Overlapping Constraint for Variational Surface Reconstruction

Henrik Aanæs¹ and Jan Erik Solem²

¹ Informatics and Mathematical Modelling,
Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark
`haa@imm.dtu.dk`

² Jan Erik Solem, School of Technology and Society,
Malmö University, 205 06 Malmö, Sweden
`jes@ts.mah.se`

Abstract. In this paper a counter example, illustrating a shortcoming in most variational formulations for 3D surface estimation, is presented. The nature of this shortcoming is a lack of an overlapping constraint. A remedy for this shortcoming is presented in the form of a penalty function with an analysis of the effects of this function on surface motion. For practical purposes, this will only have minor influence on current methods. However, the insight provided in the analysis is likely to influence future developments in the field of variational surface reconstruction.

1 Introduction

The reconstruction of surfaces from a set of calibrated cameras, or the so called (multiple view) stereo problem, is one of the main cornerstones of computer vision research. Of the magnitude of 3D reconstruction approaches it is often this surface reconstruction, which is needed in order for the end user to perceive a result as acceptable.

An approach to formulating the surface reconstruction problem, which has become popular within the last years, is by casting the problem in a variational framework c.f. e.g. [1, 2, 3, 4, 5, 6]. The problem is thus formulated using some functional, which has to be minimized. The variational approach to surface reconstruction has many advantages, where one of it's main virtues is that it ensures theoretical soundness and a clear understanding of which objective function is being used¹.

Surface reconstruction is closely related to the feature tracking problem and can in fact be viewed as registering *all* the pixels in the images across all the images, and then reconstructing the surface by simple triangulation. When extracting and matching features across images, as in e.g. [7], there is however, an implicit assumption that what is seen in one image is to some degree the

¹ Data fitting problems almost always boil down to implicitly or explicitly minimizing some objective function.

same as in the next. We will here argue that an (view) overlapping constraint of some sort similar to the tracking case is also needed in the variational approach to surface reconstruction. We will do this by demonstrating that *without* such an overlapping constraint the minimum of the variational problem is by no means the sought solution. Following this we propose a remedy addressing this issue. Since it is noted, that the present variational approaches work well the proposed overlapping constraint will have no practical implications on present day variational methods, but will address their theoretical shortcoming.

In relation to previous work it is noted, that the concept of encouraging overlap between the images is incorporated in many two view stereo algorithms as an occlusion cost, and it is e.g. eloquently formulated in [8]. This overlapping constraint has, however, seemingly been lost in the transition to the multiple view case and the corresponding variational formulations.

2 Variational Surface Estimation

As mentioned above, several variational formulations for surface estimation have been proposed in the literature. Most of these formulations fit in the general form of:

$$E(\Gamma) = \int_{\Gamma} \Phi \, d\sigma + \alpha \int_{\Gamma} d\sigma \, , \quad (1)$$

where Φ is a real-valued function, usually defined on the surface Γ (or in a neighborhood of Γ) which contains a photo consistency term and the last integral is a surface regularization term penalizing surface area. The constant α determines the weight of the regularizing term. For some methods the regularization is inherent in Φ but this does not affect the analysis below. The reader should note, that this formulation, forming the basis for the rest of this paper, is *independent* of specific choices for the function Φ , the surface representation and the numerical scheme used.

3 Counter Examples

The above mentioned methods for surface reconstruction, using (1), have proven themselves by yielding good results in practice. This is usually to a large extent due to a sufficiently good initialization, in that the minimum of (1) found is a local minimum, as described in the following.

Considering Figure 1, it is seen that there exist surfaces, Γ , which have a lower value for (1), than the 'true' underlying surface, but which has no resemblance to this true underlying surface. These undesired surfaces are constructed by placing e.g. small balls in front of each camera covering its field of view. This ensures that each camera exclusively views its part of the surface. The balls or surface area can furthermore be made arbitrary small while moving the balls closer to the camera. This construction for Γ ensures that the first photo consistency part of (1) is zero, in that a perfect match is achievable since *no two cameras view the*

Fig. 1. A stylized surface reconstruction problem, where the 4 cameras are denoted by their image plane and focal point. The true surface is depicted as the shaded area in the center of the figure and a surface, Γ , giving a better fit to (1) is represented by the circles in front of the cameras. The dotted lines are included to demonstrate that the circles block the cameras field of view

same part of the surface. Secondly the last part of (1) can be made arbitrarily small. Note that it is implicitly assumed, that the surface area is above zero, in that this is a trivial and again undesirable minimum to (1) which is always present in these formulations.

