B(0) - \sigma_{\varepsilon(n-1)}^2), \quad (19)$$

and finally replacing K_n and solving for $\sigma_{\varepsilon(n)}^2$ allow us to obtain

$$\sigma_{\varepsilon(n)}^2 = \frac{\alpha^2 \sigma_{\varepsilon(n-1)}^2 + (1 - \alpha^2)R_B(0)}{1 + \frac{\alpha^2 \beta (\sigma_{\varepsilon(n-1)}^2 - R_B(0)) + R_B(0)}{\sigma_v^2 + A^2 R_x(0)}}. \quad (20)$$

In the next section, we modified the foregoing filter to estimate the offset as a constant in noise.

4 Estimation of the Offset as a Constant in Noise

In this section the theoretical development of the main goal of this paper is finished. The idea is to develop an algorithm able to estimate the offset $B(n)$ as a constant in noise. As an example, we know this assumption is practically valid in block of frames taken within two to three minutes in several IR FPA technologies.

Then, the foregoing filter will have to be modified using $B(n) = B(n-1) = B$ a constant, i.e., $\alpha = 1$ and $\omega(n) = 0$. Then (16) is simplified to

$$\hat{B}(n) = \hat{B}(n-1) + K_n(y(n) - \beta \hat{B}(n-1)), \quad (21)$$

and equation (20) is simplified to

$$\sigma_{\varepsilon(n)}^2 = \frac{\sigma_{\varepsilon(n-1)}^2}{1 + \frac{(1-\beta)R_B(0) + \beta\sigma_{\varepsilon(n-1)}^2}{\sigma_v^2 + A^2 R_x(0)}}. \quad (22)$$

Note that if $\beta = 1$, i.e., the IR irradiance at a current frame is uncorrelated with previous frames, and equation (22) can be reduced to

$$\sigma_{\varepsilon(n)}^2 = \frac{\sigma_{\varepsilon(n-1)}^2}{1 + \frac{\sigma_{\varepsilon(n-1)}^2}{\sigma_v^2 + A^2 R_x(0)}}. \quad (23)$$

Now, each step of the recursive filter for nonuniformity correction is listed.

Algorithm:

1. Establish the range $[X_{min}, X_{max}]$ and calculate all the initial conditions.
2. Choose the length of the block of frames n_b .
3. Update $K_n = \frac{\sigma_{\varepsilon(n)}^2}{\sigma_v^2 + A^2 R_x(0)}$ and $\sigma_{\varepsilon(n)}^2 = \frac{\sigma_{\varepsilon(n-1)}^2}{1 + \frac{\sigma_{\varepsilon(n-1)}^2}{\sigma_v^2 + A^2 R_x(0)}}$ from the value $n = 1$.
4. Obtain the signal estimation $\hat{B}(n) = \hat{B}(n-1) + K_n(y(n) - \hat{B}(n-1))$.
5. Generate the corrected frame using $\hat{x}(n) = (y(n) - \hat{B}(n))/A$.
6. If $n \leq n_b$ then, go to step 3 and increase n .
7. If $n > n_b$ maintains the correction with $\hat{x}(n) = (y(n) - \hat{B}(n_b))/A$.

4.1 Initial Parameters

In order to initialize the algorithm the parameters $[X_{min}, X_{max}]$, A , $\sigma_{\varepsilon(0)}^2$, $R_B(0)$ and σ_v^2 have to be calculate for each particular IR camera. First of all, anyone with experience with his IR camera can easily propose a range $[X_{min}, X_{max}]$ for a particular block of frames. Then, $R_x(0) = \frac{(X_{max} - X_{min})^2}{12} + \frac{(X_{max} + X_{min})^2}{4}$. Also, knowing $[Y_{min}, Y_{max}]$ from the read-out IR data and employing (1) we can obtain

$$A = \frac{Y_{max} - Y_{min}}{X_{max} - X_{min}}.$$

To compute $\sigma_{\varepsilon(0)}^2$, we use (23) and.

$$\sigma_{\varepsilon(n-1)}^2 = R_B(0) - E[B(n-1)\hat{B}(n-1)],$$

assuming that no prior observations are available, i.e., $\hat{B}(-1) = 0$, then $\sigma_{\varepsilon(-1)}^2 = R_B(0)$, now replacing in (23)

$$\sigma_{\varepsilon(0)}^2 = \frac{R_B(0)}{1 + \frac{R_B(0)}{\sigma_v^2 + A^2 R_x(0)}}. \quad (24)$$

Finally, to get a good estimation of $R_B(0)$, detectors offsets of a particular camera obtained previously with another nonuniformity correction method are used [4]. Also, the variance of electronic noise σ_v^2 can easily be calculated from previous IR raw data.

