Estimation of Cardiac Electrical Propagation from Medical Image Sequence

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Abstract. A novel strategy is presented to recover cardiac electrical excitation pattern from tomographic medical image sequences. The geometrical/physical representation of the heart and the dense motion field of the myocardium are first derived from imaging data through segmentation and motion recovery. The myocardial active forces are then calculated through the law of force equilibrium from the motion field, realized with a stochastic multiframe algorithm. Since tissue active forces are physiologically driven by electrical excitations, we can readily relate the pattern of active forces to the pattern of electrical propagation in myocardium, where spatial regularization is enforced. Experiments are conducted on three-dimensional synthetic data and canine magnetic resonance image sequence with favorable results.

1 Introduction

Electrocardiograms (ECGs) have been the standard clinical tools for cardiac function assessment over the past 150 years. Recently, electrocardiographic inverse approaches are becoming of great interests to the research and clinical communities because its potential to *describe the patient-specific electrical activity of each myocyte* of the heart [1]. Based on mathematical simulations of the cardiac electrical propagation process, there are a number of efforts that aim to estimate the macroscopic cardiac electrical activities from multichannel ECG measures [1]. While such inverse approaches are invaluable, they are ill-posed because of the absence of unique mapping between the intra-cardiac electrical excitation and body surface potential distribution, and *appropriate* constraints must be adopted to ensure that solutions are available.

Since the mechanical activities of the myocardium are mainly driven by the cardiac electrical excitation, patient-specific myocardial kinematic measures should, indirectly, reflect the electrical propagation process of the heart. Over the past twenty years, there are many efforts in the medical image computing community devoted to the noninvasive recovery of cardiac motion field from imaging sequences. In general, medical images can directly provide cardiac motion information at some salient landmark points, such as the tag displacements from MR tagging images and the tissue velocities from the MR phase images.

The dense cardiac motion fields, including displacements, velocities and accelerations throughout the myocardium, can then be estimated from those coarse measurements with *a priori* constraints [2,3,4,5], for which biomechanical models have been the popular choices because of their physics meaningfulness. A recent effort is particularly related to our current work [6]. Based on probabilistic measure of the onset of regional myocardial activation, derived from 3D motion field obtained by tracking tagged MR image sequence with non-rigid registration, it combines different features, such as displacement, velocities and the magnitude of the strain, to locate the abnormal electrical conduction pathways.

Our framework is based on the physical principle that, if dense motion field (i.e. estimated from medical image sequences) and the material characteristics (i.e. the mechanical model and its parameters) of the myocardium are available, then the *internal force* (forces caused by inertia, damping, stresses) at each myocardial point can be calculated. Further, the *external force* (active forces caused by electrical excitation) could be calculated from the force equilibrium equation inside myocardium. Even though pressures do exist on the epi- and endocardial surfaces, we do not need to count the pressures when we check the force balance inside myocardium since the motion field of the whole heart already implies the presence of pressures on the cardiac surface. However, cardiac motion field estimated from images always contains noises and numerical errors. If we directly relate the onsets of electrical excitation to the onsets of active forces calculated from the estimated motion field, the pattern of electrical propagation will be corrupted by these noises and errors. Hence, electrical propagation from *a priori* information should be considered to constrain the pattern calculated from estimated motion field. In our current work, Tikhonov regularization, which is a standard technique in electrocardiographic inverse approaches [1], is applied to perform a weighted sum between predicted pattern from simulation and the pattern calculated from motion-derived active forces.

