Some Complexity Results for SystemVerilog Assertions

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Abstract. SystemVerilog Assertions (SVA) is a linear temporal logic within the recently approved IEEE 1800 SystemVerilog standard. The complexities of the satisfiability and model-checking problems are studied for a basic subset of SVA and for extensions of the basic subset obtained by adding each of the following features: local variables, regular expression intersection, quantified variables, and property declarations with arguments. It is shown that the complexities for the basic subset are PSPACE-complete, while the complexities increase to EXPSPACE-complete¹ in each of the extensions. Alternating Büchi automata constructions provide the upper bounds, while reductions from PSPACE and EXPSPACE tiling problems provide the lower bounds.

1 Introduction

SystemVerilog Assertions, abbreviated SVA, is the assertion sublanguage of the recently approved IEEE 1800 SystemVerilog standard [14]. It is a linear temporal logic that is intended to be used to represent correctness properties and functional coverage events for the validation and verification of SystemVerilog designs. Prior to revision in the IEEE P1800 committee, development of SystemVerilog was carried out within the Accellera Organization, culminating in Accellera SystemVerilog 3.1a [1]. Industrial interest in SVA has been growing, as evidenced by its support in electronic design automation (EDA) tools, its discussion among verification engineers [19], and its deployment by semiconductor companies.

SVA has been developed in parallel with another recently approved standard assertion language, IEEE 1850 Property Specification Language (PSL) [15]. The two languages share a common core based on regular expressions. The common core includes temporal properties built using implication with regular expression antecedent and the standard logical boolean operators. While SVA and PSL have syntactic differences, there has been substantial work within Accellera and IEEE committees to ensure that the languages are aligned on the semantics of the common core. The two languages differ more substantially outside the common core.

¹ EXPSPACE is defined as $\bigcup_{k \in \mathbb{N}} \text{DSPACE}(2^{n^k})$.

PSL provides the standard LTL operators, which are not in SVA, quantified variables, and numerous derived operators.² PSL, similar languages, such as ForSpec, and academic simplifications have also received attention in recent literature [5,4,9,6]. However, there remain gaps in the analysis of the complexity of these languages for model checking. SVA, on the other hand, provides local variables as one of its distinguishing features. A local variable is used to capture the value of an expression at one point within a property and hold it for later reference, after which the local variable may be reassigned. Without local variables, complex auxiliary state machines are often required to represent temporal properties of practical interest. This makes local variables extremely useful for an industrially deployed temporal logic. The semantics of local variables has been studied in the committees [10], but SVA with local variables has received little attention in the literature and the expressiveness and complexity of the logic have not been studied before. This theoretical deficit has resulted in a certain amount of confusion in the industrial verification community. EDA companies disagree on what part of SVA should be supported in model checking, and verification engineers do not know whether properties that run in simulation can reasonably be expected to be supported in formal verification.

In this paper, we fill some of the theoretical gaps by studying the expressiveness and complexity of a basic subset of SVA and several of its extensions. The basic subset is obtained from the common core by eliminating the intersection operator on regular expressions. Previous work shows that the basic subset can express all omega-regular languages [4] and that the satisfiability and model-checking problems for the basic subset are in PSPACE [7,8]. We prove a matching lower bound, hence these problems for the basic subset are PSPACE-complete.

We prove that extending the basic subset of SVA with either local variables or intersection of regular expressions increases the complexity of satisfiability and model-checking from PSPACE-complete to EXSPACE-complete. For each of these extensions, we present a construction of alternating Büchi automata that gives an algorithm with tight complexity for the satisfiability and model-checking problems. These constructions also prove that the extensions do not increase expressiveness. Adding both local variables and intersection, the complexity of these problems in the combined extension remains EXSPACE-complete. We also study extension of the basic subset with quantified variables analogous to those in PSL. The satisfiability and model-checking problems for this extension are EXSPACE-complete. Finally, we study extension of the basic subset with declared properties and arguments. Their addition also results in EXSPACE-complete complexity for satisfiability and model checking. With the exception of those involving local variables, each of the results also holds for the corresponding subset of PSL.

Our lower bounds imply that, from the point of view of complexity classes, addition to the basic subset of any one of local variables, regular expression intersection, quantified variables, or declared properties with arguments is not harder

² PSL also offers a branching subset with CTL-like syntax and semantics. In this paper we consider only the linear subset of PSL.

than another. Statements like “local variables cannot be used in model checking” and “local variables can be used in model checking only if they are restricted to analogues of PSL quantified variables” are not justified by our complexity analysis. The jump from PSPACE-hard to EXPSPACE-hard is significant, though, and model checking of arbitrary properties in these extensions is not generally considered practical. This does not mean that such features should not be supported in model-checking tools. Consider the simple use of integer constant parameters in operators, such as $[*n]$ (repeat n times). This feature also makes the model-checking problem EXPSPACE-hard because integers have logarithmic representation [2]. We do not conclude that parameters should be written in unary or that declaration and instantiation of properties with arguments should be disallowed for formal verification. Such features are considered indispensable in industrially deployed languages. Similarly, local variables, regular expression intersection, and quantified variables are in the standardized assertion languages because they have proven to be very useful for writing properties in industrial practice. By understanding how the various language features contribute to the complexity, verification engineers can make more informed decisions about trade-offs in coding styles for properties and can better estimate their tractability in formal verification. Tool builders also can better target the sources of hardness for model checking while providing a richer feature set in the property language that is supported. Hopefully, the automata constructions from this paper will provide a starting point for broader EDA tool support for formal verification using SVA.

