Deriving Class Association Rules Based on Levelwise Subspace Clustering

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Abstract. Most approaches of Class Association Rule (CAR) based classification have not intensively addressed the classification of instances including numeric attributes. In this paper, a levelwise subspace clustering method deriving hyper-rectangular clusters is proposed to efficiently provide quantitative, interpretative and accurate CARs.

1 Introduction

“*Class Association Rules (CARs)*” for classification has been proposed in recent studies [1,2,3]. These rules have the form “ $\{ \langle p_1 : q_1 \rangle, \dots, \langle p_m : q_m \rangle \} \Rightarrow cl$ ” where $\langle p : q \rangle$ is an item and cl a class. p represents an attribute and q its value. An example is “ $\{ \langle Age : [30, 39] \rangle, \langle Married : Yes \rangle, \langle NumCars : [2, 2] \rangle \} \Rightarrow Houseowner$ ” stating “A person who is in his/her thirties, married, and owns two cars belongs to the class *Houseowner*.” Here, a “*numeric item*” has a numeric interval value whereas a “*categorical item*” has a categorical value. A numeric item $\langle p : q \rangle$ in the CAR is supported by a numeric item $\langle p_t : q_t \rangle$ in an instance t if $p_t = p$ and $q_t \subseteq q$ where $\subseteq$ states that the range of q_t is within the range of q . Hence, “ $t_1 = \{ \langle Age : [35, 37] \rangle, \langle Married : Yes \rangle, \langle NumCars : [2, 2] \rangle, \langle Child : [3, 3] \rangle \}$ ” supports the aforementioned rule body, whereas “ $t_2 = \{ \langle Age : [29, 31] \rangle, \langle Married : Yes \rangle, \langle NumCars : [2, 2] \rangle, \langle Child : [3, 3] \rangle \}$ ” does not, because $\langle Age : [29, 31] \rangle$ is not within $\langle Age : [30, 39] \rangle$. Given a training data set D which is a table (or a set) of class labeled instances (transactions), let D_{cl} be a set of all instances having a class cl in D . The body of a CAR including numeric items is a “*quantitative frequent itemset*,” QFI in short, which is supported by D_{cl} more frequently than a “*minimum support (minsup)*” threshold. The numeric part of a QFI corresponds to an axis-parallel and hyper-rectangular region in an attribute subspace of D .

CBA, CMAR and CAEP are the representatives of the CAR based classification [1,2,3]. Especially CAEP, using the strength of all CARs, widely shows the best performance among many rule-based classifiers including C4.5. However, it discretizes each numeric attribute by an entropy measure without considering the dependency of the distributions among multiple attributes, and thus a cluster of instances having the same class can often be fragmented. An effective solution for this issue is the introduction of the clustering in every numeric attribute

subspace to derive strong rule bodies. CLIQUE, DOC and SUBCLU seek dense clusters in every subspace [4,5,6]. However, they do not provide QFIs covering both numeric and categorical attributes and corresponding to axis-parallel and hyper-rectangular clusters in an efficient manner. The approaches to mine quantitative association rules have addressed this issue [7,8]. However, they discretize each attribute without considering the dependency among attributes, and thus can result in the aforementioned fragmentation.

In this study, a novel and efficient approach to the exhaustive, axis-parallel and hyper-rectangular subspace clustering is proposed. Moreover, the combined use of this clustering and CAEP is evaluated. The proposed clustering algorithm has a levelwise structure. While this is similar to SUBCLU, our approach can derive clusters on both numeric and categorical items, and the numeric items having interval values can be processed.

2 CAEP

CAEP is briefly explained at first [3]. The training phase of CAEP consists of two processes. The first is to derive all rule bodies. Let the support of an itemset a by D_{cl} be $support_{D_{cl}}(a) = |\{t \in D_{cl} | a \in t\}|/|D_{cl}|$. For every cl , a set of QFIs, $LQFI(cl)$, in which every itemset a satisfies $support_{D_{cl}}(a) \geq minsup$, is derived from D_{cl} . Subsequently, for every $a \in LQFI(cl)$, the following “growth rate” for a class cl is calculated. Let $\bar{D}_{cl} = D - D_{cl}$ be the opponent instances of cl .

