Dynamic Resource Allocation Mechanism for Network Interconnection Management

Michał Karpowicz[1] and Krzysztof Malinowski[1,2]

[1] Institute of Control and Computation Engineering, Warsaw University of Technology, Nowowiejska 15/19, 00-665 Warsaw, Poland
[2] NASK (Research and Academic Computer Network), Wawozowa 18, 02-796 Warsaw, Poland
M.Karpowicz@elka.pw.edu.pl, K.Malinowski@ia.pw.edu.pl

Abstract. We propose a dynamic resource allocation mechanism which can be used in multi-agent computer network interconnection management systems. Considering a setting of multiple consumers and elastic supply we argue that interaction between autonomous system resource managers is a game. We make the following contributions. First, we analyze the stability of the Nash equilibrium point of the resource allocation game. Second, we show that in comparison to the Cournot mechanism the mechanism we propose may lead to solutions which are characterized by a larger aggregate surplus.

1 Introduction

This paper is concerned with the dynamic resource allocation problem in a large scale, decentralized systems, namely bandwidth allocation problem between Internet service providers. It is well known, that if networks are to offer global services, service providers must cooperate and exchange Internet traffic. The obvious and intensively examined question that arises here is: how can network resources be allocated in an efficient way when network providers are most likely to act in their own self interest? We apply the game theoretic approach to this problem and propose a resource allocation mechanism which can be used in multi-agent computer network interconnection management systems.

Consider a group of interconnected network service providers and focus on dynamics of interactions between their *autonomous systems*. The basic observation we make suggests that since the number of interacting autonomous systems is small, local decisions concerning network resource allocations can have a significant influence on global network performance. Thus, decision makers, i.e. resource managers, may anticipate the effects of their actions on resource prices and view these prices as functions of the actions of all decision makers. This kind of interaction between autonomous system resource managers is a game.

We follow the approach applied by Johari and Tsitsiklis [3, 4, 5]. We analyze the resource allocation game through its *Nash equilibria* and design a simple market-clearing mechanism. The approach we apply is characterized by the three salient features. First, the monetary value of resource allocation is measured by

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aggregate utility less aggregate cost. Second, the system sets the single price for resource unit to ensure demand equals supply. Third, agents anticipate the effect of their actions on market-clearing prices.

As we demonstrate in the following sections, the approach based on a simple market-clearing mechanisms may lead to solutions which are characterized by a very low computational and communicational complexity. From the network management point of view this features are crucial. Furthermore, results of Johari show that under reasonable assumptions the game we consider has a unique Nash equilibrium point. We use this results as a springboard for our mechanism design and propose a resource allocation mechanism which converges to this unique Nash equilibrium point.

This paper makes two contributions. First, we show global stability of the Nash equilibrium point of the game we consider. Second, we show that in comparison to the Cournot mechanism the mechanism we propose may lead to solutions that are characterized by a larger aggregate surplus. At this point we note that when agents are price anticipating Nash equilibria of the game we consider do not generally ensure full efficiency. Therefore, our propositions should be considered within the context of results of Johari and Tsitsiklis.

2 The Model

Following Kelly [1, 2] and Johari [3, 4, 5] suppose L agents compete for a single resource. Let a_i denote the rate allocated to agent i. First, we assume that:

Assumption 1. *For each $i \in L$, over the domain $a_i \geq 0$ the utility function $u_i(a_i)$ is concave, strictly increasing, and continuous, and over the domain $a_i > 0$, $u_i(a_i)$ is continuously differentiable. Furthermore, the right directional derivative at 0, denoted $u_i'(0^+)$, is finite.*

The total cost of using the amount $x = \sum_{i \in L} a_i$ of a resource is $c(x)$. We make the following assumption:

Assumption 2. *There exists a continuous, convex, strictly increasing price function $p(x)$ over $0 \leq x < C$ with $p(0) \geq 0$, such that for $0 \leq x < C$:*

$$c(x) = \int_0^x p(y)dy. \tag{1}$$

Thus total cost function $c(x)$ is strictly convex and strictly increasing over $0 \leq x < C$.

Suppose that the total payment agent i is willing to make for the amount a_i of resource is w_i. Let $w = [w_1, ..., w_L]^T$.