2 Preliminaries

Given a set A , A^* denotes the set of finite words over A , A^ω denotes the set of infinite words over A , and A^∞ denotes the union $A^* \cup A^\omega$. The length of word $u \in A^\infty$ is denoted $|u|$. The empty word is denoted ε . The letters of u are indicated by superscripts and are indexed consecutively beginning at zero. If $|u| > 0$, then the first letter of u is denoted u^0 ; if $|u| > 1$, then the second letter of u is denoted u^1 ; and so forth.

SVA has four language layers: boolean, sequence, property, and statement. The *boolean* layer consists of boolean expressions in which each variable referenced is either a design variable or a local variable of the assertion. Σ denotes the finite alphabet of valuations of the design variables. The *sequence* layer consists of regular expressions over the boolean layer. Every regular language of finite words over Σ can be represented by a suitably chosen sequence. The *property* layer combines sequences to create temporal logic formulas. The *statement* layer defines whether a property is to be evaluated as an obligation, an assumption, or a coverage goal. SVA statements are not discussed further in this paper.

For simplicity, we restrict each local variable to have a single-bit, boolean type (i.e., type `bit` in SystemVerilog). For a given finite set of sequences and properties, there is a finite set V of local variables that appear therein. The set of valuations of these local variables is 2^V . It is understood that V is disjoint from

the set of design variables. The set of semantic equivalence classes of boolean expressions can be identified with $2^{\Sigma \times 2^V}$. The constant “true” is denoted 1, and the constant “false” is denoted 0.

For the rest of this document we use the following notations: σ denotes a letter in Σ ; w, x, y, z denote finite or infinite words over Σ ; v denotes a one-bit local variable; b and e denote boolean expressions; R denotes a sequence; P denotes a property; and L denotes a local variable valuation in 2^V . Primes, subscripts, and superscripts are also added to these notations. The grammar for SVA sequences is

$$\begin{aligned} R ::= & b \mid (1, v = e) \mid (R) \mid R \#0 R \mid R \#1 R \mid R \text{ or } R \mid \\ & R \text{ intersect } R \mid R[*0] \mid R[*1:\$] \end{aligned}$$

$R[*0:\$]$ is an abbreviation for $R[*0]$ or $R[*1:\$]$, and, for $n > 0$, $R[*n]$ is an abbreviation for the concatenation $R \#1 \dots \#1 R$ (n copies of R).

The grammar for SVA properties is

$$P ::= R \mid (P) \mid P \text{ or } P \mid P \text{ and } P \mid R \mid\rightarrow P \mid \text{not } P$$

$R \text{ seq } P$ is an abbreviation for $\text{not}(R \mid\rightarrow (\text{not } P))$. The operator **seq** is the dual of $\mid\rightarrow$.³

Tight Satisfaction of Sequences. *Tight satisfaction* is a four-way relation, denoted $w, L_0, L_1 \models R$, that defines when a finite word w together with input local variable valuation L_0 satisfies a sequence R and yields output local variable valuation L_1 . Determination of whether the relation holds can be thought of as evaluation of R over w starting with local variable valuation L_0 . As R is evaluated, its local variables may be assigned, reassigned, and referenced at various points. In order for the result of the evaluation to be well-defined, a reference to a local variable must not be made unless the structure of R ensures that the local variable holds a well-defined value at that point. There is some subtlety to this requirement. For example, a local variable may be assigned inconsistently in the two operands of **intersect**, after which a well-defined value cannot be guaranteed.

The IEEE 1800 standard addresses this problem by restricting syntactically the places within a sequence or property at which a given local variable can be referenced.⁴ These restrictions ensure that references to local variables yield well-defined values [10]. For example, if a local variable is assigned in both operands of an **intersect**, then the local variable cannot be referenced after the **intersect** until it has been reassigned a well-defined value. We assume that all top-level properties satisfy the restrictions (e.g., as checked by a compiler). This allows us to simplify the definition of tight satisfaction from that given in [14].

For a boolean expression b and $(\sigma, L) \in \Sigma \times 2^V$, let $b[\sigma, L]$ denote the boolean value obtained by evaluating the expression b using the valuation (σ, L) of the design and local variables. The tight satisfaction relation is defined as follows:

³ “**seq**” is not an explicit operator of SVA. It is equivalent to the *follows_by* operator of ForSpec [4].

⁴ See the recursive functions *flow*, *sample*, and *block* defined in Annex E of [14].