Growth rate:

If $support_{\bar{D}_{cl}}(a) \neq 0$, $growth\_rate_{\bar{D}_{cl} \rightarrow D_{cl}}(a) = \frac{support_{D_{cl}}(a)}{support_{\bar{D}_{cl}}(a)}$,

If $support_{\bar{D}_{cl}}(a) = 0$ and $support_{D_{cl}}(a) \neq 0$, $growth\_rate_{\bar{D}_{cl} \rightarrow D_{cl}}(a) = \infty$,

Otherwise $growth\_rate_{\bar{D}_{cl} \rightarrow D_{cl}}(a) = 0$.

When the growth rate of a is more than a “growth rate threshold” $\rho(> 1)$, i.e., $growth\_rate_{\bar{D}_{cl} \rightarrow D_{cl}}(a) \geq \rho$, a is called an “emerging pattern (EP)” and selected as a rule body where its head has the class cl , i.e., $a \Rightarrow cl$. Let $LEP(cl)$ be a set of all EPs selected from $LQFI(cl)$ under this measure. The underlying principle here is to select the rule bodies having the strength to differentiate the class cl from the others. Even if the rule confidence is high in D_{cl} , the rule can match many instances in $\bar{D}_{cl}$. Such rules are weak for classification.

The second process is to derive a “base score.” First, the strength of an EP a based on the relative difference between $support_{D_{cl}}(a)$ and $support_{\bar{D}_{cl}}(a)$ is introduced as $support_{D_{cl}}(a)/(support_{D_{cl}}(a) + support_{\bar{D}_{cl}}(a)) = growth\_rate_{\bar{D}_{cl} \rightarrow D_{cl}}(a)/(growth\_rate_{\bar{D}_{cl} \rightarrow D_{cl}}(a) + 1)$. The following “aggregate score” represents the possibility of t to be classified into cl by EPs in $LEP(cl)$.

Aggregate score:

$$score(t, cl) = \sum_{a \subseteq t, a \in LEP(cl)} \frac{growth\_rate(a)}{growth\_rate(a) + 1} * support_{D_{cl}}(a). \quad (1)$$

Because the number of EPs for each cl may not be balanced, instances may get higher scores for some classes. A base score is introduced to eliminate this bias.

Base score:

$base\_score(cl)$ is the median of all aggregate scores in $\{score(t, cl) | t \in D_{cl}\}$. The testing phase uses $base\_score(cl)$, $growth\_rate(a)$ and $support_{D_{cl}}(a)$ obtained in the training phase. Given a test instance t , its aggregate score for cl , $score(t, cl)$, is computed from these results and Eq.(1). Then, it is normalized by $base\_score(cl)$ to eliminate the aforementioned bias as follows.

Normalized score:

$$norm\_score(t, cl) = \frac{score(t, cl)}{base\_score(cl)}.$$

cl having the maximum normalized score is assigned to the class of t . Except the derivation of $LQFI(cl)$ for all cl , the computational complexity of CAEP is $O(N)$ where $N = |D|$, since it scans the training data only twice.

3 Mining Rule Bodies of CARs

3.1 Levelwise Subspace Clustering

First we focus on the clustering of instances consisting of numeric items only. In our approach, the density of instances in a subspace is defined on their projected distribution to every attribute axis while ensuring the exhaustive finding of the clusters. The computational complexity is around $O(N \log N)$ as discussed later.

Let t and t' be instances sharing a numeric attribute p with interval values q and q' respectively. The “ Δ_p -neighborhood” $N_{\Delta_p}(t)$ on p is defined by $N_{\Delta_p}(t) = \{t' \in D_{cl} | Dist_p(q, q') \leq \Delta_p\}$ where “permissible range” Δ_p is a real positive number on p . If intervals q and q' overlap, then $Dist_p(q, q') = 0$, otherwise $Dist_p(q, q')$ is the distance between their boundaries facing each other. An instance $t \in D_{cl}$ is called a “core instance” on p if $N_{\Delta_p}(t)$ contains at least $MinPts$ instances, i.e., $|N_{\Delta_p}(t)| \geq MinPts$. When a core instance t is contained in $N_{\Delta_p}(t')$ of another core instance t' , t and t' have a “connection.”

Definition 1 (Density-Connected Set). A non-empty subset $C \subseteq D_{cl}$ is a “density-connected set” on p if C is the union of the Δ_p -neighborhoods of core instances where all the core instances are in a chain of connections on p .

