Stochastic Modeling and Quality Evaluation of Workflow Systems Based on QWF-Nets*

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Abstract. Quality (QOS) prediction is one of the most important research topics of workflow management system. In this paper, we propose the SWQ approach to analytically evaluate QOS of workflow systems based on QWF-net, which extends traditional WF-net by associating tasks with exponential response time and exponential TTF (time-to-failure). The comparison between simulative and analytical results in the case study indicates that the SWQ approach achieves satisfactory accuracy. The paper concludes with a comparison between the SWQ approach and other related work.

1 Introduction

With the advent and evolution of global scale economies, organizations need to be more competitive, efficient and flexible. In the past decade, workflow techniques have been widely used to address these needs.

However, among many research topics of workflow, performance/quality analysis is yet to be given the importance it deserves. Techniques and models [1-8] for QOS evaluation are still preliminary and limited. This paper introduces an analytical approach, the SWQ (meaning a stochastic approach for quality evaluation of workflow systems) to address the need for QOS evaluation. The SWQ approach is based on QWF-net (stochastic WF-net) model, which is an extension of traditional WF-net where tasks are associated with exponential response time and exponential TTF (time-to-failure). By mapping the execution of QWF-net into a homogeneous continuous Markovian process, the SWQ approach calculates its expected completion-time and reliability.

Through comparing simulative results and results obtained by SWQ approach, the case-study indicates that our approach achieves satisfactory accuracy. The paper concludes with a comparison between the SWQ approach and related work.

* Supported by the National Natural Science Foundation of China under Grant No. 60173002 and the National Grand Fundamental Research 973 Program of China under Grant No. 2002CB312004.

2 QWF-Net for Quality Prediction

The Workflow net (WF-net) proposed by van der Aalst is a high level Petri Nets with two special places i and o , which indicate the beginning and the end of the modeled process. There exist four routing patterns in WF-nets, namely sequence, parallel, selective and iterative. WF-net does not care the concept of time, but sometimes we need to consider time aspect in workflow management systems. For example, we want to know the completion time of the whole workflow net or of some subnets so that we can decide whether the arrangement of the workflow system meets constraint of time. So introducing time concept into WF-net is necessary.

This section extends WF-net to QWF-net by associating an exponential response time and exponential TTF (Time-to-failure) with each task.

Definition 1. (QWF-net) $N = (P, T, Task, \lambda, \mu)$ is a QWF-net if and only if:

1. N is structurally a WF-net
2. SPLIT/JOIN transitions (black thin bars in Fig.1) fire immediately and have response time of 0
3. SPLIT/JOIN transitions never fail
4. The set $Task \subseteq T$ denotes the set of transitions excluding SPLIT/JOIN transitions (as illustrated by white bars in Fig.1)
5. Each task has an exponential response time. A function $\lambda : Task \rightarrow Real$ is used to identify parameter (also known as **execution-rate**) of the exponential response time of each task
6. Each task fails independently and has an exponential TTF. A function $\mu : Task \rightarrow Real$ is used to identify parameter (also known as **failure-rate**) of the exponential TTF of each task

It easily follows that QWF-net is identical with WF-net in construction aspect. Therefore, structural properties of WF-net also follow in QWF-net: there should be no dead tasks; the procedure should terminate eventually; at the moment the procedure terminates there should be one token in sink place o and all the other places are empty; the definition of reachable markings and its corresponding calculation methods for WF-net can also be applied to QWF-net.

3 Quality Evaluation Based on QWF-Net

Besides assumptions about exponential task response time and TTF, this paper also assumes that:

1. The control flow randomly chooses its path on **XOR-split** according to given choice probabilities. For generality, this paper uses a function $se : Task \rightarrow Real$ to denote the probability that each task is selected when its preceding task finishes execution. Note that, if a task is not on any XOR-split, its choice probability equals 1, otherwise smaller than 1

- 2. The control flow skips **loop** when current iteration finishes according to a given probability. For generality, this paper uses a function $lo : Task \rightarrow Real$ to denote the probability that the control flow skips each task when current iteration finishes. Note that, if a task is not on any loop, its corresponding skipping probability equals 1, otherwise smaller than 1. Therefore, for a task t_i , its expected number of loop iterations is $\frac{1}{lo(t_i)}$, following a **geometric distribution**.