- $w, L_0, L_1 \models b$ iff $|w| = 1$ and $b[w^0, L_0] = 1$ and $L_1 = L_0$.
- $w, L_0, L_1 \models (1, v = e)$ iff $|w| = 1$ and L_1 results from L_0 by assigning $e[w^0, L_0]$ to v .
- $w, L_0, L_1 \models (R)$ iff $w, L_0, L_1 \models R$.
- $w, L_0, L_1 \models R_1 \text{ \#\#0 } R_2$ iff there exist x, y, z, L' such that $w = xyz$ and $|y| = 1$ and $xy, L_0, L' \models R_1$ and $yz, L', L_1 \models R_2$.
- $w, L_0, L_1 \models R_1 \text{ \#\#1 } R_2$ iff there exist x, y, L' such that $w = xy$ and $x, L_0, L' \models R_1$ and $y, L', L_1 \models R_2$.
- $w, L_0, L_1 \models R_1 \text{ or } R_2$ iff either $w, L_0, L_1 \models R_1$ or $w, L_0, L_1 \models R_2$.
- $w, L_0, L_1 \models R_1 \text{ intersect } R_2$ iff there exist L', L'' such that $w, L_0, L' \models R_1$ and $w, L_0, L'' \models R_2$ and $L_1(v) = L'(v)$ if v is assigned in R_1 , and $L_1(v) = L''(v)$ otherwise.
- $w, L_0, L_1 \models R[*0]$ iff $|w| = 0$ and $L_1 = L_0$.
- $w, L_0, L_1 \models R[*1:\$]$ iff there exist $j \geq 1$ and $L_{(0)} = L_0, w_1, L_{(1)}, w_2, L_{(2)}, \dots, w_j, L_{(j)} = L_1$ such that $w = w_1 w_2 \dots w_j$ and for every i such that $1 \leq i \leq j$, $w_i, L_{(i-1)}, L_{(i)} \models R$.

The asymmetry of the definition of L_1 in the case of **intersect** is justified as follows. If v is assigned in both R_1 and R_2 , then the syntactic restrictions bar its reference after the **intersect** until it is reassigned. Therefore, we are free to let v take either the value from L' or the value from L'' .

Satisfaction of Properties. *Satisfaction* is a three-way relation, denoted $w, L \models P$, that defines when an infinite word w together with input local variable valuation L satisfies a property P

- $w, L \models R$ iff there exist x, y, L' such that $w = xy$, $|x| > 0$, and $x, L, L' \models R$.
- $w, L \models (P)$ iff $w, L \models P$.
- $w, L \models P_1 \text{ or } P_2$ iff either $w, L \models P_1$ or $w, L \models P_2$.
- $w, L \models P_1 \text{ and } P_2$ iff both $w, L \models P_1$ and $w, L \models P_2$.
- $w, L \models R \mid\rightarrow P$ iff for all x, y, z, L' such that $w = xyz$ and $|y| = 1$ and $xy, L, L' \models R$, $yz, L' \models P$.
- $w, L \models \text{not } P$ iff $w, L \not\models P$.

Let P be a top-level property. The syntactic restrictions on references to local variables guarantee that for any $L, L', w, L \models P$ iff $w, L' \models P$. We write $w \models P$ iff for some (equivalently, for all) $L \in 2^V$, $w, L \models P$, and we let $\mathcal{L}(P)$ denote the set $\{w \in \Sigma^\omega \mid w \models P\}$. A model satisfies P iff each of its infinite computation traces satisfies P .

An SVA property is in *positive normal form (PNF)* if it does not contain any **not** operator. Positive normal form can be achieved by using DeMorgan's laws and the duality of $\mid\rightarrow$ and **seq** to push all **not** operators down until they apply only to the boolean layer, where they can be absorbed into the boolean expressions. The case of **not** R is handled by using the fact that, as property, R is equivalent to $R \text{ seq } 1$.

Automata. A *nondeterministic finite word automaton* (NFW) is a tuple $N = \langle \Sigma, S, S_0, \rho, F \rangle$, where Σ is a finite alphabet, S is a finite set of states, $\rho \subseteq S \times \Sigma \times S$ is a transition relation,⁵ $S_0 \subseteq S$ is a set of initial states, and $F \subseteq S$ is a set of accepting states. A sequence $\xi = \xi^0 \xi^1 \dots \xi^k \in S^*$ ($k \geq 0$) is a *run* of N over the finite word $w \in \Sigma^*$ provided $k = |w|$, $\xi^0 \in S_0$, and for every $0 \leq i < |w|$, $(\xi^i, w^i, \xi^{i+1}) \in \rho$. The run ξ is *accepting* if $\xi^{|w|} \in F$. An NFW N accepts a word w if there exists an accepting run of N over w . We use $\mathcal{L}(N)$ to denote the set of words in Σ^* that are accepted by N .

A *nondeterministic Büchi automaton* (NBW) is a tuple $\langle \Sigma, S, S_0, \rho, F \rangle$ defined similarly to NFW. The automaton accepts an infinite word w iff it has an infinite run over w which contains infinitely many accepting states.

For a given set X , let $\text{Bool}^+(X)$ be the set of positive Boolean formulas over X (i.e., Boolean formulas built from elements in X using $\wedge$ and $\vee$), where we also allow the formulas **true** and **false**. Let $Y \subseteq X$. We say that Y *satisfies* a formula $\theta \in \text{Bool}^+(X)$ if the truth assignment that assigns *true* to the members of Y and assigns *false* to the members of $X \setminus Y$ satisfies θ . A *tree* is a prefix-closed subset $X \subseteq \mathbb{N}^*$.