Definition 2 (Dense Cluster). A “dense cluster” $C^S \subseteq D_{cl}$ in a subspace formed by a set of numeric attributes S is defined as a maximal set of instances which is a density-connected set on every $p \in S$ in D_{cl} .

Definition 3 (Quantitative Frequent Itemset). Let $C^S \subseteq D_{cl}$ be a dense cluster in a subspace S and $a(C^S) = \{< p : q > \mid p \in S, q = [\min_p(C^S), \max_p(C^S)]\}$ an itemset where $\min_p(C^S)$ and $\max_p(C^S)$ are the minimum and the maximum interval boundaries of instances in C^S on p . If $|C^S| \geq minsup$, i.e., $support_{D_{cl}}(a(C^S)) \geq minsup$, then $a(C^S)$ is a “quantitative frequent itemset (QFI).” When the dimension of S is k , it is called a k -QFI.

A QFI is a dense, axis-parallel and monotone hyper-rectangular region having a maximal volume in the subspace. Similarly to the dense clusters of SUBCLU, the following (anti-)monotonicity property of QFIs holds.

Table 1. An example of transaction data set of $cl = \text{Houseowner}$; $D_{\text{Houseowner}}$

$t_1 = (\{ \langle \text{Age} : [20, 23] \rangle, \langle \text{Child} : [2, 3] \rangle, \langle \text{NumCars} : [2, 2] \rangle \}, \text{Houseowner})$
$t_2 = (\{ \langle \text{Age} : [30, 30] \rangle, \langle \text{Child} : [4, 5] \rangle, \langle \text{NumCars} : [1, 1] \rangle, \langle \text{Savings} : [10K, 10K] \rangle \}, \text{Houseowner})$
$t_3 = (\{ \langle \text{Age} : [30, 30] \rangle, \langle \text{Child} : [2, 2] \rangle, \langle \text{NumCars} : [5, 5] \rangle, \langle \text{Savings} : [11K, 11K] \rangle \}, \text{Houseowner})$
$t_4 = (\{ \langle \text{Age} : [30, 35] \rangle, \langle \text{Child} : [5, 5] \rangle, \langle \text{NumCars} : [1, 1] \rangle, \langle \text{Savings} : [5K, 5K] \rangle \}, \text{Houseowner})$
$t_5 = (\{ \langle \text{Age} : [35, 37] \rangle, \langle \text{Child} : [2, 2] \rangle, \langle \text{NumCars} : [2, 2] \rangle, \langle \text{Savings} : [5K, 5K] \rangle \}, \text{Houseowner})$
$t_6 = (\{ \langle \text{Age} : [36, 39] \rangle, \langle \text{Child} : [2, 2] \rangle, \langle \text{NumCars} : [2, 3] \rangle \}, \text{Houseowner})$

Table 2. Process of levelwise subspace clustering of $D_{\text{Houseowner}}$

1-QFIs
$\{ \langle \langle \text{Age} : [30, 39] \rangle \}, \{ \langle \text{Child} : [2, 5] \rangle \}, \{ \langle \text{NumCars} : [1, 3] \rangle \}, \{ \langle \text{Savings} : [10K, 11K] \rangle \} \}$
2-QFIs
$\{ \langle \langle \text{Age} : [30, 39] \rangle, \langle \text{Child} : [2, 2] \rangle \}, \{ \langle \text{Age} : [30, 35] \rangle, \langle \text{Child} : [4, 5] \rangle \}, \{ \langle \text{Age} : [30, 39] \rangle, \langle \text{NumCars} : [1, 3] \rangle \}, \{ \langle \text{Age} : [30, 30] \rangle, \langle \text{Savings} : [10K, 11K] \rangle \}, \{ \langle \text{Child} : [2, 5] \rangle, \langle \text{NumCars} : [1, 3] \rangle \} \}$
3-QFIs
$\{ \langle \langle \text{Age} : [35, 39] \rangle, \langle \text{Child} : [2, 2] \rangle, \langle \text{NumCars} : [2, 3] \rangle \}, \{ \langle \text{Age} : [30, 35] \rangle, \langle \text{Child} : [4, 5] \rangle, \langle \text{NumCars} : [1, 1] \rangle \} \}$

Lemma 1 (Monotonicity). $\forall T \subseteq S$, if $a(C^S)$ is a QFI in S , then a QFI $a(C^T)$ supported by $a(C^S)$, i.e., $a(C^S) \subseteq a(C^T)$, exists in T .