Assumption 3. *For all $w \geq 0$, the aggregate allocation $x(w) = \sum_{i \in L} a_i(w)$ is the unique solution to $\sum_{i \in L} w_i = x(w)p(x(w))$. Furthermore, for each $i \in L$:*

$$a_i(w) = \begin{cases} w_i/p(x(w)), & w_i > 0 \\ 0, & w_i = 0 \end{cases}. \tag{2}$$

Assumption 4. *$p(x)$ is differentiable and exhibits nondecreasing elasticity $\varepsilon(x) = (\partial p(x)/\partial x) \cdot (x/p(x))$ for $0 \leq x < C$.*

Now, consider a game in which, having $w_{-i} = [w_1, ..., w_{i-1}, w_{i+1}, ..., w_L]^T$ fixed, player i selects $w_i \geq 0$ to maximize his payoff:

$$Q_i(a_i(w_i, w_{-i}), w_i) = u_i(a_i(w_i, w_{-i})) - w_i. \tag{3}$$

Agent i does not know w_{-i}, but he knows that the price $p(x(w))$ and $a_i(w)$ depend on it. Furthermore, he anticipates his own influence on the price. Denote the right directional derivative of $f(x)$ at x by $\partial^+ f(x)/\partial x$ and the left directional derivative by $\partial^- f(x)/\partial x$. The following results hold:

Proposition 1. ***(Johari)*** *If Assumptions 1-3 hold, then w is a Nash equilibrium of the game defined by $(Q_i(a_i(w), w_i))_{i \in L}$, if and only if $\sum_{i \in L} w_i > 0$, and for $a_i \equiv a_i(w) > 0$ and $x \equiv x(w)$ the following two conditions hold:*

$$u_i'(a_i)\left(1 - \frac{\frac{x}{p(x)}\frac{\partial^+ p(x)}{\partial x}}{1 + \frac{x}{p(x)}\frac{\partial^+ p(x)}{\partial x}}\frac{a_i}{x}\right) \leq p(x); u_i'(a_i)\left(1 - \frac{\frac{x}{p(x)}\frac{\partial^- p(x)}{\partial x}}{1 + \frac{x}{p(x)}\frac{\partial^- p(x)}{\partial x}}\frac{a_i}{x}\right) \geq p(x). \tag{4}$$

Conversely, if $a(w) \geq 0$ and $x > 0$ satisfy (4), and $x(w) = \sum_{i \in L} a_i(w)$, then the vector $w = [p(x(w)) \cdot a_1(w), ..., p(x(w)) \cdot a_L(w)]^T$ is a Nash equilibrium.

Proposition 2. ***(Johari)*** *Suppose Assumptions 1-4 hold. Then there exists a unique Nash equilibrium $\hat{w}$ for the game defined by $(Q_i(a_i(w), w_i))_{i \in L}$.*

Proposition 3. *Suppose Assumptions 1-4 hold and $\hat{w}$ is Nash equilibrium of the game defined by $(Q_i(a_i(w), w_i))_{i \in L}$. Then the following conditions hold:*

$$(\forall i \in L) \quad p(x(\hat{w})) \leq u_i'(a_i(\hat{w})) \leq p(x(\hat{w}))(1 + \varepsilon(x(\hat{w}))). \tag{5}$$

Proof. Denote $\beta(x) = \varepsilon(x)/(1 + \varepsilon(x))$. Notice, that at equilibrium:

$$(\forall i \in L) \quad u_i'(a_i(\hat{w}))(1 - \frac{\beta(x(\hat{w}))}{x(\hat{w})}a_i(\hat{w})) - p(x(\hat{w})) = 0 \quad \Leftrightarrow$$

$$u_i'(a_i(\hat{w})) - p(x(\hat{w})) = u_i'(a_i(\hat{w}))\frac{\beta(x(\hat{w}))}{x(\hat{w})}a_i(\hat{w}) \geq 0$$

Now, suppose $u_i'(a_i(\hat{w})) > p(x(\hat{w}))(1 + \varepsilon(x(\hat{w})))$. Then we obtain:

$$(\forall i \in L) \quad p(x(\hat{w}))(1 + \varepsilon(x(\hat{w})))(1 - \frac{\beta(x(\hat{w}))}{x(\hat{w})}a_i(x(\hat{w}))) < p(x(\hat{w})) \quad \Rightarrow$$

$$1 + \varepsilon(x(\hat{w})) - \frac{\varepsilon(x(\hat{w}))}{x(\hat{w})}a_i(x(\hat{w})) < 1 \quad \Rightarrow \quad \varepsilon(x(\hat{w}))(1 - \frac{a_i(\hat{w})}{x(\hat{w})}) < 0.$$

So we obtain that $(\forall i \in L)$ $a_i(\hat{w}) > x(\hat{w}) = \sum_{j \in L} a_j(\hat{w})$, which is a contradiction. □

3 The Mechanism

We use Proposition 1 and Proposition 2 to design a resource allocation mechanism which can be used in the resource negotiation process between agents.