As such it is demonstrated that the 'true' underlying surface will *not* yield a minimum to (1) — since image noise is assumed. Furthermore, minima exist which are not the desired solutions to the problem at hand. In effect (1) does not model the problem faithfully. Note, that these undesirable minima to (1) arise by grossly ignoring the overlapping constraint, and as such this suggests that this needs to be incorporated in the modelling of the problem.

4 Overlapping Constraint

As should be apparent from the argument above, an overlapping constraint needs to be incorporated in the modelling of the surface reconstruction problem and as such also in the corresponding variational formulation. In the following analysis, an additional term $\xi(\Gamma)$ is proposed which added to (1) does exactly that.

Naturally such an overlapping constraint can be modelled or formulated in a multitude of ways. In developing $\xi(\Gamma)$ we have chosen to focus on the fact that the above mentioned methods using (1), work well in practice, and as such we want the effect of $\xi(\Gamma)$ around the 'true' surface to be zero or negligible at best. On the other hand — since we have decided on a additive term — the effect of $\xi(\Gamma)$ should be prohibitive in cases where nearly no overlap exists between the parts of Γ viewed in the various cameras. Secondly, we also note, that in many surface reconstruction tasks only part of the surface is viewed, as such we do not wish to penalize parts of Γ not being viewed in any camera. Hence we chose to take offset in the ratio between the parts of Γ viewed in *exactly one camera* and the parts of Γ where an overlap exists and Γ is viewed in *more than one camera*.

More formally, define Γ_{Single} as parts of Γ seen by exactly one camera, and Γ_{Visible} as parts of Γ visible form any camera. Hence $\Gamma_{\text{Single}} \subseteq \Gamma_{\text{Visible}} \subseteq \Gamma$. Introduce $g(\Gamma) \in [0, 1]$ as the ratio

$$g(\Gamma) = \frac{\int_{\Gamma_{\text{Single}}} d\sigma}{\int_{\Gamma_{\text{Visible}}} d\sigma} = \frac{|\Gamma_{\text{Single}}|}{|\Gamma_{\text{Visible}}|} , \quad (2)$$

where we use the notation $|\Gamma|$ to mean the area of Γ .

Let H be a smoothed version of the Heaviside step function, e.g. the C^2 approximation as defined in [9]

$$H(x) = \begin{cases} 1 & x > \gamma \\ 0 & x < -\gamma \\ \frac{1}{2}[1 + \frac{x}{\gamma} + \frac{1}{\pi} \sin(\frac{\pi x}{\gamma})] & |x| \leq \gamma \end{cases} . \quad (3)$$

The function $\xi(\Gamma)$ is then defined to be

$$\xi(\Gamma) = H(g(\Gamma) - \tau) , \quad (4)$$

where $\tau \in [0, 1]$ is a threshold level. This threshold is used to allow Γ_{Single} some predefined surface fraction.

All that is needed now is to modify (1) to take this overlapping constraint into account. This can be done by formulating the total energy functional for the problem as

$$E_{\text{Tot}}(\Gamma) = E(\Gamma) + \beta H(g(\Gamma) - \tau) , \quad (5)$$

where β is a prohibitive weighting constant. In principle one can choose other functions than "thresholding" with H , we, however, leave this up to the imagination of the reader.

