Credit Risk Evaluation Based on LINMAP

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Abstract. This paper deals with customer's credit risk assessment. We put forward a method based on the Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP). The method is used to develop a credit risk assessment model using a large sample of firms derived from the loan portfolio of a leading Construction Bank of China. The model gave us the method to determine the optimal credit evaluation weights and ideal point. Further we give an example for its application.

1 Introduction

Credit risk assessment is a significant area of financial management which is of major interest to practitioners, financiers and credit analysts. Considerable attention has been paid in this field from the theoretical and academic points during the last several decades[1-17]. Financial and operational researchers have tried to relate the characteristics of a firm (Financial ratios and strategic variables) to its credit risk[1-5]. According to this relationship the components of credit risk are identified, and decision models are developed to assess credit risk and the corresponding credit worthiness of firms as accurately as possible. The models include Z-score[6,7], discriminant analysis (DA)[8], logit analysis (LA) and probit analysis (PA) and more.

The main purpose of this paper is to investigate the potentials and the applicability of a new discrimination method in credit risk assessment, based on the linear programming technique for multidimensional analysis of preference (LINMAP)[9]. The LINMAP method employs a distance discrimination procedure to determine the class to which the firms under consideration belong.

2 The Linear Programming Technique Based on Data Mining

Supposing the commercial bank will provides a loan for m customers, to evaluate the customers' credit, they employ n indexes. So m firms and n indexes form a credit decision space[10-17]. In this credit decision space, every customer's credit corresponds to a point respectively. If the customer's best credit (the bank's preference) can be denoted with an ideal point in the decision space, the customer's credit can be

described with a weighted distance. In the credit decision space, the weighted distance between every point $(x_{i1}, x_{i2}, \dots, x_{in})$ and ideal point $(x_1^*, x_2^*, \dots, x_n^*)$ is

$$d_i = \left[\sum_{j=1}^n w_j (x_{ij} - x_j^*)^2 \right]^{\frac{1}{2}}, \quad i = 1, 2, \dots, m. \quad (1)$$

Where, $w_j, (j = 1, 2, \dots, n)$ is the i -th index's weight. It presents the importance of i -th index. At the same time, the square distance can be denoted

$$S_i = d_i^2 = \sum_{j=1}^n w_j (x_{ij} - x_j^*)^2, \quad i = 1, 2, \dots, m. \quad (2)$$

In (2), w and x^* are unknown. We can get them by using a training sample.

Using the information in the training sample, we have a sequence relation (F_k, F_l) . For simplicity, we substitute (k, l) for sequence relation (F_k, F_l) . The (k, l) means that k -th customer's credit is better than l -th customer's. Further, we can get a sequence relation set $Q = \{(k, l)\}$. Obviously, there are $n(n-1)/2$ elements in set Q , if we compare every two customer's credit.

We can calculate the weighted distance between k -th customer or l -th customer and ideal point, if we can find out w and x^* . They are

$$S_k = \sum_{j=1}^n w_j (x_{kj} - x_j^*)^2 \quad \text{and} \quad S_l = \sum_{j=1}^n w_j (x_{lj} - x_j^*)^2. \quad (3)$$

In (3), x_{kj} is the value of the k -th customer to the j -th index. Similarly the value of the l -th customer to the j -th index is x_{lj} .

In set Q , if every sequence relation (k, l) satisfied

$$S_l \geq S_k, \quad (4)$$

the k -th customer's credit is not worse than the l -th customer's. It is consistent with the information in training sample. Otherwise, if

$$S_l < S_k, \quad (5)$$

the k -th customer's credit is worse than the l -th customer's. It is not consistent with information in training sample. Apparently, if the training sample is large enough, the weighted distance should be in accordance with the information of training sample. For measuring the degree of inconsistency between the weighted distances and the training sample information, we have to give a definition.

Definition 1. let $B = \sum_{(k,l) \in Q} (S_l - S_k)^-$, where

$$(S_l - S_k)^- = \begin{cases} 0 & , \text{ if } S_l \geq S_k, \\ S_k - S_l & , \text{ if } S_l < S_k, \end{cases} = \max(0; (S_k - S_l)). \quad (6)$$

B is called inconsistent degree between the weighted distances and the training sample information.

From definition 1, we know

$$(S_l - S_k)^- = 0 \text{ if } S_l \geq S_k \text{ and } (S_l - S_k)^- > 0 \text{ if } S_l < S_k$$

The more different between S_l and S_k , the inconsistent degree is deeper.

$(S_l - S_k)^- = S_k - S_l$ can measure the inconsistent degree. To finding out optimal w^* and x^* , we have to minimize B .

Definition 2. let $G = \sum_{(k,l) \in Q} (S_l - S_k)^+$, where

$$(S_l - S_k)^+ = \begin{cases} S_l - S_k & , \text{ if } S_l \geq S_k, \\ 0 & , \text{ if } S_l < S_k, \end{cases} = \max(0; (S_l - S_k)), \quad (7)$$

calling G is consistent degree between the weighted distances and the training sample information. From definition 2, we know

$$(S_l - S_k)^- = 0 \text{ if } S_l < S_k; \quad (S_l - S_k)^- > 0 \text{ if } S_l \geq S_k.$$

The more different between S_l and S_k , the consistent degree is deeper.