3.1 Evaluating the Expected Completion Time (ECT)

This subsection presents an analytical method to calculate the expected completion time of QWF-net based on response time of each task. Let $U(t)$ denote the set of operational tasks in QWF-net at time t (execution begins at time 0), then its state-space (denoted by S) is obtained through mapping each reachable marking into a corresponding set of operational tasks.

For any reachable marking $M_1 \in [M_0 >$ where no SPLIT/JOIN transitions are activated, there exists a state which records all operational tasks in this marking. Since SPLIT/JOIN transitions fire immediately with response time 0, there is no need to generate states indicating whether SPLIT/JOIN transitions are activated or not. Take Fig.1 for example, the marking illustrated in this figure where P_1 contains a token is one such negligible marking since this marking merely indicates that $AND-split_1$ transition is activated. Also note that the marking where only sink place contains a token is mapped into an **absorbing state** which records no task is operational, meaning all tasks are idle and the control flow terminates.

The state space of $U(t)$ of Fig.1 are illustrated in Table.1. Note that, there exist more than one **initial-state** since $XOR-SPLIT_1$ may generate one token into place P_4 , P_5 or P_6 . S_{12} is the absorbing state.

Table 1. State space

state	operational tasks	state	operational tasks
S_1 (Initial-state)	$\{t_1, t_6\}$	S_7	$\{t_3, t_8\}$
S_2 (Initial-state)	$\{t_1, t_7\}$	S_8	$\{t_3\}$
S_3 (Initial-state)	$\{t_1, t_8\}$	S_9	$\{t_6\}$
S_4	$\{t_1\}$	S_{10}	$\{t_7\}$
S_5	$\{t_3, t_6\}$	S_{11}	$\{t_8\}$
S_6	$\{t_3, t_7\}$	S_{12} (Absorbing-state)	$\emptyset$

As mentioned in the assumption, a task t_i has exponential response time with parameter $\lambda(t_i)$ and the probability that control flow skips task t_i when current iteration of t_i finishes is $lo(t_i)$ ($lo(t_i) = 1$ if t_i is not on loop). The number of loop iterations of t_i , N_{t_i} , is a geometric distributed random variable with parameter $lo(t_i)$. Since task may be iteratively executed, this paper uses D_{t_i} to denote the total response time of t_i considering iterative execution and X_{t_i} to denote

Fig. 1. A QWF-net example (*Case₅*)

response time of one single iteration. According to the definition of QWF-net, X_{t_i} follows exponential distribution. Therefore, the cumulative-distribution-function (CDF) of D_{t_i} is given as

$$\begin{aligned}
 F(y) &= Prob\{D_{t_i} \leq y\} \\
 &= \sum_{K=1}^{\infty} Prob\{N_{t_i} = k\} Prob\{N_{t_i} \times X_{t_i} \leq y | N_{t_i} = K\} \\
 &= \sum_{K=1}^{\infty} lo(t_i)(1 - lo(t_i))^{K-1} E_K(y)
 \end{aligned} \tag{1}$$

where $E_K(y)$ denotes the CDF of **K-phase Erlang distribution**.

Then, the density-distribution-function (PDF) of D_{t_i} is given as

$$\begin{aligned}
 f(y) &= F'(y) \\
 &= \sum_{K=1}^{\infty} lo(t_i)(1 - lo(t_i))^{K-1} \frac{\lambda(t_i)(y\lambda(t_i))^{K-1}}{(K-1)!} e^{-\lambda(t_i)y} \\
 &= \lambda(t_i)lo(t_i)e^{-y\lambda(t_i)} \sum_{K=1}^{\infty} \frac{((1 - lo(t_i))y\lambda(t_i))^{K-1}}{(K-1)!} \\
 &= \lambda(t_i)lo(t_i)e^{-y\lambda(t_i)} \times e^{(1-lo(t_i))\lambda(t_i)y} \\
 &= \lambda(t_i)lo(t_i)e^{-\lambda(t_i)lo(t_i)y}
 \end{aligned} \tag{2}$$

where $\frac{\lambda(t_i)(y\lambda(t_i))^{K-1}}{(K-1)!} e^{-\lambda(t_i)y}$ is the PDF of the K-phase Erlang distribution.

