An Optimum RBF Network for Signal Detection in Non-Gaussian Noise

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Abstract. In this paper, we propose a radial basis function (RBF) neural network for detecting a known signal in the presence of non-Gaussian and Gaussian noise. In case of non-Gaussian noise, our study shows that RBF signal detector has significant improvement in performance characteristics; detection capability is better to those obtained with multilayer perceptrons (MLP) and the matched filter (MF) detector.

1 Introduction

Real world signals usually contain departures from the ideal signal due to non-Gaussian noise. Optimum linear detectors, under the assumption of additive Gaussian noise are suggested in [1] and [2]. A class of locally optimum detectors are used in [2]. In [3] Watterson generalizes an optimum MLP neural receiver for signal detection. Lippmann and Beckman [4] employed a neural network as a pre-processor to reduce the influence of impulsive noise components. Michalopoulou et al [5] trained a multilayer neural network to identify one of M orthogonal signals embedded in additive Gaussian noise. Gandhi and Ramamurti [6] has shown that the neural detector trained using BP algorithm gives near optimum performance. In this paper, we propose a neural network detector using RBF network and we employ this neural detector to detect the presence or absence of a known signal corrupted by Gaussian and non-Gaussian noise components. Our study shows that in non-Gaussian noise environments the RBF signal detector show better performance characteristics and good detection capability compared to neural detector using BP.

2 Preliminaries and a RBF for Signal Detection

The two commonly used measures to assess performance of a signal detector are the probability of detection P_d and the probability of false alarm P_{fa} [1]. Consider a data vector $\mathbf{X}(\mathbf{t}) = [x_1(t), x_2(t), \dots, x_N(t)]^T$ as an input to the detector in Figure 1-(a). Using the additive observational model, we have $\mathbf{X}(\mathbf{t}) = \mathbf{S}(\mathbf{t}) + \mathbf{C}(\mathbf{t})$ for the hypothesis that the target signal is present and $\mathbf{X}(\mathbf{t}) = \mathbf{C}(\mathbf{t})$ for the hypothesis that the signal is absent, where $\mathbf{S}(\mathbf{t}) = [s_1(t), s_2(t), \dots, s_N(t)]^T$ is the target signal vector and $\mathbf{C}(\mathbf{t}) = [c_1(t), c_2(t), \dots, c_N(t)]^T$ is the noise vector. With $f_N(x)$ as the marginal probability density function (pdf) of $N_i, i = 1, 2, \dots, n$, here we consider the following pdf's:

Fig. 1. Conventional and RBF signal detector

1. Gaussian pdf with $f_N(x) = e^{-x^2/2\sigma^2}/\sqrt{2\pi\sigma^2}$ and $E[N_i^2] = \sigma^2$.
2. Double exponential pdf with $f_N(x) = e^{-\|x\|/\sigma}/2\sigma$ and $E[N_i^2] = 2\sigma^2$.
3. Contaminated Gaussian pdf with $f_N(x) = (1 - \epsilon)e^{-x^2/2\sigma_0^2}/\sqrt{2\pi\sigma_0^2} + \epsilon e^{-x^2/2\sigma_1^2}/\sqrt{2\pi\sigma_1^2}$.

The structure of the signal detector based on an RBF network is shown in Figure 1-(b). The input layer has N neurons with a linear function. One hidden layer of K neurons with Gaussian transfer functions. The output layer has only one neuron whose input-output relationship approximates the two possible states. The two bias nodes are included as part of the network. A real-valued input vector $\mathbf{x}$ to a neuron of the hidden or output layer produces neural output $G(\mathbf{x})$, where $0 < G(\mathbf{x}) < 1$. The Gaussian function we choose is $G(\mathbf{x}, \mathbf{x}_i) = \exp(-1/2\sigma_i^2\|\mathbf{x} - \mathbf{x}_i\|^2)$. The RBF neural network detector test statistic $T_{NN}(\mathbf{x})$ may now be expressed as,

$$T_{NN}(\mathbf{x}) = \sum_{i=1}^K w_i \varphi_i(\mathbf{x}) + w^{b_2} \quad (1)$$

where $\{\varphi_i(\mathbf{x}), i = 1, 2, 3, \dots, K\}$ is a set of basis functions. The $\mathbf{w}$ constitutes a set of connection weights for the output layer. When using RBF the basis is

$$\varphi_i(\mathbf{x}) = G(\|\mathbf{x} - \Sigma \mathbf{t}_i\|^2) + w_i^{b_1}, i = 1, 2, 3, \dots, K \quad (2)$$

where $\mathbf{t}_i = [\mathbf{t}_{i1}, \mathbf{t}_{i2}, \dots, \mathbf{t}_{iN}]^T$ with $\mathbf{t}_i$ as unknown centers to be determined. Σ is a symmetric positive definite weighting matrix of size $N \times N$, $G(\cdot)$ represents a multivariate Gaussian distribution with mean vector $\mathbf{t}_i$ and covariance matrix Σ . By using above equations we redefine $T_{NN}(\mathbf{x})$ as

$$T_{NN}(\mathbf{x}) = \sum_{i=1}^K w_i G(\mathbf{x}, \mathbf{t}_i) = \sum_{i=1}^K w_i G(\|\mathbf{x} - \mathbf{t}_i\|). \quad (3)$$

We determine the set of weights $\mathbf{w} = [w_1, w_2, \dots, w_k]^T$ and the set $\mathbf{t}$ of vectors $\mathbf{t}_i$ of centers.

3 Simulation Results and Performance Evaluations

Neural weights are obtained by training the network at 10-dB SNR using $\theta = \sqrt{10}$ and $E[N_i^2] = 1$. During simulation, the threshold τ_{NN} is set to 0.5, and the bias weight w^{b2} value that gives a P_{fa} value in the range 0.001 – 1. For each w^{b2} value that gives a P_{fa} value in the above range, the corresponding P_d value are also simulated. The 10-dB-SNR-trained neural network is tested in the 5-dB and 10-dB SNR environment.

3.1 Performance in Gaussian Noise (Constant and Ramp Signal)

Performance characteristics of neural detectors using RBF, MLP and MF detectors are presented in Figure 2-(a) and 2-(b) for Gaussian noise. The RBF and MLP neural detectors are trained using the constant signal and ramp signal with SNR = 10 dB. And then both neural detectors and match filter detector are tested with 10-dB SNR inputs. In both cases, the RBF and MLP neural detectors performance is very close to that of the MF detector.

Fig. 2. Performance comparison in Gaussian Noise

3.2 Testing of Signal Detector in Non-Gaussian Noise

Here we illustrate the performance comparisons of the likelihood ratio (LR), MF, local optimum (LO), and neural detectors using RBF and MLP for ramp signal embedded in additive double exponential noise. Figure 3-(a) and 3-(b) show the comparison for a 10-dB-SNR-trained neural detectors operating in the 5-dB and 10-dB SNR environment.

The same experiment is repeated for the ramp signal embedded in contaminated Gaussian noise with parameters $\epsilon = 0.2$, $\sigma_0^2 = 0.25$ and $\sigma_1^2 = 4$ and the results are shown in Figure 3-(c) and 3-(d). In all these cases, the RBF based neural detector performance is superior to all other detectors (excluding LR detector which is optimal).

(a) Ramp Signal, 10 dB

(b) Ramp Signal, 5 dB

(c) 10 dB SNR

(d) 5 dB SNR

Fig. 3. Performance comparison in double exponential and contaminated Gaussian

4 Conclusion

We have proposed RBF based neural network for detection of known signal in non-Gaussian noise. Our simulation results show that its performance is superior compared to conventional detectors.

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