Modeling and Simulation of High-Speed Machining Processes Based on Matlab/Simulink

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Abstract. This paper shows the mathematical development to derive the integral-differential equations and the algorithms implemented in MATLAB to predict the cutting force in real time in high speed machining processes. This paper presents a cutting-force-based model able to describe a high-speed machining process. The model considers the cutting force as an essential output variable in the physical processes taking place in high-speed machining. For the sake of simplicity, only one type of end mill shapes is considered (i.e., cylindrical mill) for real-time implementation of the developed algorithms. The developed model is validated in slot-milling operations. The results corroborate the importance of the cutting-force variable for predicting tool wear in high-speed machining operations.

Keywords: modeling; complex systems; high-speed machining.

1 Introduction

One of the basic tasks manufacturing systems have to perform today is machining, especially high-speed machining (HSM), a real case of a complex electromechanical system [1]. The idea of characterizing the machining process using mathematical models to yield an approximate description of the physical phenomenon aroused the interest of many researchers [2]. That work has been carried on, and it has enabled computational tools to be developed for modeling and simulating the conventional machining process, relying on classic modeling and identification strategies.

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At the present time, modeling high-speed machining processes as a complex electromechanical process, especially high-speed milling, is a very active area of investigation that is peppering the scientific community with challenges [3,4]. High-speed machining has now been adopted and put into regular use at many companies, and yet certain points of how to monitor cutting-tool condition have been worked out only partially as yet, largely for lack of a mathematical model of the process that can feasibly be used in real-time applications. This paper is aimed at deriving a mathematical model and characterizing the high-speed cutting process in high-speed milling operations through the study of the dynamic behavior of cutting force. Previous work concerning conventional machining has been borne in mind as the essential starting point for this work [5].

A mathematical model is essential for understanding the dynamic behavior of metalworking processes and improving their operation on the basis of time or frequency responses. Increasingly, both experimental models and analytical models are being used in metalworking for the obtaining, manufacturing and processing (e.g., shaping and cutting) of metal materials. When a new product or metal part (e.g., mold) is being designed, the item must pass simulation testing on the process involved (e.g., the cutting process) before work ever begins on the item's physical manufacture. How useful simulations ultimately are depends largely on how faithfully the mathematical models they use describe the real behavior of the cutting process. Moreover, computational methods are essential to yield an adequate accuracy in the prediction and real-time implementation of the model.

The main goal of this paper is to derive a mathematical model from the characterization of the physical processes taking place during high-speed machining and to implement the model in MATLAB/SIMULINK. The paper is organized into five sections. Section 1 describes the state of the art, analyzes some previous work on the subject, gives a quick description of high-speed machining and finally outlines some questions related with the implementation of HSM models in MATLAB. Section 2 presents the kinematics of the model, considering the geometrical form of the helix for the cylindrical mill. Section 3 addresses the mathematical formulation of the proposed model and sets up the integral-differential equations for calculating cutting force in the time domain. Section 4 gives the results yielded by the simulations and tests in real time. Lastly, the paper frames some conclusions, looks ahead to some possible future work and discusses the lines opened to investigation.

2 Geometrical Model of the Tool

Geometrical modeling of the helical cutting edge includes the kinematic and dynamic analysis of the cutting process. Predicting cutting forces requires a system of coordinates, the helix angle and the angular distance of a point along the cutting edge [6]. The mathematical expressions that define this geometry in a global coordinate system are presented below in the geometric model, using classic vector notation.

Vector $\vec{r}(z)$ (figure 1) drawn from point O to a point P in cylindrical coordinates is expressed mathematically in equation 1.

$$\vec{r}_j = x_j\vec{i} + y_j\vec{j} + z_j\vec{k} = r(\phi_j)(\sin\phi_j\vec{i} + \cos\phi_j\vec{j}) + z(\phi_j)\vec{k} \tag{1}$$

where ϕ_j is the radial rake angle of a point P at tooth j. Point P lies at an axial depth of cut a_p in the direction of axis Z, at a radial distance $r(z)$ on the XY plane, with an axial rake angle $\kappa(z)$ and a radial lag angle of $\psi(z)$.