Proof. Because C^S is a density-connected set on $\forall p \in S$, it is a density-connected set on $\forall p \in T$, and hence $C^S \subseteq C^T$. Therefore, $\forall p \in T$, $[\min_p(C^S), \max_p(C^S)] \subseteq [\min_p(C^T), \max_p(C^T)]$, and $a(C^T)$ is supported by $a(C^S)$. ■

Accordingly, a levelwise bottom up approach is applicable to search all QFIs. We exemplify its operation by using the dataset in Table 1. Each instance (transaction) t_i consists of a numeric itemset and a class $cl = \text{Houseowner}$. We perform the clustering under parameters of $\Delta_{\text{Age}} = 5$, $\Delta_{\text{Child}} = 1$, $\Delta_{\text{NumCars}} = 1$, $\Delta_{\text{Savings}} = 1K$, $\text{MinPts} = 1$ and $\text{minsup} = 2$. First, the items in each t_i are lexicographically ordered by the attribute names. This has been already done in this table. Subsequently, 1-QFIs are searched, where the instances are maximally density-connected on an attribute. For *Age*, a 1-QFI, $\{ \langle \text{Age} : [30, 39] \rangle \}$, exists since the items densely range from 30 to 39 under $\Delta_{\text{Age}} = 5$, and its support 5 is more than minsup . This 1-QFI with its “transaction id list (TID-List)” is indicated in Table 2. Each attribute has an 1-QFI in this example.

In the next step, the levelwise search for k -QFIs ($k > 1$) starts. Index lists named *TID-List* are used to point instances in D_{cl} similarly to AprioriTid algorithm [9]. Assuming that all $(k-1)$ -QFIs are known, the following “Candidate-Generation” operation derives all candidate k -QFIs.

Definition 4 (Candidate-Generation).

Join Phase: For two $(k-1)$ -QFIs sharing $k-2$ attributes,

$$((k-1)\text{-QFI} = \{ \langle p_1 : q_1 \rangle, \langle p_2 : q_2 \rangle, \dots, \langle p_{k-2} : q_{k-2} \rangle, \langle p_{k-1} : q_{k-1} \rangle \}, \text{TID-List}),$$

$$((k-1)\text{-QFI}' = \{ \langle p_1 : q'_1 \rangle, \langle p_2 : q'_2 \rangle, \dots, \langle p_{k-2} : q'_{k-2} \rangle, \langle p_k : q'_k \rangle \}, \text{TID-List}'),$$

their join is derived as follows:

$$(\text{candidate-}k\text{-QFI} = \{ \langle p_1 : q^c_1 \rangle, \dots, \langle p_{k-1} : q^c_{k-1} \rangle, \langle p_k : q^c_k \rangle \}, \text{TID-List}^c).$$

QFI-Count(candidate - k - QFI, TID - List^c);

- (1) $k - QFIS = \phi, TIDLS = \phi;$
- (2) If $|TID - List^c| < minsup$ return $k - QFIS;$
- (3) $S = \{p \mid \langle p : q \rangle \in candidate - k - QFI, p \text{ is numeric.}\};$
- (4) $TIDLS.temp = \{TID - List^c\};$
- (5) while $TIDLS \neq TIDLS.temp$ do begin
- (6) $TIDLS = TIDLS.temp;$
- (7) forall $p \in S$ do begin
- (8) $TIDLS.temp = MDCS(TIDLS.temp, p);$
- (9) end
- (10) end
- (11) forall $TID - List \in TIDLS$ do begin
- (12) $k - QFIS = k - QFIS + (QFI(S, TID - List), TID - List);$
- (13) end
- (14) return $k - QFIS;$

Fig. 1. Algorithm of QFI-Count

- (1) For each numeric attribute, create an index list sorted with the ascending order of D . Sort items in each $t \in D$ lexicographically.
- (2) $L_1 = \{(1 - QFI, TID - List)\};$
- (3) for $(k=2; L_{k-1} \neq \phi; k++)$ do begin
- (4) $C_k = \{(candidate - k - QFI, TID - List^c)\} = Extended - Candidate - Generation(L_{k-1});$
- (5) forall (candidate - k - QFI, TID - List^c) $\in C_k$ do begin
- (6) $L_k = L_k \cup QFI - Count(candidate - k - QFI, TID - List^c)$
- (7) end
- (8) end
- (9) Answer $L = \bigcup_k L_k;$

Fig. 2. Entire algorithm

where q_i^c is the intersection of the two intervals $q_i \cap q_i'$ for $i = 1, \dots, k-2$, $q_{k-1}^c = q_{k-1}$, $q_k^c = q_k'$ and $TID - List^c = TID - List \cap TID - List'$. If some $q_i^c = \phi$, the two $(k-1)$ -QFIs are not joined.

