DYNAMICAL RECONSTRUCTION AND FEEDBACK ROBUST CONTROL OF PARABOLIC INCLUSIONS

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Abstract Two types of problems for parabolic inclusions, namely, problems of robust control under the action of uncontrolled disturbances and problems of dynamical identification of inputs, are discussed. Algorithms for solving such problems stable with respect to informational noises and computational errors are presented. The algorithms oriented to computer realization allow one to simulate a solving process in the “real time” mode. They adaptively take into account inaccurate measurements of phase trajectories and are regularizing in the following sense: the more precise is incoming information, the better is algorithm’s output. The algorithms are based on the method of auxiliary positionally-controlled models [1, 2, 4–7]. The basic elements of the algorithms are represented by stabilization procedures (functioning by the feedback principle) for appropriate Lyapunov functionals.

keywords: parabolic inclusions, feedback control, reconstruction

1. Introduction

Let a dynamical system be described by the parabolic inclusion

$$\dot{x}(t) + \partial\varphi(x(t)) \ni Bu(t) - Cv(t) + f(t), \quad t \in T = [t_0, \vartheta]. \quad (1)$$

Here $H = H^*$ is a real Hilbert space with the norm $|\cdot|_H$ and scalar product $(\cdot; \cdot)_H$, $f(\cdot) \in L_2(T; H)$ is a given function, $\varphi : H \rightarrow \overline{R} = \{r \in R : -\infty < r \leq +\infty\}$ is a lower semicontinuous convex function, $\partial\varphi$ is the subdifferential of φ . Let $x(t_0) = x_0 \in D(\varphi) = \{x \in H : \varphi(x) < +\infty\}$ be an initial state. Let $(U, |\cdot|_U)$ and $(V, |\cdot|_V)$ be uniformly convex Banach spaces; $B \in \mathcal{L}(U; H)$, $C \in \mathcal{L}(V; H)$ be linear continuous operators. It is known that there exists (for any $\{u(\cdot), v(\cdot)\} \in L_2(T; U) \times L_2(T; V)$) a unique solution

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$x(\cdot) = x(\cdot; t_0, x_0, u(\cdot), v(\cdot))$ of inclusion (1) with the following properties [3, 8]:

$$x(\cdot) \in W(T), \quad x(t) \in D(\varphi) \quad \forall t \in T, \quad t \rightarrow \varphi(x(t)) \in AC(T).$$

Here $AC(T)$ is the set of absolutely continuous functions $z(\cdot) : T \rightarrow R$, $W(T) = \{z(\cdot) \in L_2(T; H) : z_t(\cdot) \in L_2(T; H)\}$.

The paper is devoted to two problems: the problem of robust control and the problem of dynamical reconstruction of an input. Now we formulate the problems and we outline the ideas used in the solution. Let a uniform net $\Delta = \{\tau_i\}_{i=0}^m$, $\tau_i = \tau_{i-1} + \delta$, $\tau_0 = t_0$, $\tau_m = \vartheta$ with diameter $\delta = \delta(\Delta) = \tau_i - \tau_{i-1}$ be fixed on a given time interval T . Let a solution of inclusion (1) $x(\cdot) = x(\cdot; t_0, x_0, u(\cdot), v(\cdot))$ depend on a time-varying control $u(\cdot) \in L_2(T; U)$ and an unknown disturbance $v(\cdot) \in L_2(T; V)$. The function $x(\cdot)$ is unknown. At moments $\tau_i \in \Delta$ the phase state $x(\tau_i)$ is inaccurately measured. Results of measurements $\xi_i^h \in H$, $i \in [0 : m - 1]$ satisfy the inequalities

$$|\xi_i^h - x(\tau_i)|_H \leq h. \quad (2)$$

Here, $h \in (0, 1)$ is a level of informational noise.