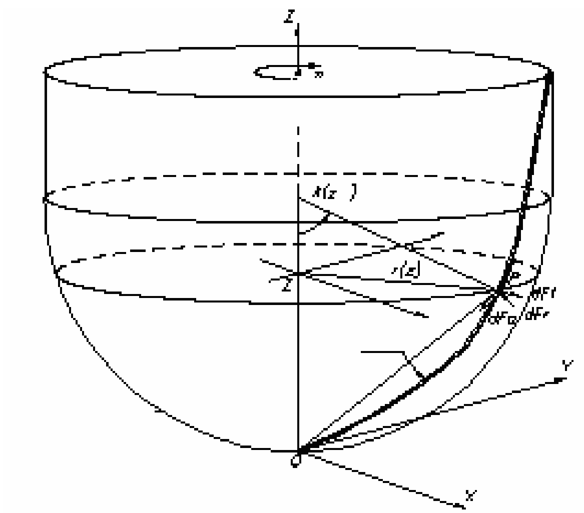

Fig. 1. Tool geometry

The geometry of the tool is represented mathematically, considering that the helical cutting edge wraps parametrically around a cylinder. The mathematical model dictated for the cutting edge considers that the edge is divided into small increments, where the cutting coefficients can vary for each location. The initial point of reference to the cutting edge of the tool (j = 1) is considered to be the angle of rotation when $z = 0$ is ϕ. The radial rake angle for the cutting edge j in a certain axial position z is expressed as:

$$\phi_j(z) = \phi + \sum_{n=1}^{j} \phi_p - \psi(z) \tag{2}$$

The lag angle $\psi(z)$ appears due to the helix angle θ. This angle is constant in the case of a cylindrical mill. In the generalized model for the geometry of a mill with helical teeth, the tool diameter may differ along the length of the tool, depending on the shape of the tool. An infinitesimal length of this cutting edge may be expressed as

$$dS = |dr| = \sqrt{r^2(\phi) + (r'(\phi))^2 + (z'(\phi))^2}\, d\phi,\ r'(\phi) = \frac{dr(\phi)}{d\phi},\ z' = \frac{dz(\phi)}{d\phi} \tag{3}$$

Chip thickness changes as a function of the radial rake (ϕ) and axial rake (κ):

$$h_j(\phi_j) = s_{tj} \sin\phi_j \cdot \sin\kappa \tag{4}$$

For a cylindrical mill, the following conditions are defined for finding the general solution:

$$r(z) = \frac{D}{2} \qquad \kappa = 90^{\circ} \quad \psi = k_\theta z \quad k_\theta = (2\tan\theta)/D \tag{5}$$

3 Dynamic Model of Cutting Forces

The force differentials (dF_t), (dF_r), (dF_a) act on an infinitesimal element of the cutting edge of the tool [7]:

$$\begin{aligned} dF_t &= K_{te} dS + K_{tc} h_j(\phi,\kappa) db \\ dF_r &= K_{re} dS + K_{rc} h_j(\phi,\kappa) db \\ dF_a &= K_{ae} dS + K_{ac} h_j(\phi,\kappa) db \end{aligned} \tag{6}$$

It is also considered that $db = \dfrac{dz}{\sin\kappa}$. In order to facilitate finding the mathematical relations inherent in this set-up, very small time increments are used. The positions of the points along the cutting edge are evaluated with the geometrical model presented herein above.

Furthermore, the characteristics of a point on the cutting surface are identified using the properties of kinematic rigidity and the displacements between the tool and the workpiece. The constants or cutting coefficients (K_{tc},, K_{rc},, K_{ac}, K_{te},, K_{re},, K_{ae}) can be found experimentally using cutting forces per tooth averaged for a specific type of tool and material [8,9]. We might point out that these coefficients are highly dependent on the location (axial depth) of the cutting edge. How these coefficients are found shall not be addressed in this paper.

Cutting forces can be evaluated employing a system of Cartesian coordinates. After transforming and total cutting forces as a function of ϕ are:

$$\begin{aligned} F_x(\phi) &= \sum_{j=1}^{Nf} (F_{xj}(\phi_j(z))) = \sum_{j=1}^{Nf} \int_{z1}^{z2} \left[-dF_{rj} \sin\phi_j \sin\kappa_j \quad -dF_{tj} \cos\phi_j \quad -dF_{aj} \sin\phi_j \cos\kappa_j \right] dz \\ F_y(\phi) &= \sum_{j=1}^{Nf} (F_{yj}(\phi_j(z))) = \sum_{j=1}^{Nf} \int_{z1}^{z2} \left[-dF_{rj} \cos\phi_j \sin\kappa_j \quad dF_{tj} \sin\phi_j \quad -dF_{aj} \cos\phi_j \cos\kappa_j \right] dz \\ F_z(\phi) &= \sum_{j=1}^{Nf} (F_{zj}(\phi_j(z))) = \sum_{j=1}^{Nf} \int_{z1}^{z2} \left[-dF_{rj} \cos\kappa_j \quad 0 \quad -dF_{aj} \sin\kappa_j \right] dz \end{aligned} \tag{7}$$

where z_1 and z_2 are the integration limits of the contact zone at each moment of cutting and can be calculated from the geometrical model described herein above. For the numerical calculation, the axial depth of cut is divided into disks having an infinitesimal height dz. The differentials of the cutting forces are calculated along the length of the cutting edge in contact, and they are summed to find the resulting forces for each axis $F_x(\phi)$, $F_y(\phi)$, $F_z(\phi)$ in an angle of rotation.

