New Approaches to Catheter Navigation for Interventional Radiology Simulation

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Abstract. For over 20 years, interventional methods have improved the outcomes of patients with cardiovascular disease. However, these procedures require an intricate combination of visual and tactile feedback and extensive training periods. In this paper, we describe a series of novel approaches that have lead to the development of a high-fidelity simulation system for interventional neuroradiology. In particular we focus on a new approach for real-time deformation of devices such as catheters and guidewires during navigation inside complex vascular networks. This approach combines a real-time incremental Finite Element Model, an optimization strategy based on substructure decomposition, and a new method for handling collision response in situations where the number of contacts points is very large. We also briefly describe other aspects of the simulation system, from patient-specific segmentation to the simulation of contrast agent propagation and fast volume rendering techniques for generating synthetic X-ray images in real-time.

1 Introduction

Stroke, the clinical manifestation of cerebrovascular disease, is the third leading cause of death in the United States. Each year, more than 700,000 strokes result in over 200,000 deaths [2]. Ischemic strokes can now be treated using interventional neuroradiologic therapies which rely on the insertion and navigation of catheters and guidewires through a complex network of arteries to restore blood flow. Because the treatment is delivered directly within the closed brain, using only image-based guidance, the dedicated skill of instrument navigation and the thorough understanding of vascular anatomy are critical to avoid devastating complications which could result from poor visualization or poor technique. This was the foundation for a recent decision by the FDA requiring all physicians who wish to treat carotid disease using catheter-based techniques to train to proficiency before performing high-risk procedures in the cerebral circulation. However, while most publications in the field of medical simulation have addressed issues related to laparoscopic training, few aspects of interventional radiology simulation have been explored. Aside of modeling the soft tissue deformation of arteries, other complex problems need to be solved to enable real-time, accurate simulation of such procedures. The one we address in this paper concerns

the simulation of non-linear deformations of wire-like structures under a large number of non-holonomic constraints, and the definition of such constraints to confine the catheter inside the vascular network. We also briefly mention other results in the areas of real-time fluid flow computation, real-time synthetic X-ray rendering, and patient-specific segmentation. Although some of these problems have been addressed in previous work [3,9,6,1], in particular in the areas of visualization and catheter modeling, many challenging problems remain, especially when trying to reach a higher level of fidelity and accuracy in the simulation.

2 Modeling of Wire-Like Structures

To control the motion of a catheter or guidewire within the vascular network, the physician can only push, pull or twist the proximal end of the device. Since such devices are constrained inside the patient's vasculature, it is the combination of input forces and contact forces that allow them to be moved toward a target. The main characteristics of wire-like structures that current models attempt to capture include geometric non-linearities, high tensile strength and low resistance to bending. We previously proposed a multibody dynamics model [5] where a set of rigid elements are connected using spherical joints [3], thus mimicking the basic behavior of such devices. Another interesting approach to modeling wire-like structures was introduced by [8]. In this model, a one-dimensional dynamic spline model is used, providing a continuous representation. Different constraints can be defined to control the model, such as sliding through fixed locations. Although real-time computation is possible, this model does not incorporate torsional energy terms. A virtual catheter model, based on a linear elasticity, was introduced by [9], using a set of finite beam elements. The choice of beams for the catheter model is natural since beam equations include cross-sectional area, cross-section moment of inertia, and polar moment of inertia, allowing solid and hollow devices of various cross-sectional geometries and mechanical properties to be modeled. The main issue is its inability at representing the large geometric non-linearities of the catheter or guidewire that occur during navigation inside the vascular network. Another approach also directly targeted at virtual catheter or guidewire modeling was proposed by [1]. In this model, only bending energies are computed, assuming no elongation and perfect torque control. The model has characteristics similar to a multi-body dynamics model but integrates more complex bending energies, as well as local springs for describing the intrinsic curvature of the catheter. Although based on a more ad-hoc representation, a good level of accuracy is obtained using this model. The main drawbacks are how collision response is handled during contact with the walls of the vessel, and computation times that are not compatible with real-time requirements.

