FORECASTING RUNOFF WITH HIGHER-EMBEDDED DIMENSIONS USING DMEP-BASED ARTIFICIAL NEURAL NETWORKS

Chuanwen Jiang *, Shuai Li *, Songhuai Du **

** Department of Electrical Engineering, Shanghai Jiaotong University
Huashan Road 1954, PR China, 200030 (email: jiangcw@sjtu.edu.cn)*

*** Department of Electrical Engineering, China Agricultural University
Beijing Tsinghua East Road 17, PR China, 100083*

Abstract: The runoff series always presents complex chaos phenomenon with higher embedded dimensions because of the influence of many complicated facts. Generally, it is an effective method to combine phase space restructures theory with artificial neural networks (ANN) model for runoff forecasting. The traditional methods that not consider the variations of the character of full Lyapunov exponential spectrum in restructure space, are proved of high precision to forecast time series with low-embedded dimension. But, they are not so effective to forecast attractors with high-embedded dimension. The paper proposes a new method, which improves the character of full Lyapunov exponential spectrum in a restructure space with high dimensions. In the mean time, an artificial neural networks model based on determinate mutation evolutionary programming (DMEP) learning algorithm is presented for chaotic runoff series forecasting. DMEP introduces chaos mapping into the mutation operation of EP, which aim to increase its convergence rate and results' precision. The test result of runoff forecasting series shows that the precision of runoff forecasting is improved by means of the new method when the embedded dimension is high.

Key words: Chaos, runoff forecasting, ANN, evolutionary programming

1. INTRODUCTION

Forecasting runoff process is always a difficult task in simulation of the whole hydrological cycle, because the runoff process refers to hydrology,

aerography, mechanics and so on. Nowadays, more and more scholars think that considering the nonlinear factors in the runoff forecasting modeling is the key to improving the runoff forecasting level. Recently, the theory of nonlinear chaos dynamics which links the determination and randomness has become the foreland of hydrology study, many scholars do lots of useful explore for hydrology runoff modeling and forecasting^{[1]-[4]}. Specially, combining phase space restructures theory with neural networks is considered as one of the most effective methods^{[5][6]}. However, the embedded dimension will be relatively high, because runoff is influenced by various intricate facts. The traditional chaos methods are proved of high precision to forecast time series with low-embedded dimension. But, they are not so effective to forecast attractors with high-embedded dimension^{[7][8][9]}. On the other hand, it is attractive to find a fast network convergent arithmetic for attractors with high-embedded dimension. The paper proposes a new method, which improves the character of full Lyapunov exponential spectrum in a restructure space with high dimensions. In the mean time, an artificial neural networks model based on determinate mutation evolutionary programming (DMEP) learning algorithm be presented for chaotic runoff series forecasting. DMEP introduces chaos mapping into the mutation operation of traditional evolutionary programming, which aim to increase its convergence rate and results' precision. The test result of runoff forecasting series shows that the precision of runoff forecasting is improved by means of the new method when the embedded dimension is high.

The paper was organized as follows: section 2 describes a artificial neural networks forecasting model based on phase space restructures theory; an improved ANN model with determinate mutation evolutionary programming learning algorithm is shown in section 3; followed by numerical examples in section 4, and conclusion in section 5.

2. ARTIFICIAL NEURAL NETWORKS FORECASTING MODEL BASED ON PHASE SPACE RESTRUCTURES THEORY

The basic theory of runoff forecasting making use of nonlinear chaos dynamics is phase space restructures theory proposed by Takens in 1981.

Consider a chaos system:

$$Y_{t+1} = f(Y_t) \quad Y_t \in R^m \quad (1)$$

where: $f : R^m \rightarrow R^m$ is a continuous smooth function.