According to the equation above, D_{t_i} follows exponential distribution with parameter $\lambda(t_i)lo(t_i)$. Since the total response time D_{t_i} of every task t_i follows exponential distribution, $U(t)$ is a homogeneous continuous Markovian process. The infinitesimal generator matrix Q of $U(t)$ is given as

$$q_{i,j} = \begin{cases} lo(t_l) \times \lambda(t_l) \times \prod_{t_m \in NEW(i,j)} se(t_m) & \text{if } S_i \xrightarrow{t_l} S_j \\ -\sum_{1 \leq r \leq W, r \neq i} q_{i,r} & \text{if } i = j \\ 0 & \text{else} \end{cases} \tag{3}$$

where $lo(t_l) \times \lambda(t_l)$ denotes the parameter of the exponential random variable D_{t_l} , W denotes the number of states in the state space, and $q_{i,j}$ denotes the transition rate from state S_i to S_j .

Relation $S_i \xrightarrow{t_l} S_j$ means that S_j is the resulting state of S_i if the operational task t_l in S_i finishes execution and becomes idle. Note that, there may exist more than one resulting states of S_i when t_l becomes idle because transition may activate choice (XOR-split). Those resulting states are viewed as different **types** in the Markovian chain according to the **phase-type** property and $\prod_{t_m \in NEW(i,j)} se(t_m)$ denotes the occurrence probability of S_j among all types, where **NEW**(i,j) denotes the set of newly-emerging operational tasks in the transition from state S_i to S_j .

Take the transition from state S_8 to S_{12} for example, we have its transition rate $q_{8,12}$ as $lo(t_3)\lambda(t_3)$ according to Eq.3.

The expected time which a state S_i takes to reach the absorbing state (time-to-termination) is defined as ETT_{S_i} , where

$$ETT_{S_i} = \begin{cases} 0 & \text{absorbing-state} \\ \frac{1}{E_{S_i}} + \sum_{1 \leq k \leq W, k \neq i} \frac{q_{i,k} \times ETT_{S_k}}{E_{S_i}} & \text{else} \end{cases} \quad (4)$$

where $E_{S_i} = \sum_{1 \leq j \leq W, j \neq i} q_{i,j}$

According to the equation above, the expected time for S_i to reach termination is the expected duration of state S_i ($\frac{1}{E_{S_i}}$) plus the averaged (by occurrence probabilities) time-to-termination of its immediate succeeding states.

Therefore, the expected completion-time of QWF-net is the averaged (by occurrence probability) time-to-termination of all initial states. Let $Init$ and $BUSY_{S_i}$ denote the set of initial states and the set of operational tasks in state S_i respectively, then the QWF-net's expected completion time ECT is

$$ECT = \sum_{S_i \in Init} (ETT_{S_i} \times \prod_{t_j \in BUSY_{S_i}} se(t_j)) \quad (5)$$

3.2 Evaluating Reliability

The reliability estimate of t_i is obtained through integrating the probability that its TTF is larger than t (meaning the probability that t_i does not fail till time t) multiplied by PDF of D_{t_i} over the interval from 0 to ∞ . Therefore, reliability of task t_i , R_{t_i} , is given by

$$\begin{aligned} R_{t_i} &= \int_0^\infty \lambda(t_i) lo(t_i) e^{-\lambda(t_i) lo(t_i) t} \times Prob\{TTF_{t_i} > t\} dt \\ &= \int_0^\infty \lambda(t_i) lo(t_i) e^{-\lambda(t_i) lo(t_i) t} \times e^{-\mu(t_i) t} dt \\ &= \frac{\lambda(t_i) lo(t_i) \int_0^\infty (\lambda(t_i) lo(t_i) + \mu(t_i)) e^{-(\lambda(t_i) lo(t_i) + \mu(t_i)) t} dt}{\lambda(t_i) lo(t_i) + \mu(t_i)} \\ &= \frac{\lambda(t_i) lo(t_i)}{\lambda(t_i) lo(t_i) + \mu(t_i)} \end{aligned} \quad (6)$$

where $\lambda(t_i) lo(t_i) e^{-\lambda(t_i) lo(t_i) t}$ is PDF of the D_{t_i} and $e^{-\mu(t_i) t}$ is the probability that TTF (time-to-failure) of t_i is greater than t .