2 Methodology

2.1 Anisotropic Representation

Given the definition of cardiac geometry and the dense motion field, a set of unstructured, adaptively sample nodes are distributed between endo- and epicardium and an anisotropic representation of cardiac fiber structures (a fiber orientation is attached to each node), geometry (sample nodes in and inside the boundary of cardiac geometry) and motion field is established by the moving least square (MLS) approximation. Let $u(x)$, $\dot{u}(x)$ and $\ddot{u}(x)$ be the displacement, velocity and acceleration of the myocardial tissue at point x . The approximated displacement, velocity and acceleration function $u_h(x)$, $\dot{u}_h(x)$ and $\ddot{u}_h(x)$ are then given: $u_h(x) = \sum_{I=1}^N \phi(x)u_I$, $\dot{u}_h(x) = \sum_{I=1}^N \phi(x)\dot{u}_I$ and $\ddot{u}_h(x) = \sum_{I=1}^N \phi(x)\ddot{u}_I$ where $\phi(x)$ is the shape function, given by MLS approximation, of node I , N is the total number of sample nodes, u_I is the nodal displacement value, $\dot{u}_I$ is the nodal velocity value and $\ddot{u}_I$ is the nodal acceleration value. Since there are not mesh constraints between nodes, refinement of framework input, motion field,

can be easily achieved by changing the density of sample nodes when new motion information are available.

2.2 Law of Force Equilibrium

After the anisotropic representation is created active forces can be calculated by the principal of force equilibrium with dense motion field and material models.

Anisotropic Composite Material Model. In our approach the heart is assumed to be elastic. For elastic materials, both isotropic and anisotropic, the stress-strain relationships obey the Hookes Law [7]:

$$\mathbf{S} = \mathbf{C}\epsilon \quad (1)$$

where $\mathbf{S} = [S_{11}, S_{22}, S_{33}, S_{12}, S_{13}, S_{23}]^T$ and $\epsilon = [\epsilon_{11}, \epsilon_{22}, \epsilon_{33}, \epsilon_{12}, \epsilon_{13}, \epsilon_{23}]^T$, with S_{ij} the components of the second Piola-Kirchhoff (PKII) stress tensor, $\mathbf{C}$ the stiffness matrix and ϵ_{ij} the components of the Green-Lagrangian strain tensor, which can be calculated from displacement [7]:

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}) \quad (2)$$

where $u_{i,j} = \partial u_i / \partial x_j$. We assume that the cardiac tissues of the myocardium are transversely isotropic, which has the same material property in the cross-section planes of the myocytes, but different material property in the direction normal to the cross-section planes. Let C_0 be the stiffness matrix of a point in the myocardium with 0° (along the x -axis) fiber orientation:

$$C_0 = \begin{bmatrix} \frac{1}{E_f} & \frac{-\nu}{E_{cf}} & \frac{-\nu}{E_{cf}} & 0 & 0 & 0 \\ \frac{-\nu}{E_{cf}} & \frac{1}{E_f} & \frac{-\nu}{E_{cf}} & 0 & 0 & 0 \\ \frac{-\nu}{E_{cf}} & \frac{-\nu}{E_{cf}} & \frac{1}{E_f} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E_{cf}} \end{bmatrix}^{-1} \quad (3)$$

where E_{cf} and E_f are the cross fiber and along fiber Young's modulus respectively. With C_0 defined, the stiffness matrix at any point with fiber orientation of θ degrees polar angle and ϕ degrees azimuthal angle can be obtained using the tensor transformation [8]:

$$C_{tran} = T^{-1}C_0T \quad (4)$$

where $T = T_{azi}(\phi)T_{pol}(\theta)$ the coordinate transformation matrices:

$$T_{azi} = \begin{bmatrix} c^2 & 0 & s^2 & 0 & 2sc & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ s^2 & 0 & c^2 & 0 & -2sc & 0 \\ 0 & 0 & 0 & c & 0 & s \\ -sc & 0 & sc & 0 & c^2 - s^2 & 0 \\ 0 & 0 & 0 & -s & 0 & c \end{bmatrix} \quad T_{azi} = \begin{bmatrix} c^2 & s^2 & 0 & 2sc & 0 & 0 \\ s^2 & c^2 & 0 & -2sc & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -sc & sc & 0 & c^2 - s^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & s \\ 0 & 0 & 0 & 0 & -s & c \end{bmatrix}$$

in which $c = \cos(\phi)$ in $T_{azi}(\phi)$ and $c = \cos(\theta)$ in $T_{pol}(\theta)$, and $s = \sin(\phi)$ in $T_{azi}(\phi)$ and $s = \sin(\theta)$ in $T_{pol}(\theta)$.