$(S_l - S_k)^- = S_l - S_k$ can measure the consistent degree.

We can minimize B , which subject to $G > B$ or $G - B = h > 0$, where h is a small positive real number. From definition 1 and definition 2, we have

$$(S_l - S_k)^+ - (S_l - S_k)^- = S_l - S_k. \quad (8)$$

Then

$$\begin{aligned} G - B &= \sum_{(k,l) \in Q} (S_l - S_k)^+ - \sum_{(k,l) \in Q} (S_l - S_k)^- \\ &= \sum_{(k,l) \in Q} [(S_l - S_k)^+ - (S_l - S_k)^-] \\ &= \sum_{(k,l) \in Q} (S_l - S_k) = h. \end{aligned} \quad (9)$$

So, we can get w and x^* by solving problem (10).

$$\begin{aligned} \min B = & \max_{(k,l) \in Q} \{0, (S_k - S_l)\} \\ s.t. & (S_l - S_k) = h. \end{aligned} \quad (10)$$

Solving problem (10) is equivalent to solving problem (11).

$$\left\{ \begin{array}{l} \min \sum_{(k,l)} \lambda_{kl} \\ s.t. \quad S_l - S_k + \lambda_{kl} \geq 0, \text{ all } (k,l) \in Q \\ \sum_{(k,l)} (S_l - S_k) = h \\ \lambda_{kl} \geq 0, \text{ all } (k,l) \in Q. \end{array} \right. \quad (11)$$

From (3), we have

$$\begin{aligned} S_l - S_k &= \sum_{j=1}^n w_j (x_{lj} - x_j^*)^2 - \sum_{j=1}^n w_j (x_{kj} - x_j^*)^2 \\ &= \sum_{j=1}^n w_j (x_{lj}^2 - x_{kj}^2) - 2 \sum_{j=1}^n w_j x_j^* (x_{lj} - x_{kj}). \end{aligned} \quad (12)$$

Because w_j and x_j^* are unknown, we institute variable v_j for $w_j x_j^*$. So, problem (11) is equivalent to problem (13)

$$\left\{ \begin{array}{l} \min_{(k,l)} \lambda_{kl} \\ s.t. \sum_{j=1}^n w_j (x_{lj}^2 - x_{kj}^2) - 2 \sum_{j=1}^n v_j (x_{lj} - x_{kj}) + \lambda_{kl} \geq 0, \\ all (k,l) \in Q, \\ \sum_{j=1}^n w_j \sum_{(k,l) \in Q} (x_{lj}^2 - x_{kj}^2) - 2 \sum_{j=1}^n v_j \sum_{(k,l) \in Q} (x_{lj} - x_{kj}) = h, \\ w_j \geq 0, j = 1, 2, \dots, n, \\ \lambda_{kl} \geq 0, all (k,l) \in Q. \end{array} \right. \quad (13)$$

By solving (13), we get all indexes' optimal weight w^* and ideal point in credit decision space x^* .

$$w^* = \begin{pmatrix} w_1^* \\ w_2^* \\ \vdots \\ w_n^* \end{pmatrix}, v^* = \begin{pmatrix} v_1^* \\ v_2^* \\ \vdots \\ v_n^* \end{pmatrix} \text{ and } x^* = \begin{pmatrix} v_1^*/w_1^* \\ v_2^*/w_2^* \\ \vdots \\ v_n^*/w_n^* \end{pmatrix}.$$

Then we can compute the weighted distance S between every customer's credit and ideal point. The value of S is smaller, the customer's credit is better.

3 An Example for Application

To illustrate the model of the LINMAP method, consider a simple example consisting of 50 firms $F_1, F_2, \dots, F_{50}$, evaluated along five financial ratios (x_1 : retained earnings/total assets, x_2 : earnings before interest and taxes/total assets, x_3 : net profit/total assets, x_4 : net profit after taxes/total assets, x_5 : liquid assets/total assets). The firms are

