An Approach of Nonlinear Model Multi-step-ahead Predictive Control Based on SVM

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Abstract. In this paper, a support vector machine (SVM) with polynomial kernel function based nonlinear model multi-step-ahead controller is presented. A SVM based multi-step-ahead predictive model is established by black-box identification. And control law is obtained by numerical and optimization methods respectively. The effect of controller is demonstrated on a recognized benchmark problem. Simulation results show that multi-step-ahead predictive controller can be well applied to nonlinear system with good performance.

1 Introduction

Model predictive control (MPC) is a class of control algorithms in which a process model is used to predict and optimize process performance^[1]. Now, attention has turned to nonlinear model predictive control (NMPC)^[2] because many industrial processes are intrinsically nonlinear. Recently a new kind of learning machine called SVM^[3,4], which is well used for function regression and time series prediction, can be used for nonlinear system identification and system control. This paper puts forward a new approach to NMPC based on SVM with quadratic polynomial kernel function. The paper is organized as follows: In section 2, a SVM with polynomial kernel function based multi-step-ahead predictive model and control algorithm for nonlinear systems are discussed. Simulation results on a recognized benchmark problem are given in section 3. In section 4, some conclusions are put forward. For more details about SVM regression with polynomial kernel function, see references^[3,4].

2 SVM Based NMPC

2.1 Nonlinear Predictive Control Model

Assume the j -step-ahead ($j=1\cdots P$, P is prediction horizon) nonlinear model can be described by SVM form:

$$\begin{aligned} y_m(k+j) &= f(I_{k+j}) = f[y(k+j-1), y(k+j-2), \dots, y(k-n+j), \\ &\quad u(k+j-1), u(k+j-2), \dots, u(k-n+j)] \\ &= \sum_{i=1}^{nsv} a_i (I_i' \cdot I_{k+j} + 1)^2 + b \\ \text{s.t.} \quad &u_{\min} \leq u \leq u_{\max} \end{aligned} \tag{1}$$

Where n , m are determined by approximation accuracy. Support values $a_i (i=1, \dots, nsv)$ and bias value b can be gotten through learning according to d pairs training data $\{I_s, y\} (s=1, \dots, d)$. I' is the set of support vectors from I_s . And assume when $j > M$, $u(k+j-1) = u(k+M-1)$, M is control horizon.

Introduce feedback correction to deal with the model error and disturbance, so the j -step-ahead closed-loop predictive output at time k is:

$$y_p(k+j) = y_m(k+j) + h_j e(k) \quad (2)$$

Where h_j is the error correcting coefficient, $e(k) = y(k) - y_m(k)$.

2.2 Nonlinear Predictive Controller

Select optimization objective function with moving horizon as below:

$$\min J(k) = \sum_{j=1}^P q_j [y_r(k+j) - y_p(k+j)]^2 \quad (3)$$

Where q_j are weighting coefficients, $y_r(k+j)$ is reference value from trajectory:

$$y_r(k+j) = a_r^j y(k) + (1 - a_r^j) y_{sp} \quad y_r(k) = y(k) \quad (4)$$

Where a_r is the coefficient related with system's robustness and y_{sp} is set point value.

Case of $P = M$:

In this case, nonlinear optimization problem (3) contains M manipulate variables and M equality constraint of model outputs. Use known $y_p(k-1+j/k-1)$ replace unknown $y(k-1+j)$, where $y_p(k-1+j/k-1)$ is the closed-loop predictive output.

$$\begin{aligned} \min J(k) = & \underbrace{q_1 [y_r(k+1) - y_p(k+1)]^2}_{J_1(k) \text{ contains } u(k)} + \underbrace{q_2 [y_r(k+2) - y_p(k+2)]^2}_{J_2(k) \text{ contains } u(k), u(k+1)} \\ & + \dots + \underbrace{q_M [y_r(k+M) - y_p(k+M)]^2}_{J_M(k) \text{ contains } u(k), u(k+1), \dots, u(k+M)} \end{aligned} \quad (5)$$

It's a common sense that the first-step controller output $u(k)$ outweighs the others, so we can first minimize $J_1(k)$ to get $u(k)$, and then substitute $u(k)$ into $J_2(k)$ to get $u(k+1)$. Recursively, we can get all $u(k+j-1)$.

$$\frac{\partial J_j(k)}{\partial u(k+j-1)} = 2q_j [y_p(k+j) - y_r(k+j)] \frac{\partial y_p(k+j)}{\partial u(k+j-1)} = 0 \quad (6)$$

It's a cubic equation with only one unknown bounded variable, and it's easily solved.

Case of $P > M$:

Consider nonlinear optimization problem (3) with $P > M$, this is a nonlinear optimal problem with equality constraint of model output and boundary constraint of controller output. In this paper, Nelder-Mead simplex direct search method is used to optimize and obtain M unknown variables. And we will use solutions obtained by algorithm in section 2.2.1 as starting point.

3 Experimental and Simulation Results

The example is a benchmark problem used by Narendra & Parthasarathy^[5], and further studied by Kambhampati^[2] using a Gaussian neural network to fulfill the one-step-ahead predictive control. Set $C = 10000$, $\varepsilon = 0.001$, $h_j = 1$, $q_j = 1$.

$$y(k+1) = \frac{y(k)}{1 + y^2(k)} + u^3(k) \quad (7)$$

where y is the plant output and $u \in [-2,2]$ is the input.

150 pairs data generated by applying by a series of random numbers between $[-2,2]$ are used to train the SVM predictive model with $m = n = 5$. And set $y_{sp} = 1.5$ and $a_r = 0.9$. Fig.2 is the system output and controller output with $P = 2, M = 2$. Fig.3 is the system output and controller output with $P = 4, M = 2$. In both cases, a disturbance rejection ($d=0.1$) is added when system is in steady state at time $k=100$, and the output response will track the set point well again quickly.

4 Conclusions

In this paper two cases of SVM with polynomial kernel function based nonlinear model multi-step-ahead predictive control strategies are investigated. The effect of both controllers is demonstrated by a benchmark problem. The analysis and experiment results show that the control strategy will provide a robust stable control of nonlinear systems with good performance in keeping reference trajectory and disturbance-rejection.

Fig. 2. System output and controller output with $P = 2, M = 2$

Fig. 3. System output and controller output with $P = 4, M = 2$

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