An *alternating Büchi word automaton* (ABW) is a tuple $B = \langle \Sigma, Z, z_0, \delta, A \rangle$, where Σ , Z , and A are as Σ , S , and F (respectively) in the definition of NFW, $z_0 \in Z$ is a single initial state, and $\delta : Z \times \Sigma \rightarrow \text{Bool}^+(Z)$ is a transition function. A *run tree* of B on an infinite word $w \in \Sigma^\omega$ is a pair (X, τ) where X is a (possibly infinite) tree and $\tau : X \rightarrow Z$ is a labeling function such that $\tau(\varepsilon) = z_0$ and such that the following holds: if $x \in X$, $|x| = i$, $\tau(x) = z$, and $\delta(z, w^i) = \theta$, then x has k children $x_1, \dots, x_k$ in X for some $0 \leq k \leq |Z|$ and $\{\tau(x_1), \dots, \tau(x_k)\}$ satisfies θ . The run tree (X, τ) is *accepting* if every infinite branch in X has infinitely many labels in A . Note that the run tree can also have finite branches: if $|x| = i$, $\tau(x) = z$, and $\delta(z, w^i) = \text{true}$, then x need not have any children. B accepts a word $w \in \Sigma^\omega$ if there exists an accepting run tree of B over w . We use $\mathcal{L}(B)$ to denote the set of words in Σ^ω that are accepted by B .

An *alternating transition system* (ATS) is a tuple $B = \langle \Sigma, Z, Z_0, \delta, A \rangle$, where Σ , Z , δ , and A are as in the definition of ABW and $Z_0 \subseteq Z$ is a set of initial states. For $z \in Z_0$, $B^{(z)}$ is the ABW that results from B by replacing Z_0 by z .

SVA Subsets. The *basic subset* of SVA, denoted SVA^b , is obtained by limiting the sequence operators to **##1**, **##0**, **or**, **[*0]**, and **[*1:\$]**. All SVA property operators are allowed, but local variables and **intersect** are excluded. Every operator in SVA^b has an equivalent operator in PSL. We consider the following extensions to the basic subset: (1) SVA^{b+l} , obtained by adding local variables; (2) SVA^{b+i} , obtained by adding the **intersect** operator; (3) SVA^{b+d} , obtained by adding declared properties with arguments; and (4) SVA^{b+q} , obtained by adding quantified variables analogous to those in PSL.⁶ SVA^{b+l+i} denotes the extension of SVA^b by adding both local variables and the **intersect** operator. The quantified variables feature is defined as follows. Let P be a property with

⁵ ρ can also be given as a function $S \times \Sigma \rightarrow 2^S$.

⁶ SVA does not have quantified variables, so SVA^{b+q} is not, strictly speaking, a subset of SVA. However, SVA^{b+q} is equivalent to a subset of PSL.

free variable x . Then “**for** x **in** S : **and** P ” is equivalent to “**and** _{$s \in S$} $P|_{x \leftarrow s}$ ” and “**for** x **in** S : **or** P ” is equivalent to “**or** _{$s \in S$} $P|_{x \leftarrow s}$ ”. $P|_{x \leftarrow s}$ results from P by replacing every occurrence of x by s .

3 Automata Constructions for SVA

In [7] it is shown how to construct an ABW for each formula from a subset of PSL. The size of the ABW is linear in the size of the formula. Every operator in SVA^b has an equivalent operator in this subset of PSL. Thus, for every property P in SVA^b there exists an ABW with size linear in the size of P that accepts $\mathcal{L}(P)$. For every property P in SVA^{b+q} or in SVA^{b+d} there is a simple exponential translation to a property in SVA^b . Therefore, there exists an ABW with size exponential in the size of P that accepts $\mathcal{L}(P)$.

The construction presented in [7] first builds particular NFWs for the sequences and then uses these NFWs to construct the ABW. The construction of the ABW can take any NFW in place of the particular ones given for sequences, and the size of the ABW is linear in the sizes of the NFWs plus the number of property operators. While the PSL subset considered in [7] does not include intersection, an exponential construction for NFWs for regular expressions with the intersect operator is given in [13]. Using this construction, it follows that for every property P in SVA^{b+i} there exists an ABW with size exponential in the size of P that accepts $\mathcal{L}(P)$. In the rest of this section we present exponential ABW constructions for the SVA^{b+l} and SVA^{b+l+i} subsets.

NFW for Sequences with Local Variables. Let R be a sequence in SVA^{b+l} . We present an inductive construction of an NFW $N(R)$ for R . The construction will arrange a function λ mapping the set of states of $N(R)$ to 2^V . The base cases are as follows.

- $N(b) = \langle \Sigma, \{0, 1\} \times 2^V, \{0\} \times 2^V, \rho, \{1\} \times 2^V \rangle$, where ρ is the set of $((0, L), \sigma, (1, L))$ such that $b[\sigma, L] = 1$. $\lambda((0, L)) = \lambda((1, L)) = L$.
- $N(R[*0]) = \langle \Sigma, \{0\} \times 2^V, \{0\} \times 2^V, \emptyset, \{0\} \times 2^V \rangle$. $\lambda((0, L)) = L$.
- $N((1, v = e)) = \langle \Sigma, \{0, 1\} \times 2^V, \{0\} \times 2^V, \rho, \{1\} \times 2^V \rangle$, where ρ is the set of $((0, L), \sigma, (1, L'))$ such that L' results from L by assigning $e[\sigma, L]$ to v . $\lambda((0, L)) = \lambda((1, L)) = L$.

For the inductive cases, assume that we have constructed NFWs $N(R_1) = \langle \Sigma, S_1, I_1, \rho_1, F_1 \rangle$ and $N(R_2) = \langle \Sigma, S_2, I_2, \rho_2, F_2 \rangle$ and associated functions λ_1, λ_2 . The sets S_1 and S_2 are assumed to have been made disjoint.

