PK+ Tree: An Improved Spatial Index Structure of PK Tree

Xiaolin Wang, Yingwei Luo*, Lishan Yu, and Zhuoqun Xu

Dept. of Computer Science and Technology, Peking University, Beijing, P.R.China, 100871
lyw@pku.edu.cn

Abstract. Spatial index is very important in GIS. PK tree and Hilbert R tree are two well-known spatial index structures. Comparison operations are very little in PK tree, while disk I/O operations are quite little in Hilbert R tree. PK+ Tree is an improved spatial index structure from PK tree. In PK+ tree, Comparison operations are less than in Hilbert R tree, while disk I/O operations are almost the same as in Hilbert R tree.

1 Introduction

With the development of information technology about data mining and multimedia, massive data processing gets more and more important in spatial processing. Spatial indexing is a key issue for massive spatial data processing.

Many kinds of spatial index structures exist nowadays. They can be divided into two main categories: partition tree and R tree. These two categories of spatial index structure have their own weakness and strongpoint. PK tree and Hilbert R tree are two efficient spatial index structures of each category.

In this paper, a new spatial index structure, PK+ tree, which are from PK tree, is introduced in order to improve storage usage, so that it may be more efficient for high dimensional data. In the next section, the problems of PK tree are analyzed. In the third section, PK+ tree is discussed in details. In the fourth section, the efficiency of storage and query of PK+ tree are compared with of PK tree and Hilbert R tree.

2 PK Tree and Its Weakness

PK tree is an efficient spatial index structure of partition tree category. In PK tree, an initial space cell C_0 are required to enclosing all the data objects to be indexed. C_0 can be split into $R=(r_x, r_y)$ sub cells C_i , and so on, so that the width and height of each C_i is $1/r_x$ and $1/r_y$ of C_{i-1} . Only those K-instantiable cells have their corresponding nodes in PK tree.

A key issue of PK tree is that it's poor efficiency of storage. The average node usage (ANU) is the percentage of valid entries to the capacity of entries in the tree. For data points in two dimensional space, each node has the capacity to hold $R(K-1)$ en-

* Corresponding author: LUO Yingwei, lyw@pku.edu.cn.

tries, but the ANU is only 40%. For the same data set, the ANU of a Hilbert R tree may reach to 84% or 87%. When processing data object other than point, for example curves or surfaces, the ANU of a PK tree can reach only 12%, while Hilbert R tree may also reach around 85%. The poor efficiency of storage leads to more disk I/O operations while access the index tree. For data point, the ANU of PK tree is half of the ANU of Hilbert R tree, but PK tree yields more efficient query performance than Hilbert R tree. But for curves or surfaces, the query performance of PK tree is much poorer than Hilbert R tree. The main reason is that the ANU of PK tree is far more less than that of Hilbert R tree. With such poor ANU, PK tree are limited to be used for high dimensional data object. To solve this problem, PK+ tree is introduced in this paper aiming at improving the ANU.

3 PK+ Tree

The ANU of PK tree drops with large $R=(r_x, r_y)$. If R can be limited to the smallest value, then the ANU might be greatly increased. In PK+ tree, R is restricted to be no more than 3. To partition a given space cell C in two-dimensional space, firstly C is split into three cells at x dimension, then at y dimension, and then at x dimension, and so on. The three cells are not symmetrical. Two cells of them are the two half of C , while the third cell is C itself! But the third cell is not the same as C , since it holds all data objects sits in C but not in the other two half cells. The third cell is called middle cell, and the two half-cells are called left cell and right cell (or bottom cell and top cell) correspondingly, as shown in figure 1.

Fig. 1. PK+ Tree Cell Splitting

PK+ tree also requires that any node in the tree is K-intantialable. Insertion or deletion is implemented with two steps. First, search from the root to find the proper node to be modified, and modify it; then, trace back to the root and checking each node on the path if it's K-intantialable, if not, some promotion, division or deletion operations will be done to keep all nodes K-intantialable. The process of K-intantialable checking in PK+ tree is the same as in PK tree, except that they apply different kinds of splitting rules.

PK+ tree split the space cell at each dimension in turn, which makes a lower degree of regularity of partition than PK tree, but the ANU is greatly increased. In PK+ tree, the extent of a middle node overlaps with its sibling left (right) node. The overlapping of nodes might reduce some query’s performance.

4 Performance Comparison

In the section, a series of experiments are done to examine the query performance of PK+ tree. Two dimensional data objects in two-dimensional space are being used in our experiment. A cluster data set from *SEQUOIA 2000 Benchmark* and a random data set are used. In these experiments, we compare PK+ tree with corresponding PK tree and 3-2 Hilbert R tree.

We count disk I/O operations and compare operations instead of measure the time cost in queries as the measurement of performance, thus we avoid the effects of implementation issues to our experiments.

Since in PK tree and PK+ tree, theoretically, the ANU will never be large than 50%. To make the storage efficiently, two kinds of methods might be applied. One method is to store each node in one or more disk pages according to the amount of valid entries in each node (SNMP, Single-node-multi-pages). The other method is to store several nodes in the same disk page if the total valid entries can be hold in one disk page (MNSP, Multi-nodes-single-page).

4.1 Storage Usage

For a given data set, the storage usage is the total valid entries vs. the total entries capacity of the index structure. ANU is used to describe the storage usage.

Table 1 shows the comparison of storage usage of PK tree, PK+ tree and Hilbert R tree. SNMP is compared with MNSP to show the effect of MNSP. Since Hilbert R tree is storage optimized, no more optimized method is considered in the comparison.

Table 1. ANU of PK+ tree, PK tree and Hilbert R tree

	SNMP		MNSP	
	Cluster data	Random data	Cluster data	Random data
PK+ tree	46.37%	50.83%	67.62%	66.78%
PK tree	12.95%	12.69%	25.49%	38.15%
Hilbert R tree	88.17%	85.52%	□	□

Table 1 shows that, MNSP outperforms SNMP, but does not make the ANU of PK+ tree reach the ANU of Hilbert R tree.

4.2 Query Performance

Our experiments show that, for both cluster data set and random data set, PK+ tree greatly reduces disk I/O operations per query than PK tree, but has almost the same

disk I/O operations per query as Hilbert R tree. Since the ANU of PK+ tree is approximately 75% of Hilbert R tree, it is obvious that PK+ tree requires much less compare operations per query than that of Hilbert R tree. In more details, for small query regions, PK+ tree yells less disk I/O operations than Hilbert R tree; for query of inside, PK+ tree yells much less disk I/O operations than Hilbert R tree.

5 Conclusion

In this paper, an improved spatial index structure based on PK tree - PK+ tree is introduced. PK+ tree outperforms PK tree when used for high dimensional object indexing. PK+ tree has a higher ANU and hence requires less disk I/O operations in queries. Though disk I/O operations in queries of PK+ tree is almost the same as that of Hilbert R tree, but Hilbert R tree yells a higher ANU, compare operations in queries is more than that of PK+ tree. PK+ tree is perfect for application that cares both disk I/O performance and CPU performance.

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