The exact solution can be found by substituting, making $\kappa = 90^\circ$, and integrating, we obtain the exact solution for cylindrical mill:

$$F_{x,j}(\phi_j(z)) = \left\{ \begin{array}{l} \frac{s_{tj}}{4k_\beta}\left[-K_{tc}\cos 2\phi_j(z) + K_{rc}\left[2\phi_j(z) - \sin 2\phi_j(z)\right]\right] \\ + \frac{1}{k_\beta}\left[K_{te}\sin\phi_j(z) - K_{re}\cos\phi_j(z)\right] \end{array} \right\}_{z_{j,1}(\phi_j(z))}^{z_{j,2}(\phi_j(z))}$$

$$F_{y,j}(\phi_j(z)) = \left\{ \begin{array}{l} \frac{-s_{tj}}{4k_\beta}\left[K_{tc}(2\phi_j(z) - \sin 2\phi_j(z) + K_{rc}\cos 2\phi_j(z)\right] \\ + \frac{1}{k_\beta}\left[K_{te}\cos\phi_j(z) - K_{re}\sin\phi_j(z)\right] \end{array} \right\}_{z_{j,1}(\phi_j(z))}^{z_{j,2}(\phi_j(z))} \quad (8)$$

$$F_{z,j}(\phi_j(z)) = \frac{1}{k_\beta}\left[K_{ac}s_{tj}\cos\phi_j(z) - K_{ae}\phi_j(z)\right]_{z_{j,1}(\phi_j(z))}^{z_{j,2}(\phi_j(z))}$$

where $z_{j,1}(\phi_j(z))$ and $z_{j,2}(\phi_j(z))$ are the lower and upper limits, respectively, that establish the axial depth of cut at lip j of the mill.

4 Simulations and Experimental Validation

The algorithms were implemented in MATLAB, drawing upon the mathematical models [10]. Despite MATLAB is not a computer algebra system, it is very useful as software tool for doing numerical computations with matrices and vectors. It can also display information graphically and includes many toolboxes for several research and applications areas. MATLAB was chosen for the advanced numerical methods that are available, the possibility it affords of running simulations and applications in real time and the portability of the programs that are developed with its use (i.e., it is possible to generate C/C++ programs from MATLAB files). Neither Matlab toolbox was used for programming the model.

The main difficulties are choosing the cutting coefficients and the properties of the materials, which were taken from earlier work on the subject [11]. The workpiece-material properties that were used for the simulation in the MATLAB environment are the properties of GGG-70 cast iron with nodular graphite. In this study, in the simulation and in the real tests, two cutting conditions for high-speed milling operations were regarded: Vc=546 m/min, sp=14500 rpm, f=1740 mm/min, a_p = 0.5 mm, a_e = 0, a_s = 12 mm, θ=30°, H = 25.0 mm, D = 12.0 mm where D is the tool diameter [mm], H is the total tool cutting-edge height [mm], θ is the helix angle [degrees], a_p is the axial depth of cut along the Z axis [mm], a_e is the radial depth of cut at the starting point (ϕ_{st}) [mm], a_s is the radial depth of cut at the stopping point (ϕ_{ex}) [mm], f is the feedrate [mm/min], Vc is the cutting speed [m/min], sp is the spindle speed in [rpm].

The constants used in the simulation and in the experimental validation were K_{tc} = 2172 N/mm^2, K_{rc} = 850 N/mm^2, K_{te}=17.3 N/mm, K_{re} = 7.8 N/mm, K_{ac} = 726 N/mm^2, K_{ae} = 6.7 N/mm. These constants or cutting coefficients referring to the material and the tool were drawn from the available literature, due to their similarity to the characteristics of the tool/material set-up used in the study in question.

The real-time model validation tests were conducted at a KONDIA HS1000 high-speed machining center equipped with a Siemens 840D open CNC. Actual cutting force signal was measured using a Kistler 9257 dynamometric platform installed on the testbed. Measurement was done by means of a DAQBOARD-2005 data-acquisition card at a sampling frequency of 50 kHz. A Karnasch 30.6472 cylindrical mill 12 mm in diameter was selected to validate the model developed under the procedure described herein. The chosen test piece, measuring 200x185x50 mm, was made of GGG-70 iron and was machined in a spiral pattern. The real cutting conditions chosen were the same as considered above for the simulation. A view of the cutting tool used in the tests (a), the workpiece and its profile to be mechanized (b) and the machine-tool laboratory (c) is shown in figure 2.

