Quantum-Behaved Particle Swarm Optimization Algorithm with Controlled Diversity

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Abstract. Premature convergence, the major problem that confronts evolutionary algorithms, is also encountered with the Particle Swarm Optimization (PSO) algorithm. In the previous work [11], [12], [13], the Quantum-behaved Particle Swarm (QPSO) is proposed. This novel algorithm is a global-convergence-guaranteed and has a better search ability than the original PSO. But like other evolutionary optimization technique, premature in the QPSO is also inevitable. In this paper, we propose a method of controlling the diversity to enable particles to escape the sub-optima more easily. Before describing the new method, we first introduce the origin and development of the PSO and QPSO. The Diversity-Controlled QPSO, along with the PSO and QPSO is tested on several benchmark functions for performance comparison. The experiment results testify that the DCQPSO outperforms the PSO and QPSO.

1 Introduction

The Particle Swarm Optimization (PSO), first introduced by Kennedy and Eberhart [5], is a stochastic optimization that can be likened to the behavior of a flock of birds. It has been used to solve a range of optimization problems, including neural network training [6] and function minimization. Since its origin in 1995, several attempts have been made to improve the performance of the original PSO ([1], [2], [4], [8], [9], [10], [17]). The original PSO, however, is not a global optimization algorithm, as has been demonstrated by F. van den Bergh in [3]. In our previous work [11], [12], [13], we proposed a novel global-convergence-guaranteed PSO, Quantum-behaved Particle Swarm Optimization (QPSO).

Like other evolutionary algorithm, the PSO as well as QPSO, confront the problem of premature convergence, which results in great performance loss and sub-optimal solutions. In the PSO or QPSO, the fast information flow between particles seems to be the reason for clustering of particles. Diversity declines rapidly, leaving the PSO or QPSO algorithm leads to low diversity with fitness stagnation as an overall result. In this paper, we propose a Diversity-Controlled Quantum-behaved Particle Swarm Optimization (DCQPSO). In the DCQPSO, a threshold value was set for population's diversity measure to prevent premature convergence and therefore enhance the overall performance of the QPSO. The rest part of the paper is organized as follows. In the

next section, the PSO is introduced. The origin and development of QPSO is described in Section 3 and the DCQPSOs are proposed in Section 4. Section 5 is the experiment results and discussion. Some conclusion remarks are given in Section 6.

2 PSO Algorithms

The PSO is a population-based optimization technique, where a population is called a swarm. Each particle represents a possible solution to the optimization task at hand. During each iteration each particle accelerates in the direction of its own personal best solution found so far, as well as in the direction of the global best position discovered so far by any of the particles in the swarm. Let M denote the swarm size. Each individual i ($1 \leq i \leq M$) has the following attributes: A current position in the search space $X_i = (x_{i,1}, x_{i,2}, \dots, x_{i,n})$, a current velocity $V_i = (v_{i,1}, v_{i,2}, \dots, v_{i,n})$, and a personal best position in the search space $P_i = (p_{i,1}, p_{i,2}, \dots, p_{i,n})$. During each iteration, each particle in the swarm is updated using (1) and (2). Assuming that the function f is to be minimized, that the swarm consists of n particles, and that $r_1 \sim U(0,1)$, $r_2 \sim U(0,1)$ are elements from two uniform random sequences in the range $(0,1)$, then

$$v_{i,j}(t+1) = w \cdot v_{i,j}(t) + c_1 \cdot r_{1,i}(t) \cdot [p_{i,j}(t) - x_{i,j}(t)] + c_2 \cdot r_{2,i}(t) \cdot [p_{g,j}(t) - x_{i,j}(t)] \quad (1)$$

for all $j \in 1, 2, \dots, n$, thus, $v_{i,j}$ is the velocity of the j th component of the velocity of the i th particle, and c_1 and c_2 denote the acceleration coefficients. P_g is the global best position found by any particle during all previous steps. The new position of a particle is calculated using

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (2)$$

The value of each component in every V_i vector can be clamped to the range $[-v_{\max}, v_{\max}]$ to reduce the likelihood of particles leaving the search space. The variable w in (1) is called the inertia weight, whose value is typically setup to vary linearly from 1 to near 0 during the course of training run. Acceleration coefficients c_1 and c_2 also control how far a particle will move in a single iteration.