Prune Phase: For all $(k-1)$ -subsets s of this candidate- k -QFI, if the following $(k-1)$ -QFI exists:

$$\forall \langle p_i : q_i^c \rangle \in s, \exists \langle p_i : q_i \rangle \in (k-1) - QFI, q_i^c \cap q_i \neq \phi, \quad (2)$$

the candidate- k -QFI is retained, and $TID - List^c$ is a candidate dense cluster $\hat{C}^S$ where $|S| = k$. Otherwise the candidate- k -QFI is pruned.

This prune phase is based on Lemma 1. As far as q_i^c intersects with q_i in Eq.(2), the possibility that s and $(k-1) - QFI$ shares more than $minsup$ transactions is not negligible. Thus the candidate k -QFI is retained under this condition. In Table 2, a candidate-2-QFI, $\{\langle Age : [30, 39] \rangle, \langle Child : [2, 5] \rangle\}$ with $TID - List^c = \{t_2, t_3, t_4, t_5, t_6\}$ is derived from two 1-QFIs, $\{\langle Age : [30, 39] \rangle\}$ and $\{\langle Child : [2, 5] \rangle\}$. This passes the prune phase.

“QFI-Count” shown in Fig.1 derives dense clusters $C^S = TID - List$ and their corresponding k -QFIs, if they exist, by assessing the density of instances in $\hat{C}^S$ based on Definition 2 and 3. In the inside loop from (7) to (9), a maximal density-connected set C is searched on p within $\hat{C}^S$ at first in a function $MDCS$ along with Definition 1 under given Δ_p and $MinPts$. Multiple C can be found when multiple dense clusters are included in $\hat{C}^S$. $MDCS$ repeats to update C on p from every C derived and kept in $TIDLS.temp$ at the previous loop iteration. C having a size less than $minsup$ is discarded in $MDCS$. This update continues in the outer loop from (5) to (10), until each C converges to C^S where each C^S is independent of the convergence process due to the (anti-)monotonicity property. In the loop from (11) to (13), each QFI corresponding to $C^S = TID - List$ is computed by Definition 3 in a function QFI and returned as the output. In the example, the candidate-2-QFI, $\{\langle Age : [30, 39] \rangle, \langle Child : [2, 5] \rangle\}$

with $TID - List^c = \{t_2, t_3, t_4, t_5, t_6\}$ is given to this QFI-Count. In the inside loop, $MDCS$ derives $TIDLS.temp = \{\{t_2, t_3, t_4, t_5, t_6\}\}$ on Age . Next, it derives $TIDLS.temp = \{\{t_3, t_5, t_6\}, \{t_2, t_4\}\}$ on $Child$. Further applications of $MDCS$ do not change $TIDLS.temp$. Since the sizes of candidates are more or equal to $minsup = 2$, two 2-QFIs, $(\langle Age : [30, 39] \rangle, \langle Child : [2, 2] \rangle, \{t_3, t_5, t_6\})$ and $(\langle Age : [30, 35] \rangle, \langle Child : [4, 5] \rangle, \{t_2, t_4\})$, are derived.

3.2 Deriving QFIs of Numeric and Categorical Items

Candidate-Generation is extended to derive QFIs consisting of numeric and categorical items. The categorical items in the joined itemset are given in the same way as in the AprioriTid algorithm. In the join phase of Definition 4, if $q_i^c = \phi$ for some numeric item or $q_i \neq q_i'$ for some categorical item, the two given $(k-1)$ -QFIs are not joined. Otherwise they are joined as $q_i^c = q_i \cap q_i'$ for each numeric item and $q_i^c = q_i = q_i'$ for each categorical item. In the prune phase, the condition $q_i^c = q_i$ for a categorical item is applied in addition to $q_i^c \cap q_i \neq \phi$ for a numeric item in Eq.(2). The algorithm QFI-Count of Fig.1 is also altered. When the candidate- k -QFI consists of categorical items only, the loop from (5) to (10) is skipped, and $TIDLS = TIDLS.temp$ is applied. The function QFI at step (12) is also altered. For a categorical attribute p_i , its value is set to be $q_i^c = q_i = q_i'$.