- $N((R_1)) = N(R_1)$. $\lambda = \lambda_1$.
- $N(R_1 \text{ ## } 0 \ R_2) = \langle \Sigma, S_1 \cup S_2, I_1, \rho_1 \cup \rho_2 \cup \rho, F_2 \rangle$, where ρ is the set of (s_1, σ, s_2) such that there exist $s' \in F_1$ and $s'' \in I_2$ such that $(s_1, \sigma, s') \in \rho_1$ and $(s'', \sigma, s_2) \in \rho_2$ and $\lambda_1(s') = \lambda_2(s'')$. $\lambda = \lambda_1 \cup \lambda_2$.
- $N(R_1 \text{ ## } 1 \ R_2) = \langle \Sigma, S_1 \cup S_2, I, \rho_1 \cup \rho_2 \cup \rho, F_2 \rangle$, where $I = I_1 \cup I_2$ if $I_1 \cap F_1 \neq \emptyset$, $I = I_1$ otherwise, and ρ is the set of (s_1, σ, s_2) such that $s_2 \in I_2$ and there exists $s' \in F_1$ such that $(s_1, \sigma, s') \in \rho_1$ and $\lambda_1(s') = \lambda_2(s_2)$. $\lambda = \lambda_1 \cup \lambda_2$.

- $N(R_1 \text{ or } R_2) = \langle \Sigma, S_1 \cup S_2, I_1 \cup I_2, \rho_1 \cup \rho_2, F_1 \cup F_2 \rangle$. $\lambda = \lambda_1 \cup \lambda_2$.
- $N(R_1 [*1:\$]) = \langle \Sigma, S_1, I_1, \rho_1 \cup \rho, F_1 \rangle$, where ρ is the set of (s, σ, s') such that $s \in F_1$ and there exists $s'' \in I_1$ such that $(s'', \sigma, s') \in \rho_1$ and $\lambda_1(s) = \lambda_1(s'')$. $\lambda = \lambda_1$.

By $N(R)|_L$ we mean the automaton obtained from $N(R)$ by eliminating from the set of initial states those that are not mapped by λ to L .

Lemma 1. *Let R be a sequence in SVA^{b+l} , and let $w \in \Sigma^*$. $w, L_0, L_1 \models R$ iff $N(R)|_{L_0}$ has an accepting run over w that ends in a state mapped by λ to L_1 .*

Lemma 2. *Let R be a sequence in SVA^{b+l} , let V be the set of local variables in R , and let $S(R)$ be the set of states in the NFW $N(R)$. Then $|S(R)| = O(|R| \cdot 2^{|V|})$.*

ABW for Properties with Local Variables. Let P be a property in SVA^{b+l} . We present an inductive construction of an ATS $B(P)$ for P , which extend the construction presented in [7]. The construction will arrange a function λ mapping the set of states of $B(P)$ to 2^V in such a way that λ induces a bijection when restricted to the set of initial states. For $L \in 2^V$, $B(P)|_L$ is the ABW that results from $B(P)$ by restricting to the single initial state that is mapped by λ to L .

We assume that the properties are in PNF. Wherever a sequence R other than 1 or 0 appears as a property, we understand it to be replaced by $R \text{ seq } 1$. The base cases of the construction are as follows.

- $B(1) = \langle \Sigma, \{0\} \times 2^V, \{0\} \times 2^V, \delta, \emptyset \rangle$, where $\delta((0, L), \sigma) = \text{true}$. $\lambda((0, L)) = L$.
- $B(0) = \langle \Sigma, \{0\} \times 2^V, \{0\} \times 2^V, \delta, \emptyset \rangle$, where $\delta((0, L), \sigma) = \text{false}$. $\lambda((0, L)) = L$.

For the inductive cases, assume that we have constructed the NFW $N(R) = \langle \Sigma, S, I, \rho, F \rangle$ and function λ_R for sequence R and, for $i = 1, 2$, the ATS $B_i = \langle \Sigma, Z_i, Z_{i0}, \delta_i, A_i \rangle$ and function λ_i for property P_i . Let ρ be given as a function $2^S \times \Sigma \rightarrow 2^S$, where $\rho(S', \sigma) = \bigcup_{s \in S'} \rho(s, \sigma)$ for $S' \subseteq S$. Assume also that the state sets S, Z_1, Z_2 have been made disjoint and are disjoint from $\{0\} \times 2^V$. Let $I|_L = I \cap \lambda_R^{-1}(L)$ and, for $i = 1, 2$, let $z_{i0,L}$ be the unique state of Z_{i0} mapped by λ_i to L .