3 Quantum-Behaved Particle Swarm Optimization

Trajectory analyses in [5] demonstrated that, to guarantee convergence of the PSO algorithm, each particle must converge to its local attractor $q = (q_1, q_2, \dots, q_D)$, of which the coordinates are:

$$q_d = (c_1 r_1 p_{id} + c_2 r_2 p_{gd}) / (c_1 r_1 + c_2 r_2), \quad (d=1, 2, \dots, n) \quad (3)$$

or

$$q_d = \varphi \cdot p_{id} + (1 - \varphi) \cdot p_{gd}, \quad \varphi \sim U(0,1), \quad (d=1, 2, \dots, n) \quad (4)$$

Assume that there is one-dimensional Delta potential well on each dimension at point q and each particle has quantum behavior. For simplicity, we consider a particle in one-dimensional space, with point q the center of potential well. Solving the *Schrödinger equation*, we can get the normalized the following probability distribution function F

$$F(y) = \int_{-\infty}^y Q(y) dy = e^{-2|q-x|/L} \quad (5)$$

where L determines search scope of each particle. Employing Monte Carlo method, we can obtain the position of the particle

$$x = q \pm \frac{L}{2} \ln(1/u) \quad u = \text{rand}(0,1) \quad (6)$$

where u is a random number uniformly distributed in $(0, 1)$.

A global point called Mainstream Thought or Mean Best Position of the population is introduced into PSO. The global point, denoted as $mbest$, is defined as the mean of the $pbest$ positions of all particles. That is

$$mbest = \frac{1}{M} \sum_{i=1}^M P_i = \left(\frac{1}{M} \sum_{i=1}^M p_{i1}, \frac{1}{M} \sum_{i=1}^M p_{i2}, \dots, \frac{1}{M} \sum_{i=1}^M p_{id} \right) \quad (7)$$

where M is the population size and P_i is the $pbest$ position of particle i . Then the value of L and the position are evaluated by

$$L = 2\beta \cdot |mbest - x| \quad (8)$$

$$x := q \pm \beta \cdot |mbest - x| \cdot \ln(1/u) \quad (9)$$

where parameter β is called Contraction-Expansion Coefficient, which could be tuned to control the convergence speed of the algorithms. The PSO with equation (19) is called Quantum-behaved Particle Swarm Optimization (QPSO).

4 Diversity-Controlled QPSO

As we know, a major problem with PSO and other evolutionary algorithms in multi-modal optimization is premature convergence, which results in great performance loss and sub-optimal solutions. In a PSO system, with the fast information flow between particles due to its collectiveness, diversity of the particle swarm declines rapidly, leaving the PSO algorithm with great difficulties of escaping local optima. Therefore, the collectiveness of particles leads to low diversity with fitness stagnation as an overall result. In QPSO, although the search space of an individual particle at each iteration is the whole feasible solution space of the problem, diversity loss of the whole population is also inevitable due to the collectiveness.

In 2002, Ursem has proposed a model called Diversity-Guided Evolutionary Algorithm (DGEA) [14], which applies diversity-decreasing operators (selection,

recombination) and diversity-increasing operators (mutation) to alternate between two modes based on a distance-to-average-point measure. The performance of the DGEA clearly shows its potential in multi-modal optimization.

Also in 2002, Riget *et al* [15] adopted the idea from Ursem into the basic PSO model with the decreasing and increasing diversity operators used to control the population. This modified model of PSO uses a diversity measure to have the algorithm alternate between exploring and exploiting behavior. They introduced two phases: attraction and repulsion and the swarm alternate between these phases according to its diversity. The improved PSO algorithm is called Attraction and Repulsion PSO (ARPSO) algorithm.

Inspired by works undertaken by Ursem and Riget *et al*, we introduce the Diversity-Controlled model in Quantum-behaved PSO. The diversity is measure by the following formula.

$$diversity(S) = \frac{1}{|S| \cdot |A|} \cdot \sum_{i=1}^{|S|} \sqrt{\sum_{j=1}^D (x_{ij} - \overline{x_j})^2} \quad (10)$$

where S is the swarm, $|S|=M$ is the population size, $|A|$ is the length of longest the diagonal in the search space, D is the dimensionality of the problem, x_{ij} is the j th value of the i th particle and $\overline{x_j}$ is the j th value of the average point.