The problem of robust control consists in the following. A nonempty set $N \subset H$ and a number $\varepsilon > 0$ are given. It is required to construct an algorithm of feedback control $u = u(t) \in P$, $t \in T$, of inclusion (1) providing fulfillment of the following condition. Whatever an unknown disturbance $v(\cdot)$, $v = v(t) \in Q$, $t \in T$, may be, the distance between the phase state $x(t) = x(t; t_0, x_0, u(\cdot), v(\cdot))$ at the moment $t = \vartheta$ and the set N should not exceed the value of ε . Here P and Q are given bounded and closed sets from spaces of controls U and V , respectively.

The problem of dynamical reconstruction of an input is as follows. Let in inclusion (1) control $u = u(t) = 0$, $t \in T$. It is required to design a dynamical algorithm of reconstruction of an unknown input $v = v(\cdot)$ in the "real time" mode.

The scheme of algorithm for solving the problem of robust control is given in the figure below [1, 4, 5].

In the beginning, an auxiliary system M (called a model) is introduced. The model has an input $\pi(\cdot)$ and an output $w(\cdot)$. The process of synchronous feedback control of inclusion (1) and M is organized on the interval T . This process is decomposed into $(m - 1)$ identical steps. At the i -th step carried out during the time interval $\delta_i = [\tau_i, \tau_{i+1})$, the following actions are fulfilled. First, at the time moment τ_i , according to the chosen rule $\mathcal{U}$, the element $u_i = \mathcal{U}(\tau_i, \xi_i^h, w_i)$ is calculated. Then (till the moment τ_{i+1}) the control $u(t) = u_i$, $\tau_i \leq t < \tau_{i+1}$, is fed onto the input of inclusion (1). The values ξ_{i+1}^h and $w_{i+1} = w(\tau_{i+1})$ are treated as algorithm's output at the i -th step.

An analogous scheme is applicable to solving reconstruction problem. In this case, an auxiliary system M (a model) is also introduced. The problem of reconstruction is substituted by the problem of designing an algorithm of feedback control of the model. This algorithm is identified with a function $\mathcal{U}$, which is chosen in such a way that the control $u = u^h(\cdot)$ approximates the unknown disturbance $v(\cdot)$: $u^h(t) = u_i = \mathcal{U}(\tau_i, \xi_i^h, w(t))$, $t \in \delta_i$.

2. Statement of the problems

Before we pass to the mathematical formulation of the problems, let us give some definitions. Denote by $u_{a,b}(\cdot)$ a function $u(t)$, $t \in [a, b]$. The symbol $P_{a,b}(\cdot)$ stands for the restriction of a set $P_T(\cdot)$ onto the segment $[a, b] \subset T$. Elements of the product $T \times H \times H$ are called positions and denote by p . Any possible function (multifunction) $\mathcal{U} : T \times H \times H \rightarrow P$ is said to be a feedback strategy. Feedback strategies correct controls at corresponding time moments. Any strongly measurable (by Lebesgue) functions $u(\cdot) : T \rightarrow P$ and $v(\cdot) : T \rightarrow Q$ are called an open-loop control and a disturbance, and are denoted by $u_T(\cdot)$ and $v_T(\cdot)$, respectively. Let a model M be described by the inclusion

$$\dot{w}(t) + \partial\varphi(w(t)) \ni \pi(t) + f(t), \quad t \in T, \quad w(t_0) = x_0, \quad (3)$$

where $\pi(\cdot) \in L_2(T; H)$ is an open-loop control. Hereinafter, denote a solution of inclusion (3) by the symbol $w(\cdot) = w(\cdot; t_0, x_0, \pi(\cdot))$. A solution $x(\cdot)$ of inclusion (1) starting from an initial state (t_*, x_*) and corresponding to a piecewise constant control $u(\cdot)$,

$$u(t) = u_i \in \mathcal{U}(p_i) \in U, \quad t \in [\tau_i, \tau_{i+1}), \quad i \in [0 : m - 1],$$

$$p_i = (\tau_i, \xi_i^h, w_i), \quad w_i = w(\tau_i; t_0, x_0, \pi(\cdot)), \quad |\xi_i^h - x(\tau_i)|_H \leq h,$$

and to a disturbance $v(\cdot) : [t_*, \vartheta] \rightarrow Q$ is called an (h, Δ, w) -motion $x_{\Delta, w}^h(\cdot; t_*, x_*, \mathcal{U}, v_{t_*, \vartheta}(\cdot))$ generated by a positional strategy $\mathcal{U}$ on a partition Δ . The set of all (h, Δ, w) -motions is denoted by $X_{[t_*, \vartheta]}(t_*, x_*, \mathcal{U}, h, \Delta, w)$.