To improve the accuracy of previously proposed models, and handle geometric non-linearities while maintaining real-time computation, we have developed a new mathematical representation based on three-dimensional beam theory, combined with an incremental approach that allows for highly non-linear behavior using a linear model, thus guarantees real-time performance. Further opti-

mizations based on substructure decomposition are introduced. Finally, a new method for correctly handling contact response in complex situations where a large number of nodes are subject to non-holonomic constraints, is presented.

2.1 Incremental Finite Element Model

To model the deformation of a catheter, guidewire, we use a representation based on three-dimensional beam theory[10], where the elementary stiffness matrix K_e is a 12×12 symmetric matrix that relates angular and spacial positions of each end of a beam element to the forces and torques applied to them:

$$K_e = \frac{E}{l} \begin{bmatrix} A & 0 & \frac{12I_z}{l^2(1+\Phi_y)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12I_z}{l^2(1+\Phi_y)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12I_y}{l^2(1+\Phi_z)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{GJ}{E} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-6I_y}{l(1+\Phi_z)} & 0 & \frac{(4+\Phi_z)I_y}{1+\Phi_z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6I_z}{l(1+\Phi_y)} & 0 & 0 & 0 & \frac{(4+\Phi_y)I_z}{1+\Phi_y} & 0 & 0 & 0 & 0 & 0 & 0 \\ -A & 0 & 0 & 0 & 0 & 0 & A & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-12I_z}{l^2(1+\Phi_y)} & 0 & 0 & 0 & \frac{-6I_z}{l(1+\Phi_y)} & 0 & \frac{12I_z}{l^2(1+\Phi_y)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-12I_y}{l^2(1+\Phi_z)} & 0 & \frac{6I_y}{l(1+\Phi_z)} & 0 & 0 & 0 & \frac{12I_y}{l^2(1+\Phi_z)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-GJ}{E} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{E} & 0 & 0 \\ 0 & 0 & \frac{6I_y}{l(1+\Phi_z)} & 0 & \frac{(2-\Phi_z)I_y}{1+\Phi_z} & 0 & 0 & 0 & \frac{6I_y}{l(1+\Phi_z)} & 0 & \frac{(4+\Phi_z)I_y}{l(1+\Phi_z)} & 0 \\ 0 & \frac{6I_z}{l(1+\Phi_y)} & 0 & 0 & 0 & \frac{(2-\Phi_y)I_z}{1+\Phi_y} & 0 & \frac{-6I_z}{l(1+\Phi_y)} & 0 & 0 & 0 & \frac{(4+\Phi_y)I_z}{1+\Phi_y} \end{bmatrix}$$

with $G = \frac{E}{2*(1+\nu)}$ where E is the Young's modulus and ν is the Poisson's ratio; A is the cross-sectional area of the beam, and l its length; I_y and I_z are cross-section moments of inertia; Φ_y and Φ_z represent shear deformation parameters and are defined as $\Phi_y = \frac{12EI_z}{GA_{sy}l^2}$ and $\Phi_z = \frac{12EI_y}{GA_{sz}l^2}$ with A_{sy} and A_{sz} the shear area in the y and z directions. For the entire structure describing a catheter or guidewire, the global stiffness matrix $[K]$ is computed by summing the contributions of each element, thus leading to the following equilibrium equation in the quasi-static case:

$$[K]U = F \quad (1)$$

where $[K]$ is a band matrix due to the serial structure of the model (one node is only shared by one or two elements). U represents a column matrix of displacements corresponding to external forces F . The matrix $[K]$ is singular unless some displacements are prescribed through boundary conditions. Such boundary conditions are naturally specified by setting the first node of the catheter (base node) to a particular translation or rotation imposed by the user.