4.2 Convergence Analysis

An important aspect for any nonuniformity correction algorithm is its convergence analysis. For this case, the convergence is determinate by $\sigma_{\varepsilon(n)}^2$, i.e., the estimation error variance. Let us define $a = \frac{R_B(0)}{\sigma_v^2 + A^2 R_x(0)}$ and replacing it in equation (24)

$$\sigma_{\varepsilon(0)}^2 = \frac{R_B(0)}{1 + a}, \quad (25)$$

then, (23) and (10) can be expressed by

$$\sigma_{\varepsilon(n)}^2 = \frac{R_B(0)}{1 + a(n+1)} \quad (26)$$

$$K_n = \frac{a}{1 + a(n+1)}, \quad (27)$$

i.e., $\sigma_{\varepsilon(n)}^2$ and K_n correspond to a decreasing succession, where a parameter is responsible of the convergence. For the case $a > 1$, the convergence is faster than when $a < 1$. In other words, for the case in which $R_B(0) > \sigma_v^2 + A^2 R_x(0)$ the convergence of the method occurs within a few frames, whereas when $R_B(0) < \sigma_v^2 + A^2 R_x(0)$ a greater number of frames is required. In infrared system, a practical consideration is that $R_B(0)$ is approximately equal to 10% of the dynamic range of the readout infrared data. Further, supposing that it is possible to consider $A^2 R_x(0) \gg \sigma_v^2$ and that $R_B(0)$ is approximately equal to 10% of $A^2 R_x(0)$, which implies that $a \approx 0.1$. Then, to obtain a reduction of until the 2% of the initial estimation error we need $n = 490$ frames. Note that for the case $a = 1$ the initial error estimation is reduced to 2% when $n = 48$ frames.

Fig. 1. Performance of the proposed method on real IR data. The first row shows the 570 – *th* frame, and the second row shows the 1570 – *th* frame of the set of IR data. (a) (d) Real corrupted frames, (b) (e) Real frames corrected by using black bodies (c) (f) Real frames corrected by the proposed method.

5 Applications to Real Infrared Image Sequences

The main goal of this section is to test the ability of the proposed method for reduce nonuniformity on real infrared video data. The sequence has been collected using a 128×128 InSb FPA cooled camera (Amber Model AE-4128) operating

in the $3 - 5\mu\text{m}$ range. In the set data, 3000 frames were collected at a rate of 30 frames per second, with 16 bits of resolution. For purposes of comparison black bodies radiator are used under laboratory conditions to generate flat-field images in the linear range of the detectors. These data is used to estimate the gain and the offset associated with each detector. With these parameters, the best correction of nonuniformity is performed, obtaining a sequence that is used like a reference.

Fig. 2. The evolution of the RMSE between the reference frames (set calibrated with black bodies) and the corrected frames by the proposed NUC method

As a quantitative measure of performance, we use the Root Mean Square Error (RMSE), which measures the difference between the reference infrared image with the corrected image using the proposed method. The RMSE is calculated by:

$$RMSE(n) = \left\{ \frac{1}{pm} \sum_{i=1}^p \sum_{j=1}^m (\hat{x}_{ij}(n) - x_{ij}(n))^2 \right\}^{1/2}, \quad (28)$$

where $p \times m$ is the number of detectors in the FPA. $\hat{x}_{ij}(n)$ is the infrared irradiance calculated with the offset estimated by the recursive filter. $x_{ij}(n)$ is the infrared irradiance calculated with the offset estimated by using the black-body radiator data (Laboratory calibration offset). A lower value of RMSE means a good correction of the frame data.

Before using the algorithm, the initial conditions are calculated. Firstly, we have assumed that the range of the input irradiance is between $[0, 255]$, this is

that all the pixels were exposed to minimum and maximum possible in the gray scale. We use a block of frames with a length of $n_b = 500$, this is approximate 17 seconds of data. With this and the raw infrared data, $R_x(0) = 21.68 \times 10^3$, $A = 257$, $R_B(0) = 1.43 \times 10^8$ and $\sigma_v^2 = 3.10 \times 10^5$. In addition, $a = 9.99 \times 10^{-2}$ and $(\%) \sigma_{\varepsilon(500)}^2 / \sigma_{\varepsilon(0)}^2 = 2.35\%$.

Figure 1 (a)(b)(c) (d) (e) (f) shows from left to right a frame of real noisy readout data, the corresponding corrected frame by using black bodies, and the corresponding corrected frame by the NUC method proposed in this paper. It can be clearly seen by using the naked eye that our method mitigate the nonuniformity noisy as well as the calibration method. The NUC performance, in this case, is evaluated employing the index root mean square error (RMSE) computed between a reference (the real IR sequence calibrated with black bodies) and our corrected IR video sequence. Figure 2 shows the calculated RMSE for each frame corrected using the proposed method. Figure 3 presents the convergence for the recursive filter. It's clear that the gain and the error variance converge to minimal value after the 500 – th frames.

Fig. 3. Convergence parameters of the proposed method. The dashed line correspond to the gain K_n and the solid line represents the error variance $\sigma_{\varepsilon(n)}^2$.

6 Conclusions

In this paper a recursive filter for NUC on IR imaging system is proposed. It was shown experimentally using real IR data that the method is able to notably reduce the non-uniformity. Indeed, the method has shown good reduction of nonuniformity after processing only around 300 frames. The main advantage of

the method is based in its simplicity using only fundamental estimation theory. Further, the method during the estimation process generates two convergence parameters, which can be used for determining the method ON-OFF time in real infrared camera operations. The keys assumptions of the proposed method are that the input irradiance at each detector is a random and uniformly distributed variable in a range that is common to all detectors in the infrared focal plane array and that input infrared sequences of frames exhibits important offset nonuniformities with slow temporal drift.

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