The problem of robust control (**Problem 1**) consists in constructing an open-loop control $\pi(\cdot) \in P_T(\cdot)$ in the model and a feedback strategy $\mathcal{U} : T \times H \times$

$H \rightarrow P$ with the following properties: whatever a value $\varepsilon > 0$ may be, one can indicate (explicitly) numbers $h_* > 0$ and $\delta_* > 0$ such that the inequalities

$$\text{dist}(x_{\Delta,w}^h(\vartheta), N) \leq \varepsilon \quad \forall x_{\Delta,w}^h(\cdot) \in X_T(t_0, x_0, \mathcal{U}, h, \Delta, w) \quad (4)$$

are fulfilled uniformly with respect to all measurements ξ_i^h with properties (2) if $h \leq h_*$ and $\delta = \delta(\Delta) \leq \delta_*$.

Here the symbol $\text{dist}(x, N)$ denotes the distance from x to N , i.e., $\text{dist}(x, N) = \inf_{y \in N} \{|x - y|_H\}$.

Let us turn to the problem of reconstruction. In this case, a disturbance $v(\cdot)$ to be reconstructed is an element of the space $L_2(T; V)$. The solution of (1) generated by a disturbance $v(\cdot) \in L_2(T; V)$ is denoted by the symbol $x(\cdot) = x(\cdot; t_0, x_0, v(\cdot))$ (we have $u(\cdot) = 0$ in (1)). A model M is described by the relation

$$\dot{w}(t) + \partial\varphi(w(t)) \ni -Cu^h(t) + f(t), \quad t \in T, \quad w(t_0) = x_0. \quad (5)$$

Let us a family of partitions

$$\Delta_h = \{\tau_{i,h}\}_{i,h=0}^{m_h}, \quad \tau_{i,h} = \tau_{i-1,h} + \delta, \quad \tau_{0,h} = t_0, \quad \tau_{m_h,h} = \vartheta \quad (6)$$

with fixed diameters $\delta = \delta(h)$ be fixed. The control $u^h(\cdot)$ is defined as follows:

$$u^h(t) = \mathcal{U}(p_i) \in V, \quad t \in \delta_{i,h} = [\tau_{i,h}, \tau_{i+1,h}), \quad (7)$$

where the position $p_i = (\tau_{i,h}, \xi_i^h, w(t))$ for $t \in \delta_{i,h}$. Let $v_*(\cdot; x(\cdot))$ be a minimal $L_2(T; V)$ -norm element of the set $V_*(x(\cdot))$ of all functions $v(\cdot) \in L_2(T; V)$ generating the solution $x(\cdot)$: $V_*(x(\cdot)) = \{\tilde{v}(\cdot) \in L_2(T; V) : x(\cdot) = x(\cdot; t_0, x_0, \tilde{v}(\cdot))\}$.

The problem of dynamical reconstruction (**Problem 2**) consists in constructing a feedback strategy $\mathcal{U} : T \times H \times H \rightarrow V$ such that the control $u = u^h(\cdot)$ defined by (7) possesses the property

$$u^h(\cdot) \rightarrow v_*(\cdot; x(\cdot)) \text{ in } L_2(T; V) \quad \text{as } h \rightarrow 0.$$

3. Algorithm for solving Problem 1

In this section, an algorithm for solving Problem 1 is designed. This algorithm is based on constructions of the theory of guaranteed control [1, 4, 5]. Let the following condition be fulfilled.