5 The Effect on Surface Motion

Adding the term $\xi(\Gamma)$ to the functional will in theory affect the surface evolution if one uses an iterative approach such as e.g. gradient descent methods. Whether

it is relevant to take this into account or not we leave for the reader to decide. For the sake of completeness, in this section we briefly derive the influence on the normal velocity in a gradient descent implementation. The Gâteaux derivative, $d\xi(\Gamma)$, of $\xi(\Gamma) = H(g(\Gamma) - \tau)$ is

$$d\xi(\Gamma) = H'(g(\Gamma) - \tau)dg(\Gamma) = H'(g(\Gamma) - \tau)\kappa \frac{(1 - g(\Gamma))}{|\Gamma_{\text{Visible}}|}, \quad (6)$$

where κ is the mean curvature of the surface and $dg(\Gamma)$ denotes the Gâteaux derivative of $g(\Gamma)$. This relation is clear from the chain rule and the fact that the Gâteaux derivative of $g(\Gamma)$ is

$$dg(\Gamma) = \frac{\kappa_{\text{Single}}|\Gamma_{\text{Visible}}| - |\Gamma_{\text{Single}}|\kappa_{\text{Visible}}}{|\Gamma_{\text{Visible}}|^2} = \kappa \frac{\left(1 - \frac{|\Gamma_{\text{Single}}|}{|\Gamma_{\text{Visible}}|}\right)}{|\Gamma_{\text{Visible}}|} = \kappa \frac{(1 - g(\Gamma))}{|\Gamma_{\text{Visible}}|}, \quad (7)$$

where we have used the product rule and the fact that $\kappa_{\text{Single}} = \kappa_{\text{Visible}} = \kappa$ on Γ and the well known fact that the Gâteaux derivative of area functionals give the mean curvature, cf. [10]. The result is that if one wants to incorporate this in the gradient descent using any surface representation, an extra term $H'(g(\Gamma) - \tau)\kappa(1 - g(\Gamma))/|\Gamma_{\text{Visible}}|$ in the normal velocity is needed.

6 Discussion and Conclusion

The above mentioned surface reconstruction methods employing a variational formulation, all work well without our proposed overlapping constraint (5). One could then ask oneself why this constraint is relevant, since it seemingly has no practical relevance. Firstly, a correct modelling of the problem is important in its own right, and especially with a variational formulation, which is often used to ensure theoretical soundness of ones method. Secondly in the further development of surface estimation methods the insight accompanying the proposed overlapping constraint is likely to become a real concern. An example of the latter is; that the idea to this constraint came while discussing if a certain surface reconstruction algorithm actually reached the global minimum.

In conclusion, an overlapping constraint has been proposed, which helps ensure that the global minimum to functionals used for surface estimation is in fact what is desired. This constraint also encapsulates a better modelling of the surface reconstruction problem, such that it better reflects our implicit expectations.

References

1. Faugeras, O., Keriven, R.: Complete dense stereovision using level set methods. Computer Vision - ECCV'98. 5th European Conference on Computer Vision. Proceedings (1998) 379–93 vol.1

2. Faugeras, O., Keriven, R.: Variational principles, surface evolution, pdes, level set methods, and the stereo problem. *Image Processing, IEEE Transactions on* 7 (1998) 336–344
3. Jin, H., Yezzi, A., Soatto, S.: Variational multiframe stereo in the presence of specular reflections. Technical Report TR01-0017, UCLA (2001)
4. Jin, H., Soatto, S., Yezzi, A.: Multi-view stereo beyond lambert. *Computer Vision and Pattern Recognition, 2003. Proceedings. 2003 IEEE Computer Society Conference on* 1 (2003) 171–178
5. Yezzi, A., Soatto, S.: Structure from motion for scenes without features. *Computer Vision and Pattern Recognition, 2003. Proceedings. 2003 IEEE Computer Society Conference on* 1 (2003) 525–532
6. Yezzi, A., Soatto, S.: Stereoscopic segmentation. *International Journal of Computer Vision* 53 (2003) 31–43
7. Hartley, R.I., Zisserman, A.: *Multiple View Geometry*. Cambridge University Press, The Edinburgh Building, Cambridge CB2 2RU, UK (2000)
8. Cox, I., Higorani, S., S.B.Rao, Maggs, B.: A maximum likelihood stereo algorithm. *Computer Vision and Image Understanding* 63 (1996) 542–67
9. Zhao, H., Chan, T., Merriman, B., Osher, S.: A variational level set approach to multiphase motion. *J. Computational Physics* 127 (1996) 179–195
10. do Carmo, M.: *Differential Geometry of Curves and Surfaces*. Prentice-Hall (1976)