- $B((P_1)) = B(P_1)$. $\lambda = \lambda_1$.
- $B(R \mapsto P_1) = \langle \Sigma, (\{0\} \times 2^V) \cup S \cup Z_1, \{0\} \times 2^V, \delta \cup \delta_1, S \cup A_1 \rangle$, where δ is defined over $((\{0\} \times 2^V) \cup S) \times \Sigma$ as follows.
 - $\delta((0, L), \sigma) = \bigwedge \rho(I|_L, \sigma) \wedge \bigwedge \delta_1(Z, \sigma)$, where Z is the set of $z \in Z_{10}$ such that $\lambda_1(z) \in \lambda_R(\rho(I|_L, \sigma) \cap F)$.
 - $\delta(s, \sigma) = \bigwedge \rho(s, \sigma) \wedge \bigwedge \delta_1(Z, \sigma)$, where Z is the set of $z \in Z_{10}$ such that $\lambda_1(z) \in \lambda_R(\rho(s, \sigma) \cap F)$. $\lambda = \lambda' \cup \lambda_R \cup \lambda_1$, where λ' maps $\{0\} \times 2^V$ by $((0, L)) \mapsto L$.
- $B(R \text{ seq } P_1) = \langle \Sigma, (\{0\} \times 2^V) \cup S \cup Z_1, \{0\} \times 2^V, \delta \cup \delta_1, A_1 \rangle$, where δ is defined over $((\{0\} \times 2^V) \cup S) \times \Sigma$ as follows.
 - $\delta((0, L), \sigma) = \bigvee \rho(I|_L, \sigma) \vee \bigvee \delta_1(Z, \sigma)$, where Z is the set of $z \in Z_{10}$ such that $\lambda_1(z) \in \lambda_R(\rho(I|_L, \sigma) \cap F)$.
 - $\delta(s, \sigma) = \bigvee \rho(s, \sigma) \vee \bigvee \delta_1(Z, \sigma)$, where Z is the set of $z \in Z_{10}$ such that $\lambda_1(z) \in \lambda_R(\rho(s, \sigma) \cap F)$.

- $\lambda = \lambda' \cup \lambda_R \cup \lambda_1$, where λ' maps $\{0\} \times 2^V$ by $((0, L)) \mapsto L$.
- $B(P_1 \text{ or } P_2) = \langle \Sigma, (\{0\} \times 2^V) \cup Z_1 \cup Z_2, \{0\} \times 2^V, \delta \cup \delta_1 \cup \delta_2, A_1 \cup A_2 \rangle$, where δ is defined over $(\{0\} \times 2^V) \times \Sigma$ by $\delta((0, L), \sigma) = \delta_1(z_{10,L}, \sigma) \vee \delta_2(z_{20,L}, \sigma)$.
 - $B(P_1 \text{ and } P_2) = \langle \Sigma, (\{0\} \times 2^V) \cup Z_1 \cup Z_2, \{0\} \times 2^V, \delta \cup \delta_1 \cup \delta_2, A_1 \cup A_2 \rangle$, where δ is defined over $(\{0\} \times 2^V) \times \Sigma$ by $\delta((0, L), \sigma) = \delta_1(z_{10,L}, \sigma) \wedge \delta_2(z_{20,L}, \sigma)$.
- $\lambda = \lambda' \cup \lambda_1 \cup \lambda_2$, where λ' maps $\{0\} \times 2^V$ by $((0, L)) \mapsto L$.

Lemma 3. *Let P be a property in SVA^{b+l} in PNF, and let $w \in \Sigma^\omega$. $w, L \models P$ iff $w \in \mathcal{L}(B(P)|_L)$.*

Lemma 4. *Let P be a property in SVA^{b+l} in PNF, let V be the set of local variables in P , and let $Z(P)$ be the set of states in the ATS $B(P)$. Then $|Z(P)| = O(|P| \cdot 2^{|V|})$.*

The Intersect Operator. In this section we extend the construction of ABWs for properties in SVA^{b+l} to SVA^{b+l+i} . Assume that for the sequences R_1 and R_2 we have constructed the NFWs $N_1 = \langle \Sigma, S_1, I_1, \rho_1, F_1 \rangle$ with mapping $\lambda_1: S_1 \rightarrow 2^V$ and $N_2 = \langle \Sigma, S_2, I_2, \rho_2, F_2 \rangle$ with mapping $\lambda_2: S_2 \rightarrow 2^V$, respectively. We define $N(R_1 \text{ intersect } R_2) = \langle \Sigma, S_1 \times S_2, I, \rho, F_1 \times F_2 \rangle$, where $I = \{(i_1, i_2) \in I_1 \times I_2 \mid \lambda_1(i_1) = \lambda_2(i_2)\}$ and

$$\rho = \{((s_1, s_2), \sigma, (s'_1, s'_2)) \mid (s_1, \sigma, s'_1) \in \rho_1 \text{ and } (s_2, \sigma, s'_2) \in \rho_2\}$$

and we define $\lambda: S_1 \times S_2 \rightarrow 2^V$ by $\lambda((s_1, s_2))(v) = \lambda_1(s_1)(v)$ if v is assigned in R_1 and $\lambda((s_1, s_2))(v) = \lambda_2(s_2)(v)$ otherwise. The following lemma extends the result of Lemma 1 to SVA^{b+l+i} .

Lemma 5. *Let R be a sequence in SVA^{b+l+i} , and let $w \in \Sigma^*$. $w, L_0, L_1 \models R$ iff $N(R)|_{L_0}$ has an accepting run over w that ends in a state mapped by λ to L_1 .*

Lemma 6. *Let R be a sequence in SVA^{b+l+i} , let V be the set of local variables in R , and let $S(R)$ be the set of states in the NFW $N(R)$. If V is non-empty, then $|S(R)| = O(2^{|R| \cdot |V|})$. Otherwise $|S(R)| = O(2^{|R|})$.*

Lemma 7. *Let P be a property in SVA^{b+l+i} in PNF, let V be the set of local variables in P , and let $Z(P)$ be the set of states in the ATS $B(P)$. If V is non-empty, then $|Z(P)| = O(2^{|P| \cdot |V|})$. Otherwise $|Z(P)| = O(2^{|P|})$.*

Upper Bound Complexity. Following the automata-theoretic approach of [17], the satisfiability and model-checking problems for linear temporal logics are solved by representing the property by an ABW and then translating the ABW into an exponentially larger NBW [12]. Satisfiability is solved by checking for emptiness of the NBW, and model-checking is solved by checking for emptiness of the product of the model with the NBW for the negation of the property. Since the emptiness problem for NBWs is in NLOGSPACE [18], the complexity of these problems is in PSPACE with respect to the number of states of the ABW [17]. These observations and the automata constructions of the preceding sections lead to the following theorem.