But unlike to their works, we only set a low bound to the diversity of the population. The procedure of the algorithm is as follows. After initialization, the algorithm is running in attraction mode that guaranteed by setting the value of β smaller than 1.0. In [13], using stochastic simulation, we found out that the particle will converge when $\beta < 1.778$ and otherwise will diverge. In this paper, the attraction mode is realized by varying β from 1.0 to 0.5 over the running. That is,

$$\beta = (1.0 - 0.5) \times (MAXITER - t) / MAXITER - 0.5 \quad (11)$$

where $MAXITER$ is the maximum number of iterations and t is the number of current iteration. In the course of the running of the algorithm, if the diversity measure of the population declines to below the threshold value d_{low} , the particles will explode to increase the diversity until it is larger than d_{low} . There are two method of making the particle to explode. One method is to control the Contraction-Expansion Coefficient β . That is, we can set $\beta = \beta_0$ ($\beta_0 > 1.778$) once the diversity is lower than the threshold value d_{low} . In this paper the DCQPSO using this method is called DCQPSO1. The other method of increasing the diversity is initializing the Mean Best Position ($mbest$) of the population across the search space once the diversity is smaller than d_{low} . The reason for the initialization of $mbest$ is that when the diversity is low, the distance between the particle and the $mbest$ position is too small for the particle to escape the local optima as can be seen from equation (9). Therefore initializing the $mbest$ could enlarge the gaps between particles and the $mbest$ position, consequently making particles explode temporarily. The DCQPSO with this operation is called DCQPSO2. The DCQPSO algorithms are formulated as follows.