If the system can be observed through an observer:
 $g: R^m \rightarrow R^L$ ($L \leq m$), then:

$$x_t = g(Y_t) \quad x_t \in R^L \quad (2)$$

To simplify the presentation, let $L=1$, then:

$$X_t = (x_t, x_{t-1}, \dots, x_{t-d+1}) \quad (3)$$

where X_t is one state in the d -order state space. According to the famous Takens Theory^[10], when $d \geq 2m + 1$ (m is the order of the attractor), there exists a deterministic mapping with dimension being d : $F^{(d)}: R^d \rightarrow R^d$. It describes the evolving track of X_t in the state space and the mapping has the same geometric structure and topology with the original system. So:

$$X_{t+1} = F^{(d)}(X_t) \quad (4)$$

(4) can also be presented as:

$$x_{t+1} = \tilde{F}^{(d)}(x_t, x_{t-1}, \dots, x_{t-d+1}) \quad (5)$$

The state space described by $\tilde{F}^{(d)}$ is called as the restructured state space. d is called as the embedded dimension, which is the dimension of the minimal state space that can completely contain the attractor sets comprised by state transition. Because $\tilde{F}^{(d)}$ is deterministic, we are only needed to made an estimate on $\tilde{F}^{(d)}$ in order to forecast x_{t+1} . Currently, fitting methods to $\tilde{F}^{(d)}$ are local linear model, artificial neural networks model and so on. The forecasting model combining phase space restructures theory and artificial neural networks as follows:

Figure 1. ANN Forecasting Model

By selecting a suitable set of weights and transfer functions, it is known that the ANN can approximate any smooth, measurable function between the input and output vectors. In this paper, a output mean squared error (MSE) of ANN is considered and defined as:

$$MSE = \frac{1}{N} \sum_{s=1}^N \left[\sum_{k=1}^M (o_{sk} - l_{sk})^2 \right] \quad (6)$$

where o_{sk} is the expected output, l_{sk} is the predicted output, M is the number of output neurons, and N is the number of training set samples.

The above-mentioned method is very easy and feasible, but it still has two disadvantages: firstly, the current forecast methods have good forecast precision when the embedded dimension is low. However, when the dimension is high, the result is not satisfied. Secondly, the networks will converge very slowly when using common training arithmetic, especially, when the dimension d becomes enough large, the phenomenon is obvious. The following of this paper will mainly discuss the two problems.

3. AN IMPROVED ANN MODEL FOR RUNOFF FORECASTING

3.1 Improvement Of The Global Lyapunov Exponent Spectrum Of High Embedded Dimension Reconstructed State Space

Generally, the more information used for forecasting, the higher the forecasting precision. When using the current chaos forecasting methods to forecast, it is found that the forecasting precision will decrease with the increase of the embedded dimension after it reached some dimension. The reason for such phenomena lies in the change of the global Lyapunov exponent spectrum of the reconstructed state space^[8]. According to the definition of Lyapunov exponent spectrum, it is known that the global Lyapunov exponent spectrum of the reconstructed state space will change with the change of embedded dimension. With the increase of the embedded dimension, the elements with small subscripts in the reconstructed state space have positive larger Lyapunov exponents which make the global Lyapunov exponent spectrum of the reconstructed state space increase. One

way to decrease the global Lyapunov spectrum of the reconstructed state space is to let the nearby two tracks and time into the exponential separation relationship by taking the following form:

$$(e^{-\lambda d\tau} x_{t-d+1}, e^{-\lambda(d-1)\tau} x_{t-d+2}, \dots, e^{-\lambda\tau} x_1) \quad (7)$$

When the embedded dimension d is large enough,

$$\sum_{i=1}^d e^{-\lambda\tau i} \approx \frac{e^{-\lambda\tau}}{1 - e^{-\lambda\tau}} = 1 \quad (8)$$

$$\text{So: } \lambda\tau = \ln 2 \rightarrow e^{-\lambda\tau} = \frac{1}{2}.$$

This means that we can introduce the effect of $(\frac{1}{2})^{d(i)}$ in the reconstructed state space. The reconstructed state space is transformed to:

$$X_i = ((\frac{1}{2})x_i, (\frac{1}{2})^2 x_{i-1}, \dots, (\frac{1}{2})^d x_{i-d+1}) \quad (9)$$

This method can improve the forecasting error problem caused by the increase of embedded dimension.