Theorem 1

- The satisfiability and model-checking problems for properties in any of the extensions SVA^{b+i} , SVA^{b+q} , SVA^{b+d} are in EXPSPACE with respect to $|P|$.
- The satisfiability and model-checking problems for properties in SVA^{b+l} are in EXPSPACE with respect to $|V|$ and in PSPACE with respect to $|P|$.
- The satisfiability and model-checking problems for properties in SVA^{b+l+i} are in EXPSPACE with respect to $|V| \cdot |P|$.

4 Lower Bounds

In this section we explore some of the “sources of hardness” of SVA satisfiability and model-checking. In the previous section we showed that the satisfiability and model checking problems for properties in SVA^b are in PSPACE . A reduction from the PSPACE tiling problem [20,11,16] can be used to prove the following.

Proposition 1. *The satisfiability and model checking problems for properties in SVA^b are PSPACE -hard.*

When SVA^b is extended by local variables, regular expression intersection, declared properties with arguments, or quantified variables, the complexity of the model-checking and satisfiability problems become EXPSPACE -hard.

Theorem 2. *The satisfiability and model checking problems for properties in SVA^{b+l} , SVA^{b+i} , SVA^{b+d} , and SVA^{b+q} are EXPSPACE -hard.*

We present a proof sketch by outlining a reduction from a version of the EXPSPACE tiling problem [20,11,16] to each of the subsets. In the *EXPSPACE tiling problem*, the following are given: a finite set T of “tiles”, vertical and horizontal restrictive relations $\mathcal{V} \subseteq T \times T$ and $\mathcal{H} \subseteq T \times T$, an initial tile $\tau_0 \in T$, a final tile $\tau_a \in T$, and an integer $n > 0$. The problem is to decide whether there exists $m > 0$ such that there exists a tiling of a $2^n \times m$ grid such that the following hold: (1) τ_0 is in the bottom left corner; (2) the first occurrence of τ_a is in the top left corner; (3) every pair of horizontally neighboring tiles is in $\mathcal{H}$; and (4) every pair of vertically neighboring tiles is in $\mathcal{V}$.

Given an EXPSPACE tiling problem $\mathcal{T} = \langle T, \mathcal{V}, \mathcal{H}, \tau_0, \tau_a, n \rangle$, we define for each subset a property P , polynomial⁷ in n , such that there exists an infinite word w that satisfies P iff there exists a tiling for $\mathcal{T} = \langle T, \mathcal{V}, \mathcal{H}, \tau_0, \tau_a, n \rangle$. The idea is to partition w into substrings of length n called “blocks” such that every block represents one tile. The n letters in a block provide a binary encoding of a number between 0 and $2^n - 1$, inclusive. We refer to this number as the *counter value* of the block. The blocks are enumerated in the order of increasing counter values modulo 2^n , starting from 0. Pairs of consecutive blocks (except those enumerated $(2^n - 1, 0)$) must satisfy $\mathcal{H}$, and pairs of blocks whose counter values are equal and are separated by $2^n - 1$ intermediate blocks must satisfy $\mathcal{V}$.

⁷ Quadratic for SVA^{b+i} and SVA^{b+d} , linear for the other extensions.

We construct the properties for the reductions to the different subsets in two parts. The *shared part* is common to all the properties and is in SVA^b . For each subset we construct its *specific part* using the extending feature of the subset.

The alphabet for all properties is $2^{\{\clubsuit, \diamond, c, z\}} \times T$, where $\clubsuit$ is a special signal that marks the beginning of a block, $\diamond$ is a special signal that is first high one letter after the last block of the tiling, c is a one-bit signal used to represent the counter value of the block, and z is an auxiliary variable used to represent the carry over when incrementing the counter value. We use the convention that the value of c in the first (resp., last) letter of a block is the least (resp., most) significant bit of the counter value. The set T of tiles is of constant size. The T component of a letter is referenced by the signal t . The tile represented by a block is understood to be the value of t in its last letter.

For lack of space, we do not present the shared part. It is obtained by creating properties in SVA^b of size $O(n)$ requiring that (1) a prefix of w be partitioned into blocks of length n , the first letter of each being indicated by $\clubsuit$; (2) the counter values of the blocks increment modulo 2^n starting at 0; (3) $\diamond$ appear for the first time after a block with counter value $2^n - 1$ and mark the first letter after the end of the prefix; (4) the first block's tile value be τ_0 ; (5) the tile value of the first block in the last row be τ_a ; and (6) every two consecutive blocks in the same row be in $\mathcal{H}$. It remains to construct for each of the extensions the specific part of the property requiring that a pair of blocks with the same counter value that are separated by $2^n - 1$ intermediate blocks be in $\mathcal{V}$.