Except Green-Lagrangian strain tensor deformation gradient $\mathbf{F}$ also can be used to describe the deformation [7]:

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad (5)$$

where F_{ij} the components of deformation gradient, x_i the coordinates in deformed configuration and X_i the coordinates in reference configuration. With PKII stress $\mathbf{S}$ and deformation gradient $\mathbf{F}$ nominal stress $\mathbf{P}$ can be defined [7]:

$$\mathbf{P} = \mathbf{S} \cdot \mathbf{F}^T \quad (6)$$

Force Equilibrium Equation. Considering a dynamic analysis under finite deformation without damping in an arbitrary domain Ω with boundary Γ in Fig. 1, law of force equilibrium should be obeyed everywhere inside domain Ω in each time instant [7]:

$$\frac{P_{ji}}{\partial X_j} + f_i^B = \rho_0 a_i \quad (7)$$

with natural (force) boundary conditions f_i^S on S_f and the essential (displacement) boundary conditions u_i^S on S_u (Fig. 1). S_f and S_u are the location that natural boundary conditions and essential boundary conditions are applied respectively, ρ_0 is the density of mass in reference configuration and a_i are the components of acceleration. X_j are the coordinates in reference configuration, P_{ji} are the components of nominal stress and f^B is density of body force. Because it is difficult to define damping force in a point inside myocardium and the movements of myocardium are mainly driven by active forces, the effect of damping forces could be neglected in equation (7). Though we assume nominal stresses in equation (7), it is not difficult to prove that equation (7) also exists by replacing $\mathbf{P}$ with cauchy stress τ under infinitesimal deformation condition [7]:

$$\tau_{ij,j} + f_i^B = \rho a_i \quad (8)$$

where τ_{ij} are the components of cauchy stress, $\tau_{ij,j} = \partial \tau_{ij} / \partial x_j$ and ρ is the density of mass.

A simple one-dimensional uniform bar can be used to demonstrate the law of force equilibrium. Consider this one-dimensional bar is subjected to a distributed load $f^B(x) = x$ and a concentrated load R at it right end and its left end is fixed in Fig. 1. We assume the bar has a constant cross sectional area A and Young's modulus E . The law of force equilibrium gives the governing equations of the bar:

$$EA \frac{d^2 u}{dx^2} + f^B = 0 \quad (9)$$

$$u|_{x=0} = 0 \quad EA \frac{du}{dx}|_{x=L} = R \quad (10)$$

Fig. 1.

It is easy to prove that the analytical solution of displacements in equation (9) is $u(x) = (-x^3/6 + Rx + \frac{1}{2}aL^2x)/EA$ by $d^2u(x)/d^2x + f^B(x) = -x + x = 0$. If we consider a small part in the middle of the bar, we only need to consider the internal forces caused by stresses and body forces $f^B(x)$ to check the force balance because natural boundary condition, the concentrated load R , has been implied by the displacement field of whole bar.

Though there could be other components in f^B inside myocardium except active forces, the majority of f^B are the active forces caused by electrical excitations. In our work we assume that f^B only consist of active forces. We have not considered damping forces because it is difficult to define a damping force in one particular point inside myocardium and the contributions of damping forces could be neglected during cardiac contraction. Let's go back to equation (7), stresses can be calculated from displacements and material models, and inertial forces can be calculated with accelerations and densities so that active forces could be easily calculated from equation (7). The movements of heart are driven by many kinds of forces, such as pressures inside heart and outside heart, but as we demonstrated in the one-dimensional bar example, the loading R will not affected the recovery of body forces inside the bar since displacements of bar have implied the presence of loading R in the end of bar. We assume only active forces can exist as external forces inside myocardium so that the active forces can be recovered from the motion field by the law of force equilibrium and material models (stress-strain relations and densities), as we have explained above that natural and essential boundary conditions have been implicitly enforced into motion field.