3.2 A Deterministic Mutation Evolutionary Programming (DMEP) Algorithm For ANN

Evolutionary programming methods whose search direction was based on survival of the fittest were becoming a widely used method for complex optimal problems. EP seeks the solution of an optimization problem by evolving a population of candidate solutions over a number of generations or iterations. A new population was generated from an existing population through the use of a mutation operator. Assuming the EP population size is m , x_i is the i_{th} individual before mutation, x'_i is the i_{th} individual after mutation, they are related to a vector with n components, $x_i(j)$ is the j_{th} component of x_i , $x'_i(j)$ is the j_{th} component of x'_i , $\sigma_i(j)$ is the approximate mutation scale of the j_{th} component. CEP with Gaussian mutation can be expressed as:

$$\begin{aligned}
 x'_i(j) &= x_i(j) + \sigma_i(j)N_j(0,1) \\
 j &= 1, 2, \dots, n \quad i = 1, 2, \dots, m
 \end{aligned} \tag{10}$$

Where, $N(0,1)$ indicates a random Normal Distribution value with a mean value 0, variances 1.

FEP with Cauchy mutation can be expressed as:

$$\begin{aligned}
 x'_i(j) &= x_i(j) + \sigma_i(j)C_j(0,1) \\
 j &= 1, 2, \dots, n \quad i = 1, 2, \dots, m
 \end{aligned} \tag{11}$$

Where, $C(0,1)$ indicates a random Cauchy-distribution value with a center 0, scale 1.

MFEP with mean of Gaussian and Cauchy mutations can be expressed as:

$$\begin{aligned}
 x'_i(j) &= x_i(j) + \frac{1}{2}\sigma_i(j)[C_j(0,1) + N_j(0,1)] \\
 j &= 1, 2, \dots, n \quad i = 1, 2, \dots, m
 \end{aligned} \tag{12}$$

The above three algorithms adopt a fixed-distribution random for mutation. These mutations cannot guarantee that the new individuals are all in the solution space. To avoid the illegal solutions, this paper implements a new chaos mutation operator considering the inner random and ergodic properties of a chaos dynamics system. It is well known that a chaotic motion can traverse every state in a certain region by its own regularity, and every state is visited only once therefore no precise periodicity. The unique ergodicity and the irregularity of the series generated by chaos make the chaotic dynamics a potential candidate in the field of global optimization^{[11]-[14]}. The new mutation formula is:

$$\begin{aligned}
 x'_i(j) &= x_i(j) + \frac{1}{2}\sigma_i(j)[R^{(1)}(j) + R^{(2)}(j)] \\
 j &= 1, 2, \dots, n \quad i = 1, 2, \dots, m
 \end{aligned} \tag{13}$$

$$\begin{aligned}
 R^{(1)}(j+1) &= 4.0y(j) \quad (1-y(j)) \quad y(j) \in (0,1) \\
 R^{(2)}(j+1) &= \sin(2/y'(j)) \quad y'(j) \in (0,1]
 \end{aligned} \tag{14}$$

Where, $R^{(1)}$ and $R^{(2)}$ are two famous chaos expressions.

It is assumed that the three-layered perceptrons are chosen for all application cases in this study. W_1 is the connection weight matrix between the input layer and the hidden layer, W_2 is the connection weight matrix between the hidden layer and the output layer.

The performance of each individual is measured according to a fitness function. The fitness function can be calculated by

$$f(W_1, W_2) = \text{MSE}(W_1, W_2) \quad (15)$$

The DMEP procedure for (15) can be described as follows:

Step 1 Initiation: Let $X_i = \{W_1^i, W_2^i\}$ be an individual set, create an initial group with m individuals. All individuals are set between the lower and upper bonds and then calculate fitness J .

Step 2 Mutation: every father individual X_i generates a child individual X'_i by using (13).

Step 3 Calculate fitness: all father and child individuals form a group with $2m$ members. For each member A , select q members from the group with a certain probability and compare them with the individual A ; if A 's fitness is smaller than that of the counterpart individual (for minimal questions), A wins. Here q is a prescribed integer which is called competition scale.

Step 4 Selection: select m individuals with the most winnings from the group to be the next father generation.