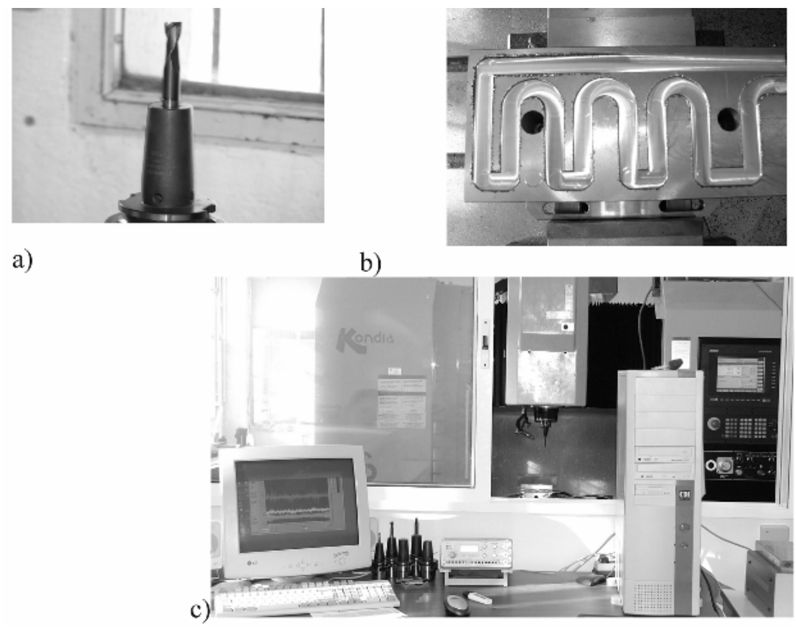

Fig. 2. Cutting tool for experiments, b) experimental piece, c) partial view of the Laboratory for machine tool research

Figure 3 shows the real behavior of cutting forces F_x, F_y and F_z and the resulting force F_{qT} for the case analyzed. The model's response is shown as a solid line.

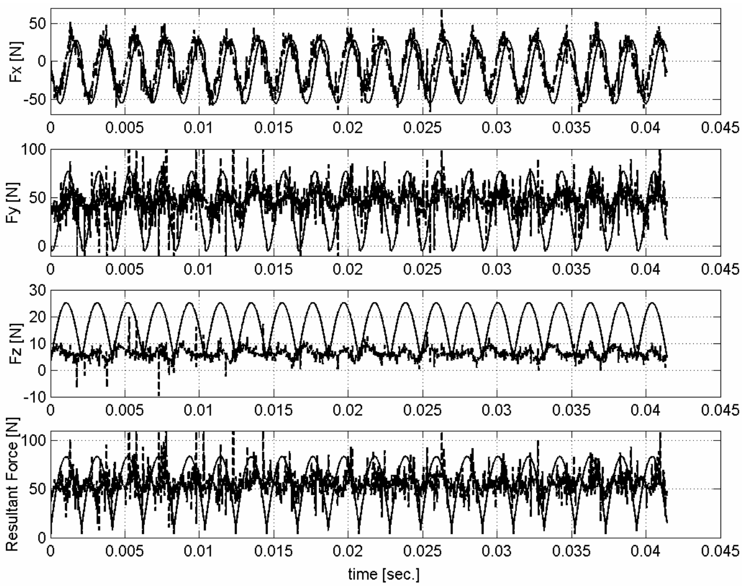

Fig. 3. Measured (straight line) and predicted by model (dashed line) cutting force for a new tool in high speed slot cutting

The average resulting cutting force $\overline{F}_e$ estimated by the model is 56.7N and the average resulting cutting force $\overline{F}_{qT}$ measured in a real high-speed cutting operation is 55.6N. The error criterion $\overline{E} = \left(\overline{F}_{qT} - \overline{F}_e\right) \cdot 100 / \overline{F}_{qT}$, is 4.5%.

5 Conclusions

This paper reports some results in modeling the high-speed cutting process and the validation of the model in question. The mathematical model implemented and assessed in MATLAB has two fundamental, co-dependent parts. The first is a multiple-input system that defines the model's kinematics, where constants and variables describe the tool geometry, the material type and the cutting parameters. The second addresses the dynamics, represented by integral-differential equations. In the case of cylindrical mill the exact analytical solution is found, inasmuch as the limits of integration and the boundary conditions along the tool geometry can be pre-established.

On the basis of the literature to which we have had access, this paper constitutes the first successful attempt in the implementation and application of computationally efficient algorithms for predicting cutting force in high-speed machining processes. With such models available, there is now a short-term way of dealing with essential questions concerning the surface finish and stability of high-speed cutting operations.

Despite the success in the application of Matlab, computer algebra in the formulation and solution of more complex cutting tools, as well as the use of the interface to communicate Mathematica and Matlab will be explored during future research.

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