Definition 1. *We define operator $\mathcal{R}_{x_i}[f(x_1, ..., x_N)]$ as follows:*

$$\hat{x}_i \in \mathcal{R}_{x_i}[f(x_1, ..., x_N)] \neq \varnothing \quad \Leftrightarrow \quad f(x_1, ..., \hat{x}_i, ..., x_N) = 0. \tag{6}$$

Definition 2. ***(Resource Allocation Mechanism)***
Payment rule:

$$w_i^{k+1} = p^k \cdot \hat{a}_i^k \qquad (\forall i \in L), \tag{7}$$

where

$$\hat{a}_i^k = \mathcal{R}_{a_i}\left[u_i'(a_i)(1 - \frac{\beta^k \cdot a_i}{x^k}) - p^k\right]. \tag{8}$$

Allocation rule:

$$a_i^{k+1} = \mathbf{1}(w_i^{k+1}) \cdot \frac{w_i^{k+1}}{p^{k+1}} \qquad (\forall i \in L), \tag{9}$$

where

$$x^{k+1} = \mathcal{R}_x\left[\sum_{i \in L} w_i^{k+1} - xp(x)\right], \quad p^{k+1} = p(x^{k+1}). \tag{10}$$

The mechanism we introduce allocates resources according to Assumption 3, i.e. it sets the price $p(x(w))$ to ensure demand $\sum_{i \in L} w_i/p(x)$ equals supply x. Strategies w_i received by the mechanism denote *willingness* each agent has to pay for the rate a_i of resource at the unit price $p(x(w))$. These are equal to $p(x(w)) \cdot \hat{a}_i$, where $\hat{a}_i$ maximizes payoffs $Q_i(a_i(w), w_i)$ at the price $p(x(w))$.

We now analyze convergence properties of mechanism (7) -(10) and examine the stability of the Nash equilibrium point $\hat{w}$.

Lemma 1. *Let $\hat{p} = \sum_{i \in L} \hat{w}_i/x(\hat{w})$, where $\hat{w}$ is Nash equilibrium of the game defined by $(Q_i(a_i(w), w_i))_{i \in L}$. If Assumptions 1-4 hold, then:*

$$(p(x(w)) - \hat{p})\left[\sum_{i \in L} \hat{a}_i(w) - a_i(w)\right] \leq 0. \tag{11}$$

Proof. Denote $p \equiv p(x(w))$. Suppose that $p \leq \hat{p}$. The optimal response to price p must lead to allocation for which the following condition holds: $\sum_{i \in L} \hat{a}_i(w)$ $-a_i(w) \geq 0$. Conversely, if $p > \hat{p}$ then $\sum_{i \in L} \hat{a}_i(w) - a_i(w) < 0$. Now, condition $\sum_{i \in L} \hat{a}_i(w) - a_i(w) \geq 0$ implies that optimal price $\hat{p}$ must be higher than or equal to $p = p(x(w))$, and if $\sum_{i \in L} \hat{a}_i(w) - a_i(w) < 0$ then $p > \hat{p}$. □

Proposition 4. *Let $\mathcal{F}(w(t)) = p(x(w(t)))\hat{a}(w(t)) - w(t)$, where:*

$$(\forall i \in L) \quad \hat{a}(w(t)) = \mathcal{R}_{a_i}\left[u_i'(a_i)(1 - \frac{\beta(x(w(t))) \cdot a_i}{x(w(t))}) - p(x(w(t)))\right].$$