Step 5 Stop criteria: if the stop criteria be satisfied, the steps are stopped, else, go to step 2.

4. NUMERICAL EXAMPLES

To forecast runoff using chaos theory, it is necessary to examine the chaos characteristics of runoff. Chaos has three apparent characteristics: (1) sensitive dependence on initial conditions; (2) having non-cyclic characteristics; (3) strange attractors. In order to identify the numerical characters of chaos, many methods are proposed based on the above three characteristics. For example, the reconstructed state space theory and Poincaré section methods were used to describe the strange attractor of chaos movement; Lyapunov exponent was introduced to describe the sensitivity on initial state; several kinds of dimensions can be defined to describe the non-cyclic property.

Case studies for the proposed method were carried out for runoff forecasting using different historical data of Ge-Zhou Dam of Yangtze River in China based on modifying Takens restructure theory that showed in Equation (9). Dimensions of attractors are calculated by means of G-P algorithm which is put up by Grassberger and Procaccia, and the method which is put up by Wolf^[15] to calculate Lyapunov exponential is also utilized.

Case 1: Daily runoff data of Ge-Zhou Dam of Yangtze (From 1960/1/1 to 1969/12/31) is shown in Figure 2.

Maximum Lyapunov exponential: $\lambda_{\max} = 0.004883362$

Embedded dimension: $d = 6$

Time delay: $\tau = 1$

Figure 2. Daily runoff Poincare section of Ge-Zhou Dam
(From 1960/1/1 to 1969/12/31)

The Poincare section of daily runoff is shown in Figure 3.

Figure 3. Daily runoff Poincare Section of Ge-Zhou Dam

(From 1960/1/1 to 1969/12/31)

Case II: Daily runoff data of Ge-Zhou Dam of Yangtze (From 1985/1/1 to 1998/12/31) is shown in Figure 4, the Poincare section of daily runoff is shown in Figure 5.

Maximum Lyapunov exponent: $\lambda_{\max} = 0.002814374$

Embedded dimension: $d = 6$

Time delay: $\tau = 1$

Figure 4. Daily runoff curve of Ge-Zhou Dam of Yangtze (From 1985/1/1 to 19998/12/31)

Figure 5. Daily runoff Poincare Section of Ge-Zhou Dam (From 1985/1/1 to 1998/12/31)

The demonstration of computing the Lyapunov exponent of Case 1 and Case 2 proves that there are chaos in these runoff time series.

Applying the data of Case 1, the paper forecasted for 24 consecutive days based on the above three methods in different embedded dimension. The results were analyzed based on the Relative Percentage Error Index (RPEI):

$$\text{RPEI} = \frac{1}{N} \sum_{i=1}^N \frac{|x'_i - \hat{x}'_i|}{x'_i} \times 100$$

(15)

The relative percentage forecasting errors are shown in Table 1.

Table 1 Forecasting Results

Embedded Dimension	<i>D</i> =5	<i>D</i> =6	<i>D</i> =8	<i>D</i> =10
Chaos model(local linear)	5.85%	5.51%	5.68%	6.31%
Phase space+BP	5.74%	5.43%	5.47%	5.80%
New method	5.70%	4.27%	4.50%	5.05%

The iteration performances of the DMEP and CEP are showed in figure 6.

Figure 6. Iteration performances of the DMEP and CEP

Analysis of different historical runoff time series found that the forecasting precision for the method proposed in this paper is better than that for the traditional methods.

5. CONCLUSIONS

The chaotic runoff time series with high-embedded dimension is very common in the nature. Therefore it is useful to study the forecasting methods of high-embedded dimension. The paper proposes a new method, which improves the character of full Lyapunov exponential spectrum in a restructure space with high dimensions. In the mean time, an artificial neural networks model based on determinate mutation evolutionary programming (DMEP) learning algorithm be presented for chaotic runoff series forecasting.

DMEP introduces chaos mapping into the mutation operation of traditional evolutionary programming, which aim to increase its convergence rate and results' precision. The results show that the method plays an important role to improve the precision of forecasting of runoff series with higher embedded dimensions.

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