Condition 1 ("Entire domination") *There exists a closed set $D \subset H$ such that $BP = CQ + D$.*

Here we use the following notation: $BP = \{Bu : u \in P\}$, $CQ = \{Cv : v \in Q\}$, $CQ + D = \{u : u = u_1 + u_2, u_1 \in CQ, u_2 \in D\}$.

Let us describe the procedure used in the construction of the (h, Δ, w) -motion $x_{\Delta, w}^h(t; t_0, x_0, \mathcal{U}, v_{t_0, t}(\cdot))$ corresponding to a fixed partition Δ and a strategy $\mathcal{U}$ of the form:

$$\mathcal{U}(t, x, w) = \arg \min \{(x - w(t), Bu)_H : u \in P\}. \quad (8)$$

We assume $u_0^h \in P$ in the interval $[t_0, \tau_1]$. Under the action of this control as well as of an unknown open-loop disturbance $v_{t_0, \tau_1}(\cdot)$, some (h, Δ, w) -motion $\{x_{\Delta, w}^h(\cdot; t_0, x_0, u_0^h, v_{t_0, \tau_1}(\cdot))\}_{t_0, \tau_1}$ is realized. At the moment $t = \tau_1$ we find u_1 from the condition

$$u_1^h \in \mathcal{U}(p_1), \quad p_1 = (\tau_1, \xi_1^h, w(\tau_1)), \quad |\xi_1^h - x_{\Delta, w}^h(\tau_1)|_H \leq h.$$

Then we calculate the realization of the (h, Δ, w) -motion $\{x_{\Delta, w}^h(\cdot; \tau_1, x_{\Delta, w}^h(\tau_1), u_1^h, v_{\tau_1, \tau_2}(\cdot))\}_{\tau_1, \tau_2}$. Let the (h, Δ, w) -motion $x_{\Delta, w}^h(\cdot)$ be defined in the interval $[t_0, \tau_i]$, $(\tau_i = \tau_{i, h})$. At the moment $t = \tau_i$ we assume

$$u_i^h \in \mathcal{U}(p_i), \quad p_i = (\tau_i, \xi_i^h, w(\tau_i)), \quad |\xi_i^h - x_{\Delta, w}^h(\tau_i)|_H \leq h.$$

As the result of the action of this control and of an unknown open-loop disturbance $v_{\tau_i, \tau_{i+1}}(\cdot)$ the (h, Δ, w) -motion of system (1) $\{x_{\Delta, w}^h(\cdot; \tau_i, x_{\Delta, w}^h(\tau_i), u_i^h, v_{\tau_i, \tau_{i+1}}(\cdot))\}_{\tau_i, \tau_{i+1}}$ is realized in the interval $[\tau_i, \tau_{i+1}]$. The procedure of forming the (h, Δ, w) -motion stops at the moment ϑ .

Let $W(t; t_0, x_0)$ be the set of attainability for inclusion (3), i.e.,

$$W(t; t_0, x_0) = \{w(t) : w(t) = w(t; t_0, x_0, \pi_{t_0, t}(\cdot)), \pi_{t_0, t}(\cdot) \in D_{t_0, t}(\cdot)\},$$

$$D_{t_0, t}(\cdot) = \{u(\cdot) \in L_2([t_0, t]; H) : u(\tau) \in D \text{ for a. a. } \tau \in [t_0, t]\}.$$

THEOREM 1 *If $W_D(\vartheta; t_0, x_0) \cap N \neq \emptyset$ then Problem 1 has a solution and the positional strategy $\mathcal{U} : T \times H \times H \rightarrow P$ defined according to (8) solves this Problem. Otherwise, Problem 1 does not have a solution.*