2.3 Recovery of Electrical Propagation Patterns

Since a certain delay is always assumed between the onsets of active forces and the instants of depolarization of myofibers, we can relate the pattern of active forces to the pattern of electrical propagation. But the onsets of active forces are not so easy to detect we adopt the peaks of active forces as the signs of activation of myofibers and a constant delay still can be assumed between the peaks of active forces and the instants of depolarization of myofibers. Finally we can attain patient specific the pattern of electrical propagation from a dense motion fields by the law of force equilibrium. Nevertheless such approach is purely based on image data without any priori constraints. Law of force equilibrium should be

obeyed all the time, but the connection between the onsets of active forces and onsets of electrical excitations could be interrupted by errors or noises spatially.

In this paper Tikhonov regularization [1] is applied to this connection with a priori constraint, which is generated from a modified FitzHugh-Nagumo model [9]. The classical Tikhonov regularized solution is obtained through minimization of the following equation:

$$J_T = (Mea - H \times Est)^T (Mea - H \times Est) + \lambda (Est - Pre)^T Q (Est - Pre) \quad (11)$$

where Mea measurement vector, Est estimation vector and Pre prediction vector. λ and Q are referred to the regularization parameter and regularization matrix, H is the mapping matrix between measurement and estimation, and $(*)^T$ denotes the transpose operation. There are many ways to determine the λ and Q , but we can interpret Tikhonov regularization as a stochastic framework by λ represents the measurement noise variance and estimation is the realization of a Gauss vector with a mean value equal to prediction and covariance matrix equal to the inverse of Q if Gaussian white noises are assumed. The main advantage of Tikhonov regularization is that the solution of equation (11) can be expressed in closed form as:

$$Est = (H^T H + \lambda Q)^{-1} (H^T \times Mea + \lambda Q \times Pre) \quad (12)$$

Measurements are the instants of depolarization of myofibers calculated from motion field and predictions are the instants of depolarization of myofibers from simulation in equation 12. Since measurement and prediction are the same data, the onsets of electrical excitation in myofibers, H matrix is a identity matrix.

3 Results

We verify our algorithm by using experiments on a deformable cube with two different fiber directions and left face is fixed. The cube is excited by electrical excitation initiated from its left face and deforms according to the propagation of the electrical excitation under finite deformation condition, and nodal contractile active forces and deformations of 100 time steps in one cycle are captured as the ground truth. In the experiments the ground truth contractile stresses are added with noises at 10dB SNR and the nodal displacements on the right face of the cube are added with noises at 1dB SNR to provided the noisy measurements. From the results in the first row in fig. 2, we can see that estimated displacements could be recovered from noisy measurements quite well in our implementation. From the results in the second row in fig. 2, we can see the a simulation of propagation of electrical excitation in the cube, noisy pattern of propagation recovered from estimated motion field, which caused by numerical errors and noises, and smooth pattern of propagation with Tikhonov regularization.

Experiments also have been done on cardiac MR images. The MR images contain a sequence of sixteen images of a canine heart in one cardiac cycle,

Fig. 2. Cube geometry with fiber orientations and displacement magnitude maps in the first row, and patterns of propagation in second row. Left to right in first row: cube geometry, ground truth, mutliframe estimation. Left to right in second row: simulation, pattern without regularization, pattern with regularization. Color map in first row for displacement magnitude map. Color map in the second row for instants of depolarization map.

Fig. 3. From left to right in the first row: cardiac geometry with fiber orientations, circumferential strain map calculated from estimated displacement of frame 8, color map for strain magnitude map, estimated displacement magnitude map of frame 8 and color map for displacement magnitude map. From left to right in the second row : simulation, pattern without regularization, pattern with regularization and color map in the second row for depolarization map.

providing three-dimensional data for analysis. The fiber orientations of the heart are obtained by mapping the data from the Auckland University Bioengineering Institute¹, using the iterative closest points algorithm [10]. The results show that our algorithm is clinically relevant (Fig. 3).

4 Conclusions and Future Work

The inverse approach from MR images to electrical propagation is very novel, but difficult due to complicated processes from electrical excitation to heart

¹ <http://www.bioeng.auckland.ac.nz>

contraction. Reasonable, but computation feasible models with physics meaning should be considered to help the recovery of electrical propagation. Currently we only apply spatial constraint from simulated pattern to pattern from estimated motion field. A temporary constraint or new measurements (such as ECG signals) should increase the accuracies of our approach and provide more information of electrical propagation.

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