For the local variable extension, we let $R_{c \neq v}$ denote a sequence that is tightly satisfied by a block iff the counter value of the block is different than the binary value represented by the local variables $v_0, v_1, \dots, v_{n-1}$. Such a sequence can be created with size $O(n)$. Also, we let $R_{c=v}$ denote the sequence

$c == v_0 \ \#\#1 \ c == v_1 \ \#\#1 \ \dots \ \#\#1 \ c == v_{n-1},$

which is tightly satisfied by a block iff the counter value of the block is equal to the binary value represent by $v_0, v_1, \dots, v_{n-1}$. The specific part P_Y^l for the local variable extension is as follows:

```
(1[*n] ##0 t!= $\tau_a$ )[*0:$] ##1
(1,  $v_0=c$ ) ##1 (1,  $v_1=c$ ) ##1  $\dots$  ##1 (1,  $v_{n-1}=c$ ) ##0 t!= $\tau_a$  |->
and $_{\tau_1 \in T}$  (t== $\tau_1$  |-> 1 ##1  $R_{c \neq v}$  [*1:$] ##1  $R_{c=v}$  ##0 || $_{\mathcal{V}(\tau_1, \tau_2)}$  t== $\tau_2$ )
```

After any number of blocks with tile other than τ_a , the counter value is stored in $v_0, v_1, \dots, v_{n-1}$. Then a conjunction over $\tau_1 \in T$ of implications whose antecedents test $t == \tau_1$ ensures that $(\tau_1, \tau_2) \in \mathcal{V}$, where τ_2 is the tile of the next block at which the counter value equals the binary value saved in $v_0, v_1, \dots, v_{n-1}$. The size of P_Y^l is $O(n)$.

The specific part for the extension by quantified variables is similar:

```
for  $v_0, \dots, v_{n-1}$  in bit: and
(1[*n] ##0 t!= $\tau_a$ )[*0:$] ##1  $R_{c=v}$  ##0 t!= $\tau_a$  |->
and $_{\tau_1 \in T}$  (t== $\tau_1$  |-> 1 ##1  $R_{c \neq v}$  [*1:$] ##1  $R_{c=v}$  ##0 || $_{\mathcal{V}(\tau_1, \tau_2)}$  t== $\tau_2$ )
```

Instances of properties with arguments can be used to universally quantify the arguments. In this way, we can adapt the specific part with quantified

variables to serve as the specific part using declared properties with arguments. Here is an example to illustrate. Let $P(a, b, c)$ be a declared property with arguments. Define $P_1(b, c) = P(1, b, c)$ and $P(0, b, c)$, $P_2(c) = P_1(1, c)$ and $P_1(0, c)$, $P_3 = P_2(1)$ and $P_2(0)$. The property P_3 is equivalent to $\forall a \forall b \forall c : P(a, b, c)$. The adaptation involves n declared properties, each of size $O(n)$, so the specific part is of size $O(n^2)$.

For the specific part using intersect, we assume that the following auxiliary sequences of size $O(n)$ have been constructed. For $0 \leq i \leq n - 2$, R_i is a sequence that is tightly satisfied by a series of two or more blocks provided the i th bits of the counters of the first and last blocks are equal. R_{n-1} is a sequence that is tightly satisfied by a series of two or more blocks provided the $(n-1)$ st bits of the counters of the first and last blocks are equal and, in addition, counter bit $n - 1$ changes its value exactly twice in the intermediate blocks. The minimum number of intermediate blocks is 2^{n-1} , and the maximum number is $3 \cdot 2^{n-1} + 2$. Inside these boundaries, distinct blocks have distinct counter values. The sequence $R_{\text{tiles in } \mathcal{V}}$ is tightly satisfied by a series of blocks provided the pair formed by the tiles of the first and last blocks is in $\mathcal{V}$. From these auxiliary sequences, we construct $R_{\text{next}} = R_0 \text{ intersect } \cdots \text{ intersect } R_{n-1}$, which is tightly satisfied by a series of blocks provided the first and last blocks have the same counter value and there are exactly $2^n - 1$ intermediate blocks. R_{next} is of size $O(n^2)$. The specific part $P_{\mathcal{V}}^i$ is as follows:

```
(1[*n - 1] ##1 t!=τa) [*0:$] ##1 1 |->
(1[*n - 1] ##1 t==τa) or (Rnext intersect Rtiles in V)
```

The consequent is tightly satisfied by a series of two or more blocks provided that either the tile of the first block is τ_a or the first and last blocks have the same counter value, their tiles are in $\mathcal{V}$, and there are exactly $2^n - 1$ intermediate blocks. The property $P_{\mathcal{V}}^i$ requires the consequent for every block before τ_a occurs.

5 Conclusions and Related Work

There has been some prior work on the complexity of satisfiability and model checking of PSL and similar languages. In [4], the complexity of the ForSpec language is shown to be PSPACE-complete. In [3] there is a discussion of the complexity of the *reset operator* in Sugar and ForSpec. Symbolic verification of PSL properties restricted to negated sequences is explored in [6]. A construction of alternating Büchi automata for a large subset of PSL is presented in [7]. The automata from this construction run over both finite and infinite words, and the construction accounts for the neutral semantic variant for finite words.

In this paper we have explored the complexity of SVA. We have shown that when features like local variables, regular expression intersection, and declared properties with argument are used, the complexity of the language becomes EXPSPACE-complete. These results invite further investigation of the practical usage of these features and the performance of model-checking algorithms that support them. Other theoretical gaps in the complexity analysis remain. For

example, the expressiveness and complexity of SVA with recursive properties and the complexity of checking SVA in simulation are still open.

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