Proof (outline). The proof of the first part of this theorem is performed according to the known scheme (see, for example, [4]). The basic elements of the algorithms are represented by stabilization procedures (functioning by the feedback principle) for appropriate Lyapunov functionals. Let $\pi^*(\cdot) \in D_T(\cdot)$ be an open-loop control with the following property: $w(\vartheta; t_0, x_0, \pi^*(\cdot)) \in N$. Here $w(\cdot) = w(\cdot; t_0, x_0, \pi^*(\cdot))$ denotes the solution of (3) for $\pi(\cdot) \equiv \pi^*(\cdot)$. The cornerstone is the proof of "smallness" of variation of the Lyapunov functional $\varepsilon(t) = |x_{\Delta, w}^h(t) - w(t)|_H^2$ in the interval T . Namely, the strategy $\mathcal{U}$ (8) provides fulfillment of the inequalities

$$\varepsilon(t) \leq \varepsilon(\tau_i) + C(t - \tau_i)(h + (t - \tau_i)^{1/2})$$

for $t \in [\tau_i, \tau_{i+1}]$, $i \in [0 : m - 1]$. The inequalities, in turn, imply the estimate $\varepsilon(t) \leq C(h + \delta)$, where a constant C does not depend on $x_{\Delta, w}^h(\cdot)$ and $w(\cdot)$. Hence, in virtue of condition 1, it follows that relation (4) is true.

Let us prove the second part of the theorem. For this purpose, it is sufficient to indicate a rule $\mathcal{V} : T \times H \times H \rightarrow Q$ forming a disturbance $v(\cdot)$ such that, for any feedback control $u(\cdot)$, the solution of inclusion (1) avoids the set N at the moment ϑ . Along with inclusion (1), we consider the inclusion

$$\dot{y}(t) + \partial\varphi(y(t)) \ni Bu^e(t) - Cv^e(t) + f(t). \quad (9)$$

We construct the procedure of synchronous feedback control of (1) and (9). Let us fix a partition Δ of the interval T . In the interval $[t_0, \tau_1]$, we assume $u^e(t) = u^{(0)} \in P$, $v^e(t) = v^{(0)} \in Q$, $v(t) = v_0 \in Q$, where $u^{(0)}$, v_0 are arbitrary elements from the sets P and Q , $v^{(0)}$ is an element such that $Bu^{(0)} - Cv^{(0)} \in D$. Under the action of these constant controls and of an unknown control $u_{t_0, \tau_1}(\cdot)$, some solution $\{x(\cdot; t_0, x_0, u_{t_0, \tau_1}(\cdot), v_0)\}_{t_0, \tau_1}$ of inclusion (1) and some solution $y_{\Delta}^h(t) = y(t; t_0, x_0, u^{(0)}, v^{(0)})$, $t_0 \leq t \leq \tau_1$, of inclusion (9) are realized. By analogy with (h, Δ, w) -motion $x_{\Delta, w}^h$, the former is denoted by $x_{\Delta, y}^h$ and is called an (h, Δ, y) -motion: $x_{\Delta, y}^h(\cdot) = x(\cdot; t_0, x_0, u_{t_0, y}(\cdot), \mathcal{V})$ and the latter is called an (h, Δ) -motion of the model. The set of all $x_{\Delta, y}^h(\cdot)$ is denoted by $X_T(t_0, x_0, \mathcal{V}, \Delta, y)$. Let the (h, Δ) -motion $y_{\Delta}^h(\cdot)$ and (h, Δ, y) -motion $x_{\Delta, y}^h(\cdot)$ be defined in the interval $[t_0, \tau_i]$. To form $\{y_{\Delta}^h(\cdot)\}_{\tau_i, \tau_{i+1}}$, $\{x_{\Delta, y}^h(\cdot)\}_{\tau_i, \tau_{i+1}}$, we proceed in the following way. At the moment $t = \tau_i$ we find $u^{(i)}$, $v^{(i)}$, and v_i from the rule

$$u^{(i)} = \arg \min \{(y_{\Delta}^h(\tau_i) - \xi_i^h, Bu)_H : u \in P\}, \quad |\xi_i^h - x_{\Delta, y}^h(\tau_i)|_H \leq h,$$