The entire algorithm to derive QFIs from D is indicated in Fig.2. Required parameters are Δ_p for all numeric attributes, $MinPts$ and $minsup$. First, some index lists are created for the efficient processing in Extended-Candidate-Generation and QFI-Count. Subsequently, all QFIs are computed in L by the adaptation of the AprioriTid Algorithm. In the implementation, the inversed indexing $(t_i, \{candidate - k - QFI\})$ from each t_i to its containing candidate- k -QFIs is used instead of $(candidate - k - QFI, TID - List^c)$ similarly to the standard AprioriTid. This approach is applied to D_{cl} of every class cl to derive $LQFI(cl)$ required by CAEP described in the previous section.

4 Experimental Evaluation

4.1 Computational Efficiency

The most expensive task is the derivation of QFIs for CAR's bodies. Thus, its computational efficiency is evaluated by using artificial data sets. First, a set of seed items, SSI , is randomly generated where $r_n\%$ of them are numeric and the rest categorical. Second, a set of seed QFIs, $SQFI$, is generated by randomly selecting seed items from SSI . The size of each QFI is determined by uniform random distribution having its average at $\overline{|QFI|}$. Third, a set of instances (transactions) D is generated where each instance t is made by randomly selecting a QFI from $SQFI$ and further randomly adding extra $2\overline{|QFI|}$ seed items taken from SSI in the average. Finally, the values of numeric items in each t are distorted by introducing Gaussian noise having 5% amplitude. Our algorithm is tested on a Pentium 4 2.7 GHz PC with 2GB RAM. The default parameters for the test are $|SSI| = 1000$, $r_n = 50\%$, $|SQFI| = 10$, $\overline{|t|} = 12$, $N = |D| = 40000$,

Table 3. Complexity of clustering

Parameter	Range of Assessment	Dependency of	
		Comp.Time	Mem.Cons.
$ SSI $	[20, 20000]	constant	constant
r_n	[0%, 100%]	constant	constant
$ SQFI $	[1, 50]	$O(SQFI)$	$O(SQFI)$
$ t $	[8, 100]	exp. inc.	exp. inc.
$minsup$	[0.2%, 10%]	exp. dec.	exp. dec.
Δ_p	[0.1%, 100%]	inc. const.	inc. const.
$MinPts$	[1, 8000]	dec. const.	dec. const.
N	[200, 10^6]	$O(N \log N)$	$O(N)$

exp. inc./dec. : exponential increase/decrease.
inc./dec. const. : increase/decrease and saturation.

Fig. 3. N vs. Comp. Time

$minsup = 5\%$, $MinPts = 1$ and $\Delta_p = 20\%$ (in relative width of the maximum and minimum values of the instances on each numeric attribute).

The qualitative dependencies of computation time and memory consumption on the parameters summarized in Table 3 are similar to the AprioriTid. Their rapid increases are observed under the small values of Δ_p and $MinPts$ where the values are comparable with the average gap among two different instances values. The dependency of the computation time on N shown in Fig.3 is almost $O(N \log N)$ up to 1 million instances, while SUBCLU requiring the range query of each instance shows $O(N^2)$ where a dense cluster is such that for each instance in the dense cluster the neighborhood of a given radius ϵ has to contain at least a minimum number of $MinPts$ instances [6]. The memory consumption is $O(N)$, because the size of the inversed TID-List is proportional to N .

4.2 Classification Performance and Interpretability

CAEP combined with QFI derivation is called LSC-CAEP in this paper. Its 10CV accuracy has been compared with C4.5, CBA, CMAR and the original CAEP by using the UCI repository data as indicated in Table 4. These data sets were selected so that the accuracy by various approaches has been reported [1,2,3]. The parameters of LSC-CAEP was set as $minsup = 12\%$, $MinPts = 1$, $\Delta_p = 16\%$ and $\rho = 1.1$, where their optimality has been confirmed through empirical surveys. The bold faces in the table are the best. The standard deviations of the accuracies over 10CV by C4.5, CBA and LSC-CAEP are 2.6%, 3.4% and 4.0% respectively, while the average discrepancies of the accuracies of LSC-CAEP are 5.6% from C4.5 and 4.5% from CBA in Table 4. LSC-CAEP performs moderately better than C4.5 and better or equal at least to CBA. As shown in the last column of Table 4, the computation times of LSC-CAEP ranges from 0.1 to 87sec, while C4.5 and CBA are from 0.3 to 2.2sec. The speed of LSC-CAEP usually remains practical due to its aforementioned good scalability, though it is affected by the dependency among attributes similarly to the Apriori algorithm.