There is, however, one main drawback in using directly such a model: it is linear and therefore cannot represent the geometric non-linearities that a typical wire-like object exhibits. Therefore we propose to update $[K_e]$ at every time step, by using the solution obtained at the previous time step. The new set of local

stiffness matrices are then assembled in $[K_t]$. Here, we do not use the initial configuration as the reference state, but instead use the previously computed solution. By controlling when each new $[K_t]$ is going to be computed, we can ensure we remain in the linear domain for each incremental step, leading to a correct, global deformation. One potential drawback of this approach is that the model could exhibit a inelastic behavior, i.e. in the absence of forces or torques, the model would only return to the previous state, not the reference configuration. We overcome this problem by computing a force F_t defined as $F_t = -\alpha[K_t](X_t - X_0)$ with $0 < \alpha \leq 1$. This force is added to the external forces F before solving the linear system, and it can be shown that it acts as a damping force, where α relates to the damping coefficient of the model.

To simulate accurately a device such as a guidewire or catheter in the context of interventional neuroradiology, we need to have a large number (from 100 to 200) of beam elements in the model. Although solving linear systems with about 1,000 unknowns can be done in real-time using iterative methods, when integrating non-holonomic constraints, real-time computation on a single-processor workstation is no longer possible. To improve speed and handle accurately collision response, we propose the following optimizations.

2.2 Optimization Using Substructures Analysis

To optimize the computation of a wire-like object composed of multiple beams, we decompose the object in a set of substructures. Each substructure can be constituted of one or several beam elements, and is analyzed independently, assuming that all common boundaries (joints) with the adjacent substructures are fixed (see Figure 1(a)). By doing this, we isolate each substructure from the rest of the model. In a second phase, the boundary conditions are relaxed by propagating from the base to the tip of the catheter (see Figure 1(b)). The actual local compliance is determined from equilibrium equations at each boundary joint. The total deformation of the structure can be calculated from the superposition of two displacement such that:

$$U = U^l + U^c \quad (2)$$

By specifying the displacement of the first node of each beam to zero, one can compute the local displacement of the second node U^l , due to external force applied on it. Then, U^c represents the necessary corrections to allow boundary displacements. The FEM model of each beam i gives the stiffness between node i and node $i + 1$:

$$\begin{bmatrix} (K_i)_{(1,1)} & (K_i)_{(1,2)} \\ (K_i)_{(2,1)} & (K_i)_{(2,2)} \end{bmatrix} \begin{bmatrix} U_i \\ U_{i+1} \end{bmatrix} = \begin{bmatrix} F_i \\ F_{i+1} \end{bmatrix} \quad (3)$$

Setting Boundary Conditions. When applying boundaries conditions to the first node of each beam, we obtain $U_i^l = 0$ in equation (3). Then, the local displacement of node $i + 1$ subject to an external force F_{i+1} is:

$$U_{i+1}^l = [(K_i)_{(2,2)}]^{-1} F_{i+1} \quad (4)$$

Fig. 1. (a) *Setting boundary conditions:* the object is split in a series of substructures, and local displacements and forces are computed after constraining the first node of each substructure; (b) *Relaxing boundary conditions:* correction displacements are applied recursively, starting from node 1, at each first node of each substructure.

And the reaction on point i , due to this displacement is:

$$R_i = [(K_i)_{(1,2)}]U_{i+1}^l = -\underbrace{(-[(K_i)_{(1,2)}][(K_i)_{(2,2)}]^{-1})}_{[H_i]^T} F_{i+1} \quad (5)$$

Relaxing Boundary Conditions. In this second phase, the external force applied at node $i + 1$ has already been taken into account, therefore $F_{i+1}^c = 0$ and only R_i remains. The correction displacement at node $i + 1$ is linked to the correction displacement at node i by equation (7):

$$F_{i+1}^c = [(K_i)_{(2,1)}]U_i^c + [(K_i)_{(2,2)}]U_{i+1}^c = 0 \quad (6)$$

$$U_{i+1}^c = -\underbrace{[(K_i)_{(2,2)}]^{-1}[(K_i)_{(2,1)}]}_{[H_i]} U_i^c ; \quad U_i^c = -[K^{-1}]R_i \quad (7)$$

When relaxing boundary conditions, all joints are taken into account except the first node (base) of the model. Doing so eliminates rigid body motion and leads to a non-singular system.