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Initialize the population
for t=1 to MAXITER
    Compute the mbest of the population;
    measure the diversity of the population by (10)
     $\beta = (1.0 - 0.5) * (\text{MAXITER} - t) / \text{MAXITER} + 0.5$ ; (in attraction
mode)
    if (diversity < dlow) (in explosion mode)
         $\beta = \beta 0$ ; (for DCQPSO1)
        (or initializing the mbest; (for DCQPSO2));
    endif
    for i=1 to population size M;
        if  $f(P_i) < f(X_i)$  then  $X_i = P_i$ 
        pg=argmin( $P_i$ );
        for d=1 to dimension n
             $f_i = \text{rand}(0,1)$ ;
             $q = f_i * p_{id} + (1 - f_i) * p_{gd}$ ;
             $u = \text{rand}(0,1)$ ;
            if  $\text{rand}(0,1) > 0.5$ ;
                 $X_{id} = q - \beta * \text{abs}(m_{bestd} - x_{id}) * \log(1/u)$ ;
            else
                 $X_{id} = q + \beta * \text{abs}(m_{bestd} - x_{id}) * \log(1/u)$ ;
            endif
        endfor
    endfor
endfor

```

It can be seen that the DCQPSO runs in attraction mode during the most of the iterations. Only when the diversity falls below the *dlow* do the particles fly in explosion mode. The explosion process is transitory, and once diversity is over the threshold, the population will return to attraction mode again.

5 Experiment Results and Discussion

We have tested the QPSO and DCQPSO on four widely known benchmark functions for testing the performance of different evolutionary optimization strategies. These functions are all minimization problems with minimum value zero. The four test functions are listed in Table 1. In all performance tests, the initial range of the population in all cases listed in Table 1 is asymmetry. Table 1 also lists $V_{\max}$ for original PSO.

The fitness value is set as function value and the neighborhood of a particle is the whole population. The population size is set to be 20 for all cases. We test original PSO with inertia weight (called Standard PSO or SPSO), QPSO and DCQPSOs for performance Comparison. We had 50 trial runs for every instance and recorded mean best fitness. The maximum number of iterations is set as 1000, 1500 and 2000 generations corresponding to the dimensions 10, 20 and 30 for first three functions, respectively, The dimension of the last functions is 2 and the maximum number of iterations is 2000 for this function. In performance test of the SPSO, the inertia weight

Table 1. The table lists expression of benchmark function, the initial range of the population in the performance tests. The third column is the upbound of the velocity of the particle in the case of the SPSO.

Functions	Formulations	Initial Range	$X_{\max}$	$V_{\max}$
Rosenbrock function f_2	$f(x)_2 = \sum_{i=1}^n (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$	(15, 30)	100	100
Rastrigrin function f_3	$f(x)_3 = \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i) + 10)$	(2.56, 5.12)	10	10
Griewank function f_4	$f(x)_4 = \frac{1}{4000} \sum_{i=1}^n (x_i - 100)^2 - \prod_{i=1}^n \cos(\frac{x_i - 100}{\sqrt{i}}) + 1$	(300, 600)	600	600
Shaffer's function f_5	$f(x)_7 = 0.5 + \frac{(\sin \sqrt{x^2 + y^2})^2 - 0.5}{(1.0 + 0.001(x^2 + y^2))^2}$	(30, 100)	100	100

Table 2. Average best fitness and standard deviation of all algorithms on Rosenbrock function

	Dimension	MAXITER	Mean Value	St. Dev.
SPSO	10	1000	94.1276	194.3648
	20	1500	204.337	293.4544
	30	2000	313.734	547.2635
QPSO	10	1000	59.4764	153.0842
	20	1500	110.664	149.5483
	30	2000	147.609	210.3262
DCQPSO1	10	1000	19.0109	29.5079
	20	1500	82.0134	84.4259
	30	2000	111.9926	164.3119
DCQPSO2	10	1000	34.6391	63.4889
	20	1500	102.1606	178.6908
	30	2000	128.0084	160.3456

Table 3. Average best fitness and standard deviation of all algorithms on Rastrigrin function

	Dimension	MAXITER	Mean Value	St. Dev.
SPSO	10			
		1000	5.5382	3.0477
	20	1500	23.1544	10.4739
QPSO	30	2000	47.4168	17.1595
	10	1000	5.2543	2.8952
	20	1500	16.2673	5.9771
DCQPSO1	30	2000	31.4576	7.6882
	10	1000	4.8308	2.4628
	20	1500	13.0424	4.8795
DCQPSO2	30	2000	22.7104	5.5532
	10	1000	4.6831	3.6387
	20	1500	15.3056	11.8478
	30	2000	24.2655	6.4856

w is decreases linearly from 0.9 to 0.4 as in [17]. In performance tests for QPSO and DCQPSOs, the Contraction-Expansion Coefficient β varies from 1.0 to 0.5 linearly when the algorithms are running. For DCQPSOs, this parameter control means that the population is in attraction mode. The threshold value of the diversity measure d_{low} is to be 0.005. Moreover, in DCQPSO1, the value of β_0 is set as 2.0.

The mean values and standard deviations of best fitness for 50 runs of each function are recorded in Table 2 to Table 5. The numerical results show that the DCQPSOs works better on the first three functions than QPSO and SPSO. DCQPSO1 has slightly better performance than DCQPSO2. On Shaffer's function, the performances of DCQPSOs are not improved.

Table 4. Average best fitness and standard deviation of all algorithms on Greiwank function

	Dimension	MAIXTER	Mean Value	St. Dev.
		1000		
SPSO	10		0.09217	0.0833
	20	1500	0.03002	0.03255
	30	2000	0.01811	0.02477
QPSO	10	1000	0.08331	0.06805
	20	1500	0.02033	0.02257
	30	2000	0.01119	0.01462
DCQPSO1	10	1000	0.0781	0.0745
	20	1500	0.0189	0.0229
	30	2000	0.0090	0.0132
DCQPSO2	10	1000	0.0655	0.0464
	20	1500	0.0202	0.0204
	30	2000	0.0087	0.0122

Table 5. Average best fitness and standard deviation of all algorithms on Shaffer's function

	Dimension	MAXITER	Mean Value	St. Dev.
SPSO	2	2000	2.78E-04	0.001284
QPSO	2	2000	0.001361	0.003405
DCQPSO1	2	2000	0.0012	0.0032
DCQPSO2	2	2000	0.0019	0.0039

6 Conclusion

In this paper, we proposed a method of controlling the diversity measure of the QPSO. The methodology of DCQPSO is setting a threshold value for the diversity to prevent the particles from clustering. Controlling of the diversity has been testified to be a good technique for enhancing the performance of the QPSO. However, for other benchmark functions, such as Sphere function and Ackley function, the results of this method is poor, since the optima of these functions is easy to find and the local search

ability is key to the performance of the algorithm. Annealing the diversity threshold d_{low} could lead to improvements, because it may be an advantage to decrease d_{low} near the end of the optimization.

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