Table 1. Data in the training sample

Indexes x_{ij} Firms						Categories
	x_1	x_2	x_3	x_4	x_5	
F1	0.02	0.03	0.01	0.61	0.03	1.00
F2	0.06	0.13	0.03	0.66	0.11	1.00
F3	0.07	-0.13	-0.13	0.29	0.21	1.00
F4	0.11	0.08	0.01	0.46	0.57	1.00
F5	0.13	0.07	-0.03	0.55	0.53	1.00
F6	0.04	0.06	0.02	0.26	0.29	2.00
F7	0.05	0.32	0.01	0.24	0.17	2.00
F8	0.07	0.08	0.01	0.26	0.33	2.00
F9	0.11	0.01	0.00	0.26	0.36	2.00
F10	0.12	0.04	0.01	0.16	0.28	2.00
F11	0.20	0.08	0.00	0.66	0.79	3.00
F12	0.21	-0.08	-0.02	0.79	0.99	3.00
F13	0.23	0.00	0.00	0.77	1.00	3.00
F14	0.42	-0.04	0.00	0.81	0.99	3.00
F15	0.45	0.12	0.09	0.47	0.92	3.00
F16	0.08	0.18	0.14	0.54	0.62	4.00
F17	0.25	0.30	0.01	0.18	0.43	4.00
F18	0.32	0.17	0.11	0.52	0.84	4.00
F19	0.34	0.13	0.17	0.17	0.51	4.00
F20	0.35	0.02	0.14	0.65	1.00	4.00
F21	0.19	0.10	0.03	0.57	0.75	5.00
F22	0.40	0.09	-0.01	0.33	0.73	5.00
F23	0.42	0.37	0.10	0.50	0.92	5.00
F24	0.49	0.05	0.03	0.38	0.88	5.00
F25	0.57	0.01	0.01	0.24	0.81	5.00

Table 2. Square distance of firms in training sample

Firms	Square distance					Categories
F1~F5	0.0954	0.0966	0.1011	0.1049	0.1036	1
F6~F10	0.1050	0.1058	0.1055	0.1058	0.1058	2
F11~F15	0.1063	0.1072	0.1081	0.1058	0.1093	3
F16~F20	0.1068	0.1092	0.1086	0.1083	0.1083	4
F21~F25	0.1068	0.1083	0.1115	0.1092	0.1097	5

Table 3. Data in examination sample

Firms	Indexes x_{ij}	x_1	x_2	x_3	x_4	x_5	Categories
F26		0.00	0.04	0.04	0.82	0.62	1
F27		0.03	0.06	0.01	0.74	0.37	1
F28		0.05	0.00	0.01	0.57	0.62	1
F29		0.07	0.18	-0.03	0.58	0.64	1
F30		0.07	0.01	0.00	0.72	0.59	1
F31		0.04	0.00	0.00	0.40	0.44	2
F32		0.04	0.10	0.01	0.50	0.34	2
F33		0.07	0.10	0.01	0.17	0.24	2
F34		0.07	0.12	0.04	0.45	0.45	2
F35		0.13	0.04	0.04	0.23	0.36	2
F36		0.09	0.08	0.00	0.70	0.79	3
F37		0.15	0.09	0.00	0.65	0.80	3
F38		0.17	-0.12	0.00	0.65	0.82	3
F39		0.41	0.09	-0.01	0.58	0.71	3
F40		0.46	-0.04	-0.01	0.79	0.99	3
F41		0.03	0.08	0.04	0.42	0.71	4
F42		0.13	0.16	0.14	0.23	0.36	4
F43		0.14	0.11	0.04	0.38	0.53	4
F44		0.17	0.12	0.18	0.24	0.41	4
F45		0.19	0.08	0.08	0.34	0.53	4
F46		0.23	0.11	0.01	0.27	0.50	5
F47		0.33	0.08	0.07	0.46	0.79	5
F48		0.37	0.19	0.01	0.62	0.99	5
F49		0.45	0.00	0.06	0.43	0.89	5
F50		0.55	0.18	0.05	0.26	0.80	5

Table 4. Square distance of firms in examination sample

Firms	Square distance					Categories
F26~F30	0.1038	0.0999	0.1051	0.1023	0.1029	1
F31~F35	0.1045	0.1042	0.1060	0.1047	0.1056	2
F36~F40	0.1070	0.1072	0.1054	0.1068	0.1057	3
F41~F45	0.1069	0.1067	0.1063	0.1065	0.1062	4
F46~F50	0.1097	0.1076	0.1099	0.1083	0.1108	5

divided into two groups (a training sample and a examination sample) and classified into five categories. Table 1 illustrates the performances of the firms according to each ratio and their classification.

From Table 1, we have a sequence relation set. So, we have a linear programming problem. Solving it, we can get the optimal weights w^* and ideal point x^* .

$$w^* = \begin{pmatrix} 0.0699 \\ 0.0100 \\ 0.0100 \\ 0.1154 \\ 0.0100 \end{pmatrix}, \quad x^* = \begin{pmatrix} 0.8735 \\ -4.6423 \\ 0.3161 \\ 1.0913 \\ -8.9744 \end{pmatrix}.$$

Table 2 gives the square distance of firms in training sample. Then, we use an examination sample (as Table3) for model validation. Easily, we can obtain the results as table4 shows. Combining Table4 with Table3 and Table2, we can find that the model developed provided high classification accuracy. Only credit categories of F28, F33, F35, F36, F37 and F39 are different from examination sample information.

4 Conclusions

This paper focused on the identification of commercial bank customer's credit. The approach employed for developing credit assessment model is based on LINMAP approach. The method employs mathematical programming techniques to develop credit rating model. In this paper the objective was to find out a method of credit discrimination. Such a discrimination method supports credit analysts in identifying potential defaulters, thus facilitating credit-granting decisions. Of course, the applications of the LINMAP method are not only limited to credit assessment; it also involves other financial risk management fields.

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