Solving for the Entire Structure. Wire-like objects being composed of serially-linked elements, a bottom to top propagation strategy allows to solve the entire structure. The local compliance $[(K^{-1})_{(i,i)}]$ of each node i will be first computed (we recall that $U_i = [(K^{-1})_{(i,i)}]F_i$). Gathering equations (4), (5) and (7), we obtain:

$$[K^{-1}]_{(i+1,i+1)} = [(K_i)_{(2,2)}]^{-1} + [H_i]^T [(K^{-1})_{(i,i)}] [H_i] \quad (8)$$

To initialize the computation, we note that since all 6 DOFs of the first node are fixed, $(K^{-1})_{(1,1)} = 0$. The beam model then leads to a global structural analysis: the force applied on one node induces the displacement of all other nodes. Then, a top to bottom function (Algorithm 1) computes the displacement contribution of each force F_i on the current and previous nodes, and a bottom to top function (Algorithm 2) computes the contribution on following nodes.

2.3 Contact Response

Another important problem in simulating catheter or guidewire navigation is solving the contacts between the virtual device and the vessels wall. Because

Algorithm 1: Force accumulation

Input: $[(K^{-1})_{(2,2)}] \dots [(K^{-1})_{(n,n)}], F, U$
Output: U
 $f_{acc} = 0;$
for $i = n \dots 2$ **do**
 $f_{acc} = f_{acc} + F_i;$
 $U_i = [(K^{-1})_{(i,i)}] f_{acc};$
 $f_{acc} = [H_{i-1}]^T f_{acc};$
end

Algorithm 2: Dpt accumulation

Input: $[(K^{-1})_{(2,2)}] \dots [(K^{-1})_{(n,n)}], F, U$
Output: U
 $u_{acc} = 0;$
for $i = 2 \dots n - 1$ **do**
 $U_i = U_i + u_{acc};$
 $u_{node} = [(K^{-1})_{(i,i)}] F_i;$
 $u_{acc} = [H_i](u_{acc} + u_{node});$
end
 $U_n = U_n + u_{acc};$

sliding occurs at the point of contact, Lagrange multiplier techniques, simple penalty forces or quadratic programming approaches will not constrain the flexible body properly. To solve this problem, we compute the local compliance of the node in the contact space defined by $\mathbf{n}$, where $\mathbf{n}$ is the normal at each contact point. The local compliance of the contact α , at node i , is:

$$W_{\alpha\alpha} = [\mathbf{n}]^T (K^{-1})_{(i,i)} [\mathbf{n}] \quad (9)$$

The computation of $W_{\alpha\alpha}$ is very fast, as $(K^{-1})_{(i,i)}$ is already known from equation (8). The problem of multiple sliding contact, can be solved using a Gauss-Seidel like algorithm [4]. Considering a contact α , among m instantaneous contacts, one can write the linearized behavior of the catheter in the form:

$$\underbrace{\delta_\alpha - [W_{\alpha\alpha}]f_\alpha}_{\text{unknown}} = \underbrace{\sum_{\beta=1}^{\alpha-1} \underbrace{[W_{\alpha\beta}]f_\beta}_{\text{from algorithm 2}} + \sum_{\beta=\alpha+1}^m \underbrace{[W_{\alpha\beta}]f_\beta}_{\text{from algorithm 1}}}_{\text{frozen}} + \delta_\alpha^{\text{free}} \quad (10)$$

where $[W_{\alpha\beta}]$ is a compliance matrix that models the coupling between contact points α and β . For each contact α , this method solves the contact and friction equations by considering the others contact points ($\alpha \neq \beta$) "frozen". The solution of these equations (contact, friction) is non-linear and can be computed using an iterative method [7]. Contributions of frozen nodes are accumulated according to algorithms (1) and (2). By sorting nodes in decreasing order, the contribution of contact force f_α on the current and previous nodes is computed by algorithm 1. Its contribution on following nodes is computed at the end of Gauss-Seidel iteration, by using algorithm 2. This leads to a precise and very fast method for computing the entire structure.