If Assumptions 1-4 hold, then Nash equilibrium point of the game defined by $(Q_i(a_i(w), w_i))_{i\in L}$ *is an globally asymptotically stable equilibrium point of the system:*

$$\dot{w}(t) = \mathcal{F}(w(t)). \tag{12}$$

Proof. First notice, that the only stationary point of system $\mathcal{F}(w(t))$ is $w = p(x(w))\hat{a}(w)$. Now, we show that $p(x(w(t)))$ converges to equilibrium price $\hat{p} = \sum_{i\in L} \hat{w}_i / x(\hat{w})$ when agents set their payments according to $\mathcal{F}(w(t))$. We have:

$$\dot{p}(x(w(t))) = p'(x(w(t))) \sum_{i\in L} \frac{\partial x(w)}{\partial w_i} \dot{w}_i \leq p'(x(w(t))) \sum_{i\in L} \frac{p(x(w(t)))\hat{a}_i(w(t)) - w_i(t)}{p(x(w(t)))} =$$

$$= p'(x(w(t))) \sum_{i\in L} (\hat{a}_i(w(t)) - a_i(w(t))).$$

Consider dynamical system $\dot{p}(x(w(t)))$ and define the Lyapunov function $V(t) = [p(w(t)) - \hat{p}]^2$.

$$\dot{V}(t) = 2[p(w(t)) - \hat{p}]\dot{p}(x(w(t)))$$
$$\leq 2p'(x(w(t)))[p(x(w(t))) - \hat{p}] \sum_{i\in L} (\hat{a}_i(w(t)) - a_i(w(t))) \leq 0.$$

So, for $w(0) = w_0$ price $p(x(w(t)))$ converges to $\hat{p}$. Under assumptions of the proposition this implies that $w(t)$ converges to $\hat{w}$. □

4 Comparison

Consider now the Cournot competition model and related resource allocation game with payoffs defined by:

$$\tilde{Q}_i(a, a_i) = u_i(a_i) - a_i p(\sum_{j\in L} a_j) \quad (i \in L).$$

We examine relations between allocations achieved at the Cournot equilibrium point $\tilde{a}$ and at the Nash equilibrium point $a(\hat{w})$. We remind that $\tilde{a} = [\tilde{a}_1, ..., \tilde{a}_L]$ is a Cournot equilibrium if:

$$(\forall i \in L) \quad \tilde{a}_i \in \arg\max_{a_i} \tilde{Q}_i(a, a_i).$$

Proposition 5. *Consider payoff function* $\tilde{Q}_i(a, a_i) = u_i(a_i) - a_i p(\sum_{i\in L} a_i)$. *Let* $\tilde{a}_i = \mathcal{R}_{a_i}\left[\partial \tilde{Q}_i / \partial a_i\right]$ *and let* $\hat{w}$ *be the Nash equilibrium of the game defined by payoffs* $(Q_i(a_i(w), w_i))_{i\in L}$. *If Assumptions 1-4 hold, then:*

$$a_i(\hat{w}) \geq \tilde{a}_i \quad (\forall i \in L). \tag{13}$$

Proof. We show that at $\tilde{w}$ such that $\tilde{a} = a(\tilde{w})$ we have:

$$\frac{\partial Q_i(a_i(\tilde{w}), \tilde{w}_i)}{\partial w_i} \geq 0.$$

We can write $\tilde{a}_i = \mathcal{R}_{a_i}[\partial\tilde{Q}_i/\partial a_i] = (u_i^{'}(\tilde{a}_i) - p(\tilde{x}))/p'(\tilde{x})$. For $a(\tilde{w}) = \tilde{a}$ from the equation $\partial Q_i(a_i(\tilde{w}), \tilde{w}_i)/\partial w_i$, with $\tilde{x} = \sum_{i \in L} \tilde{a}_i$, we obtain:

$$\frac{u_i^{'}(\tilde{a}_i)}{p(\tilde{x})}\left(1 - \frac{\beta(\tilde{x})}{\tilde{x}}\tilde{a}_i\right) - 1 = \frac{u_i^{'}(\tilde{a}_i)}{p(\tilde{x})}\left(1 - \frac{u_i^{'}(\tilde{a}_i) - p(\tilde{x})}{p(\tilde{x})(1+\varepsilon(\tilde{x}))}\right) - 1.$$

Now, suppose that $a_i(\hat{w}) < \tilde{a}_i$. This implies that $u_i^{'}(a_i(\hat{w})) > u_i^{'}(\tilde{a}_i)$ and that:

$$\frac{\partial Q_i(a_i(\tilde{w}), \tilde{w}_i)}{\partial w_i} = \frac{u_i^{'}(\tilde{a}_i)}{p(\tilde{x})}\left(1 - \frac{u_i^{'}(\tilde{a}_i) - p(\tilde{x})}{p(\tilde{x})(1+\varepsilon(\tilde{x}))}\right) - 1 < 0.$$