Therefore, QWF-net's reliability is the averaged reliability of all initial states

$$Reliability = \sum_{S_i \in Init} (R_{S_i} \times \prod_{t_j \in BUSY_{S_i}} se(t_j)) \quad (7)$$

where R_{S_i} denotes the reliability of state S_i

$$R_{S_i} = \begin{cases} 1 & \text{absorbing-state} \\ \sum_{\text{Every } S_j \text{ satisfying } S_i \xrightarrow{t_l} S_j} R_{t_l} \times R_{S_j} \times \frac{q_{ij}}{E_{S_i}} & \text{else} \end{cases} \quad (8)$$

where $E_{S_i} = \sum_{1 \leq j \leq W, j \neq i} q_{i,j}$

4 Case Study and Simulation

This section applies the SWQ approach to some examples and studies its accuracy through a comparison with Monte-Carlo simulation. The examples are given by Fig.2. Tasks involved are listed in Table.2. *Case*₁₋₄ are four simple examples dealing with sequential, parallel, selective and iterative routing modes, respectively. *Case*₅ (given earlier in Fig.1) and *Case*₆ are more complex examples featured by all routing modes.

A simulation procedure is developed to evaluate expected completion time and reliability of QWF-net. The simulation procedure is similar to simulation

Fig. 2. Cases

Table 2. Tasks involved in the cases

Tasks	λ	μ	se	lo	Tasks	λ	μ	se	lo
t_1	0.3	0.0026	1	1	t_7	0.3	0.0013	0.2	1
t_2	0.2	0.0028	1	1	t_8	0.45	0.0014	0.1	1
t_3	0.4	0.0053	1	0.33	t_9	0.3	0.0064	1	1
t_4	0.65	0.0041	1	1	t_{10}	0.45	0.0054	1	1
t_5	0.6	0.0051	1	1	t_{11}	0.8	0.0023	1	1
t_6	0.2	0.0037	0.7	1	t_{12}	0.25	0.0058	1	1

algorithms for SPN (stochastic Petri-net) performance analysis, which use random variable generators to decide firing delay of each transition at each simulation run. The procedure is executed for sufficiently many times. At each run, the procedure randomly chooses a path along XOR-split and decides the numbers of loop iterations of loops according to predefined probabilities. Then the procedure uses exponential-distribution random variable generators to generate response time of each task. Also, the program uses random variable generators to generate the TTF of every task. At each run, if all tasks' TTF are greater than their response time (meaning no failure happens during the execution of this task), a success is recorded. The simulative estimates of completion-time is obtained through averaging completion-time of all runs. The simulative estimate of reliability is the ratio of successes to the number of all runs.

Results obtained by simulation (illustrated in normal style) are compared with those by the SWQ approach (illustrated in bold style) in Table.3. As shown, analytical results is pretty close to simulative results. It indicates that SWQ approach achieves satisfactory accuracy.

Table 3. Comparison between simulative and SWQ results

case	ECT	Reliability	case	ECT	Reliability
<i>Case₁</i>	9.8879/ 9.8718	97.25%/ 97.16%	<i>Case₄</i>	7.4942/ 7.5000	96.07%/ 96.13%
<i>Case₂</i>	6.5456/ 6.5480	95.42%/ 95.32%	<i>Case₅</i>	11.8686/ 11.8661	93.94%/ 93.99%
<i>Case₃</i>	4.3883/ 4.3889	98.60%/ 98.61%	<i>Case₆</i>	25.5926/ 25.5539	86.07%/ 86.11%

5 Comparison with Related Work

Research of [2,3] uses a reduction technique to simplify sequential, parallel, selective and iterative routing patterns into a single transition with equivalent QOS estimate, however these methods are not very realistic since they assume tasks in WF-net have deterministic response time rather than nondeterministic as this paper does.

Methods of [1,7,8] are similar to the SWQ approach in that they also model the control flow as continuous Markov chains. These models map execution of each task into each state of CTMC chain and analytically evaluate transition probability or transition rate between states. However these methods can not model parallel execution of more than one tasks in their Markov chains.

Method proposed by [5] develops a simplification technique to simplify four basic routing patterns into a single task with approximate equivalent performance and assumes that the **equivalent task** still follows exponential distribution if all tasks have exponential response time. However, its assumption that the simplified construct still follows exponential distribution is obviously inaccurate and unrealistic. For instance, *n* tasks with exponential response time of the same execution rate arranged by sequential routing pattern should be simplified into a single equivalent task of *n*-phase Erlang response time (which is obviously not exponential).

6 Conclusion

This paper proposes the SWQ method to analytically evaluate QOS (expected-completion-time and reliability) of workflow systems based on the QWF-net model, which extends traditional WF-net by associating tasks with exponential response time and TTF. This paper also develops a simulation procedure to calculate the simulative QOS results. The comparison between simulative and analytical results in the case study indicates that SWQ approach achieves satisfactory accuracy.

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