The interpretability of the rules is important but a quite subjective matter. The following two QFIs having large support values are found by LSC-CAEP in Labor Relations Database in UCI repository.

Table 4. Comparison of accuracies

dataset	num. of records	num. of attributes(numeric)	num. of classes	C4.5	CBA	CMAR	CAEP	LSC-CAEP [comp. time (sec)]
Cleve	303	13(5)	2	.782	.828	.822	.833	.789 [38]
Ecoli	336	8(7)	8	.824	-	-	-	.831 [22]
Heart	270	13(6)	2	.808	.819	.822	.837	.845 [87]
Hepatitis	155	19(6)	2	.806	.818	.805	.830	.852 [26]
Iris	150	4(4)	3	.953	.947	.940	.947	.967 [0.1]
Glass	214	9(9)	7	.687	.739	.701	-	.681 [19]
Labor	57	16(8)	2	.793	.863	.897	-	.943 [0.1]
Wine	178	13(13)	3	.927	.950	.950	.971	.972 [52]
Zoo	101	16(0)	7	.922	.968	.971	-	.911 [19]

support=19: {class:good, duration-years:[2,2], working-hours:[33,40], wage-inc.-2nd-year(%):[4.0,5.8]}.

support=16: {class:good, duration-years:[3,3], working-hours:[35,40], wage-inc.-2nd-year(%):[3.5,5.0]}.

These QFIs suggest an assumption that the increase of job stability from 2 years to 3 years balances with admitting slightly longer working hours and 0.5% ~ 0.8% less wage increase. The following two CARs having high growth rates are found in Iris data. The insights on the species of iris can be learned.

growth rate=4.5: petal width:[1.4-2.5] → class:virginica

growth rate=1.9: sepal length:[4.9-7.0], sepal width:[2.0-3.4] → class:setosa.

The fine granularity of the interval boundaries helps the interpretation.

5 Discussion and Conclusion

To check the applicability of LSC-CAEP to large data sets, LSC-CAEP, C4.5 and CBA were applied to Census-Income data containing 199523 instances and 40 attributes (numeric: 7 categorical: 33) in UCI KDD Archive, and confirmed that the accuracy achieved by LSC-CAEP is 92.4%, which is comparable with 94.3% and 94.0% of C4.5 and CBA respectively. Further improvement of the performance of LSC-CAEP will be addressed in future study.

The most expensive tasks in LSC-CAEP are the sort which is $O(N \log N)$ and QFI-Count of Fig.1. The maximal density-connected sets on every numeric attribute p are easily derived in one scan of *TIDLS.temp* in *MDCS* by using the sort index list built at the first step in Fig.2. Hence it is $O(N)$ at maximum. The iteration of the outer loop from (5) to (10) in QFI-Count varies extensively. In the worst case, an instance is removed in each loop path from the edge of a region where instances are ranged in a periodic manner, and the loop becomes $O(N^2)$. However, in the most likely case which is an exponential density distribution, a portion $0 < r < 1$ of the instances in the average are retained in each loop path. The loop finishes by the time $r^m N$ becomes less than *minsup* where m is the number of loop paths. Thus $\text{minsup} \leq r^m N$, and m is around $O(\log N)$. Accordingly, the entire algorithm is expected to be $O(N \log N)$.

Our proposal enabled efficient subspace clustering on the mixtures of numeric and categorical data in a levelwise algorithm. Further new approaches of clustering and classification for large data sets can be developed along this line.

Acknowledgement. The authors wish to thank Dr. Alexandre Termier in ISIR, Osaka Univ. for his extensive support to write this paper. This research was partially supported by the Japan Society for the Promotion of Science (JSPS) Grant-in-Aid for Scientific Research (B), 16300045, 2005.

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