We obtain:

$$1 - \frac{u_i^{'}(\tilde{a}_i) - p(\tilde{x})}{p(\tilde{x})(1+\varepsilon(\tilde{x}))} - \frac{p(\tilde{x})}{u_i^{'}(\tilde{a}_i)} < 0 \quad \Leftrightarrow \quad \frac{u_i^{'}(\tilde{a}_i) - p(\tilde{x})}{u_i^{'}(\tilde{a}_i)} < \frac{u_i^{'}(\tilde{a}_i) - p(\tilde{x})}{p(\tilde{x})(1+\varepsilon(\tilde{x}))} \quad \Leftrightarrow$$

$$u_i^{'}(a_i(\hat{w})) > u_i^{'}(\tilde{a}_i) > p(\tilde{x})(1+\varepsilon(\tilde{x})) > p(x(\hat{w}))(1+\varepsilon(x(\hat{w}))).$$

This is a contradiction. According to Proposition 3 equilibrium allocation $a_i(\hat{w})$ can only occur when $u_i^{'}(a_i(\hat{w})) \leq p(x(\hat{w}))(1 + \varepsilon(x(\hat{w})))$, so $\partial Q_i(a_i(\tilde{w}), \tilde{w}_i)/\partial w_i \geq 0$. This proves that $a_i(\hat{w}) \geq \tilde{a}_i \quad (\forall i \in L)$. □

Proposition 6. *Consider payoff function* $\tilde{Q}_i(a, a_i) = u_i(a_i) - a_i p(\sum_{i \in L} a_i)$. *Let* $\tilde{a}_i = \mathcal{R}_{a_i}\left[\partial\tilde{Q}_i/\partial a_i\right]$ *and let* $\hat{w}$ *be the Nash equilibrium of the game defined by payoffs* $(Q_i(a_i(w), w_i))_{i \in L}$. *If Assumptions 1-4 hold, then:*

$$\sum_{i \in L} u_i(a_i(\hat{w})) - c(\sum_{i \in L} a_i(\hat{w})) \geq \sum_{i \in L} u_i(\tilde{a}) - c(\sum_{i \in L} \tilde{a}_i). \tag{14}$$

Proof. First, notice that from Proposition 3 we have:

$$u_i^{'}(\tilde{a}_i) \geq p(\sum_{i \in L} \tilde{a}_i) = p(\tilde{x}), \qquad u_i^{'}(a_i(\hat{w})) \geq p(\sum_{i \in L} a_i(\hat{w})) = p(x).$$

Furthermore, from the Proposition 3 and Proposition 5 we obtain: $u_i^{'}(\tilde{a}) \geq p(x) = p(\sum_{i \in L} a_i(\hat{w}))$. Now, we have:

$$\sum_{i \in L} \int_0^{a_i(\hat{w})} u_i^{'}(a)da - \sum_{i \in L} \int_0^{\tilde{a}_i} u_i^{'}(a)da = \sum_{i \in L} \int_{\tilde{a}_i}^{a_i(\hat{w})} u_i^{'}(a)da \geq$$

$$\geq p(x) \sum_{i \in L} \int_{\tilde{a}_i}^{a_i(\hat{w})} da = p(x)(x - \tilde{x}) \geq \int_{\tilde{x}}^{x} p(y)dy = c(x) - c(\tilde{x}),$$

which completes the proof. □

Proposition 5 shows that in comparison to the Cournot model the proposed mechanism (7)-(10) allocates more resources. Furthermore, Proposition 6 proves that it may lead to the higher aggregate surplus.

Figure 1 shows optimal response curves for two agents. Notice that since $a_i(a_j(w)) \geq a_i(a_j)$, at intersection point E^w both agents obtain more resources then at E^a. Figure 2 demonstrates convergence of the mechanism (7)-(10).

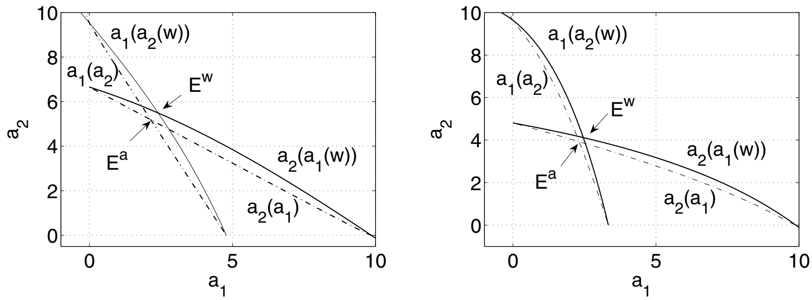

Fig. 1. Optimal response curves for $u_i(a_i) = \gamma_i \log(a_i + 1)$ (left), $u_i(a_i) = \gamma_i \arctan(a_i)$ (right) and $p(x) = (C - x)^{-1}$, $C = 10$