$v^{(i)}$ is an arbitrary element from the set Q such that $Bu^{(i)} - Cv^{(i)} \in D$,

$$v_i \in \mathcal{V}(t, \xi_i^h, y_{\Delta}^h(\tau_i)),$$

$$\mathcal{V}(\tau_i, x, y_{\Delta}^h(\tau_i)) = \arg \min \{(y_{\Delta}^h(\tau_i) - x, Cv)_H : v \in Q\}.$$

After that, we assume that $u^e(t) = u^{(i)}$, $v^e(t) = v^{(i)}$ for $t \in [\tau_i, \tau_{i+1}]$ in (9), then we calculate the (h, Δ) -motion in the interval $[\tau_i, \tau_{i+1}]$: $y_{\Delta}^h(t) = y(t; \tau_i, y_{\Delta}^h(\tau_i), u^{(i)}, v^{(i)})$, $\tau_i \leq t \leq \tau_{i+1}$. Simultaneously, the control $v(t) = v_i$ for $t \in [\tau_i, \tau_{i+1}]$ is fed onto the input of (1). As a result of the action of this control and of an unknown control $u_{\tau_i, \tau_{i+1}}(\cdot)$, some (h, Δ, y) -motion $\{x_{\Delta, y}^h(\cdot; \tau_i, x_{\Delta, y}^h(\tau_i), u_{\tau_i, \tau_{i+1}}(\cdot), v_i)\}_{\tau_i, \tau_{i+1}}$ is realized in the interval $[\tau_i, \tau_{i+1}]$. This procedure stops at the moment ϑ . Analogously to [4], we establish that for every $\varepsilon \in (0, \varepsilon_*)$ one can indicate (explicitly) numbers $h_1 = h_1(\varepsilon) > 0$ and $\delta_1 = \delta_1(\varepsilon) > 0$ such that the inequality

$$\text{dist}(x_{\Delta, y}^h(\vartheta), N) \geq \varepsilon \quad \forall x_{\Delta, y}^h(\cdot) \in X_T(t_0, x_0, \mathcal{V}, \Delta, y)$$

is fulfilled if $h \leq h_1$, $\delta = \delta(\Delta) \leq \delta_1$. Toward this aim, the Lyapunov functional $\varepsilon^{(1)}(t) = |x_{\Delta,y}^h(t) - y_{\Delta}^h(t)|_H^2$ is estimated in the interval T , and its "small" variation is established. The theorem is proved.

4. Algorithms for solving Problem 2

Let us describe algorithms for solving Problem 2. Recall that we consider inclusion (1) with unknown $v(\cdot)$ and $u(\cdot) = 0$. In this section, we assume that U is a Hilbert space with the scalar product $(\cdot, \cdot)_U$. Constructions described below are based on the approach developed in [2, 6, 7].

Let a partition Δ_h of the form (6) and a function $\alpha(h) : (0, 1) \rightarrow R^+$ be fixed. Let the following condition be fulfilled:

$$\alpha(h) \rightarrow 0, \quad \delta(h) \rightarrow 0, \quad \delta(h)\alpha^{-2}(h) \rightarrow 0, \quad (10)$$

$$h^2\delta^{-1}(h)\alpha^{-1}(h) \rightarrow 0 \text{ as } h \rightarrow 0.$$

A positional strategy $\mathcal{U} : T \times H \times H \rightarrow V$ is defined by the rule

$$\mathcal{U}(p_i) = \alpha^{-1}(h)C^*(\xi_i^h - w(t)), \quad (11)$$

where $p_i = (\tau_i, \xi_i^h, w(t))$ is the position for $t \in \delta_i = [\tau_i, \tau_{i+1})$, $\tau_i = \tau_{i,h}$, $w(\cdot)$ is the solution of inclusion (5) with $u(\cdot)$ defined by (7), (11).

THEOREM 2 *Let condition (10) be fulfilled. Then the positional strategy $\mathcal{U}$ of the form (7), (11) solves Problem 2.*

The proof of this theorem is performed according to the scheme from [2].

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