3 Interventional Neuroradiology Simulation

In addition to the models and algorithms described previously, we have addressed several other aspects of the simulation of interventional radiology procedures. We briefly describe here these results, more details can be found in [11].

A first problem we addressed concerns arterial flow computation, which impacts both catheter navigation and contrast agent propagation. Blood flow in

Fig. 2. Catheter navigation inside the cerebrovascular network. Complex, non-linear deformations are correctly represented by the incremental FEM model. Collision detection and collision response allow the catheter to stay within the lumen of the vessels.

each vessel is modeled as an incompressible viscous fluid flowing through a cylindrical pipe, and can be calculated from a simplified Navier-Stokes equation called Poiseuille's Law (11). It relates the flow Q in each vessel to the pressure gradient ΔP , viscosity of the fluid η , radius r , and length L of the vessel.

$$Q = \frac{\Delta P}{R} \quad \text{with} \quad R = \frac{8\eta L}{\pi r^4} \quad (11)$$

Solving this equation for the whole vascular system is equivalent to solving a linear FEM model, and, assuming there is no deformation of the vessels, the global resistance matrix $[R]$ can be pre-inverted, allowing for real-time simulation of vascular flow. An example of flow computation is illustrated in Figure 3.

Two other important aspects which relate to visual feedback are X-ray simulation and contrast agent propagation. Both are used during interventional procedures to visualize blood vessels, as well as the anatomy. Contrast agent propagation is simulated using an advection equation where the concentration of contrast agent $C(x, t)$ is a function of the injection rate $r(t)$ and the averaged laminar flow velocity $u(x, t)$.

$$\frac{\partial C(x, t)}{\partial t} + u(x, t) \frac{\partial C(x, t)}{\partial x} = r(t) \quad (12)$$

We numerically solve (12) using a finite difference scheme. The resulting angiogram is visualized using 3D particles which intensity values are a function of $C(x, t)$. Similarly, we have developed a new volume rendering approach that can render CT datasets as fluoroscopic or X-ray images, in real-time. The X-ray attenuation process simulates X-ray beam attenuation as described by a discretized Beer's Law [11]. This method greatly reduces computation times by using OpenGL and specific texture map operations.

Finally, we propose a multi-modal representation of the vascular system, generated from skeletonization of patient specific CTA datasets, as a mean to ensure consistency in the simulation. The segmentation task is particularly challenging due to the small vessel diameter and the close proximity of vessels to the skull,

Fig. 3. *Left:* Color-coded vascular model obtained from patient data, warmer colors correspond to increased blood flow; *Center:* Contrast agent injection using particle-based rendering; *Right:* Real-time catheter navigation and fluoroscopic rendering

and thus remain semi-automatic. After applying a combination of anisotropic filtering and morphological operators, medial axis and local radius information is computed. This information is then used to automatically generate a graph of the vascular network for fluid flow computation, a set of 3D particles for contrast agent rendering, a discretization of the centerlines for contrast agent numerical computation, and a C1-continuous triangulation of the surface of the vasculature optimized for fast collision detection. An example of segmented vascular network is presented in Figure 3.

4 Conclusions and Future Work

In this paper we propose a series of new approaches for fast and accurate simulation of catheter or guidewire navigation within a vascular network. The model we introduce for simulating wire-like structures is able to represent the complex behavior of medical devices and handles collision responses in an effective manner. We also describe results related to the development of a full simulator for interventional neuroradiology. The system runs in real-time on a single processor machine and is capable of simulating realistically key aspects of a diagnostic procedure. Current results for a 100 node model show a computation time of 25 ms for one time step (including the computation of K_e , substructure analysis, collision detection, and contact response) on a Pentium 4 2.6 GHz processor. Going forward, we will address the issue of simulating devices such as stents and coils, and develop a set of metrics for measuring performance during training.

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