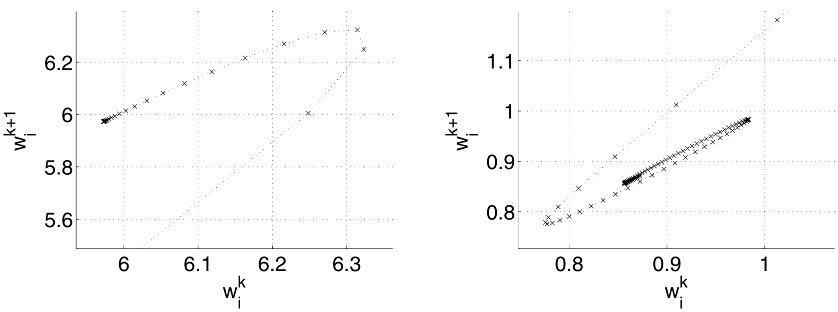

Fig. 2. Payment trajectories in phase space. Responses to p^k (left) and to $\sum_{k \in K} p^k/K$ (right) for $u_i(a_i) = \gamma_i \log(a_i + 1)$

5 Example

In this section we illustrate the results presented above. Suppose that L agents negotiate bandwidth allocations with local resource manger and that their utility functions are given by:

$$u_i(a_i(w)) = \gamma_i \log(a_i(w) + 1), \quad (\forall i \in L).$$

Suppose also that $p(x) = (C - x)^{-1}$, where C is effective bandwidth. Now, consider a provisioning problem. Suppose resource manager wants to sell a bandwidth of $C = 45$ Mbps to three groups of agents: $L_1 = \{1..10\}$, $L_{1.5} = \{11..20\}$ and $L_{25} = \{21, 22\}$. Group L_1 needs a peak rate of 1Mbps, group $L_{1.5}$ a peak rate of 1.5Mbps and group L_{25} a peak rate of 25Mbps. Let γ_i be random variables, independent and uniformly distributed on intervals $[3, 5]$, $[5, 7]$ and $[20, 21]$ for $i \in \{L_1, L_{1.5}, L_{25}\}$ respectively. We simulate the following scenario:

- $t < 0$: equilibrium state
- $t = 1$: agent $i = 21$ departs;
- $t = 2$: agent $i = 21$ joins, agents $\{11..18\}$ depart;

- $t = 3$: agents $\{1..10\}$ depart;
- $t = 4$: all agents join competition.

The scenario events take place when $\|a - \hat{a}\| < 0.00001$. We compare results obtained from mechanism (7)-(10) to results obtained from the Cournot mechanism. Notice that in the Cournot model agents submit allocations they want to buy at price $p(x(a))$. In case of mechanism (7)-(10) agents submit payments for allocations they need at price $p(x(w))$. Figures 3 present results of the simulation. Indeed, mechanism (7)-(10) allocates more resources than the Cournot mechanism and generates higher aggregate surplus.

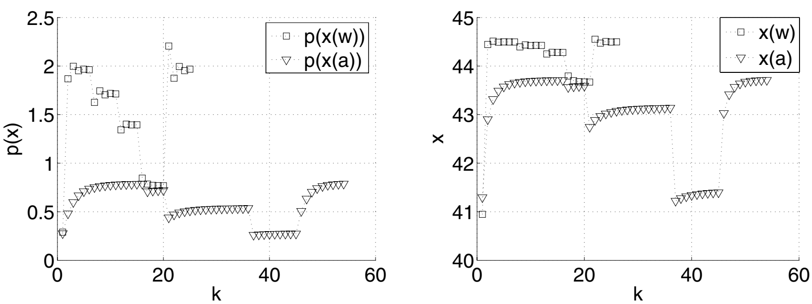

Fig. 3. Simulation results. Mechanism (7)-(10) allocates more resources ($x(w) = \sum_{i \in L} a_i(w) > \sum_{i \in L} a_i = x(a)$). This implies that $p(x(w)) > p(x(a))$. Notice that convergence rate of mechanism (7)-(10) is higher.

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