DESIGN OF INTEGRAL VARIABLE STRUCTURE CONTROL FOR NONLINEAR SYSTEM BASED ON CMAC NEURAL NETWORK AND REFERENCE MODEL

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Abstract: Because mismatch of models of uncertain nonlinear control system and external disturbance etc., these will make effects of controller designed badly in uncertain nonlinear system. In this paper, we put forward a method, which can improve performance of uncertain nonlinear control system. We divide nonlinear control system into reference model and uncertain parts two respectively design and last synthesize control law of two parts. We build up approximate reference model through information known, make use of integral variable structure control of exponential approach method, and adopt to integral variable structure control based on CMAC neural network for uncertain part. We make use of method of extension controller's design based on extension set theory while uncertain boundary unknown. The simulate experiment validate this control method can improve control quality in an extent.

Keywords: CMAC neural network; integral variable structure control; uncertain nonlinear system; robust control; reference model

1. INTRODUCTION

Because there are uncertainty and nonlinear property of system when design actual engineering controller, which makes design become difficult. We make use of method of Integral Variable Structure Control (IVSC)^[1], while we have known estimative parameter and uncertain boundary of system model; we may make use of method of extension controller's design^[2] based on extension set theory while uncertain boundary unknown. We

both suppose that system model completely cannot be determined and design controller above two control method. In fact, sometimes we may make full use of information known to build up approximate system model. Although mismatch of model with actual system may be comparatively big, it can reflect some effective characteristic of actual system more or less. We call approximate system model to build up as reference model [3]. In this paper, we put forward integral variable structure control method of nonlinear system based on CMAC neural network and reference model. We confirm its validity through simulate and experiment.

2. THE THEORY AND SYSTEM DESCRIPTION OF NONLINEAR CONTROL SYSTEM

Define nonlinear system as: $\dot{x}^{(n)} = f(\underline{x}) + bu$ (1)

In formula (1), $\underline{x} = (x_1, x_2, \dots, x_n)^T = (x, \dot{x}, \ddot{x}, \dots, x^{(n-1)})^T$ is state vector, f is continuous bounded function, b is unknown time-varying positive real even $0 < b_{\min} \leq b \leq b_{\max}$, and $u \in R$ is input. Object of control is to supply an appropriate control input u that it makes distance of error vector $\underline{e} = \underline{x} - \underline{x}_d = (e_1, e_2, \dots, e_n)^T = (e, \dot{e}, \ddot{e}, \dots, e^{(n-1)})^T$ and zero vector become minimum. In this formula, $\underline{x}_d = (x_{d1}, x_{d2}, \dots, x_{dn})^T = (x_d, \dot{x}_d, \ddot{x}_d, \dots, x_d^{(n-1)})^T$ is state vector of antipant object.

Define reference model of nonlinear system built as follow:

$$\dot{x}_m^{(n)} = f_m(x_m) + b_m u_m \quad (2)$$

Describe mismatch of model and actual system as follow:

$$\dot{x}_\Delta^{(n)} = f_\Delta(\underline{x}_\Delta) + b_m u_\Delta \quad (3)$$

In above two formulas, b_m is parameter estimate of b , let $b_m = \sqrt{b_{\min} \cdot b_{\max}}$, $\underline{x}_m$ is state variable of reference model, and $\underline{x}_\Delta$ is dispersion between state variable of reference model and state variable $\underline{x}$ of actual system. Also, suppose that the relations as follows are tenable.

$$\underline{x}_\Delta = \underline{x} - \underline{x}_m \quad (4)$$

$$f_\Delta(\underline{x}_\Delta) = f(\underline{x}) - f_m(\underline{x}_m) \quad (5)$$

In condition of suppose, nonlinear system may be divided into two respective parts: reference model as formula (2) and model mismatch as formula (3). We may use different control law u_m and u_Δ for two parts to get optimal effect of control.

3. STRUCTURE OF CONTROL SYSTEM

P When f_m is known and f_Δ is uncertain, we may use approach method to resolve Integral Variable Structure Control (IVSC) u_m , use method of extension controller design u_Δ , and finally combine both as the ultimate control law. The control system structure is shown in figure 1[4]. The model controller makes state input $\underline{x}_m$ of reference model trace anticipant state vector $\underline{x}_d$. That is to make error vector $\underline{e}_m = \underline{x}_m - \underline{x}_d$ full approach to zero. If f_m known, we may directly use exponential approach method to resolve IVSC u_m

Figure 1. Structure of control system

4. DESIGNING CONTROL LAW OF NONLINEAR CONTROL SYSTEM

The switching function usually select linear function:

$$S_m(e_m) = e_m^{(n-1)} + a_1 e_m^{(n-2)} + \dots + a_{n-1} e_m + K_{lm} Z_m \quad (6)$$

In above formula, define $Z_m = e_m$. Resolve derivative for (6) and get:

$$S_m(e_m) = e_m^{(n)} + a_1 e_m^{(n-1)} + \dots + a_{n-1} e_m + K_{lm} e_m \quad (7)$$

Let S_m and exponential approach law identical:

$$S_m = -\varepsilon \operatorname{sgn}(S_m) - k S_m \quad (8)$$

We may get u_m from (7), (8) and (2) formula as follows:

$$e_m^{(n)} + a_1 e_m^{(n-1)} + \dots + a_{n-1} \dot{e}_m + K_{lm} e_m = -\varepsilon \operatorname{sgn}(S_m) - k S_m \quad (9)$$

$$\begin{aligned} x_m^{(n)} - x_d^{(n)} + a_1 e_m^{(n-1)} + \dots + a_{n-1} \dot{e}_m + K_{lm} e_m &= -\varepsilon \operatorname{sgn}(S_m) - k S_m \\ f_m(x_m) + b_m u_m - x_d^{(n)} + a_1 e_m^{(n-1)} + \dots + a_{n-1} \dot{e}_m \\ &+ K_{lm} e_m = -\varepsilon \operatorname{sgn}(S_m) - k S_m \end{aligned} \quad (10)$$

Get from above formula as follows:

$$u_m = b_m^{-1} \left\{ u_m - f_m(x_m) - \varepsilon \operatorname{sgn}(S_m) - k S_m \right\} \quad (11)$$

$$\hat{u}_m = x_d^{(n)} - a_1 e_m^{(n-1)} - \dots - a_{n-1} \dot{e}_m - K_{lm} e_m$$

Designing compensatory controller may adopt method to design extension controller [2]. The designing object of control law u_Δ is to compensate for dispersion between model's state vector $\underline{x}_m$ and actual system's state vector $\underline{x}$, which is to make $\underline{x}_\Delta$ full closer to anticipant value $\underline{x}_{\Delta d}$. According to Integral Variable Structure principle, define switch surface as follow:

$$S(e_\Delta) = e_\Delta^{(n-1)} + a_1 e_\Delta^{(n-2)} + \dots + a_{n-1} e_\Delta + K_I Z_\Delta \quad (12)$$

$e_\Delta = \underline{x}_\Delta - \underline{x}_{\Delta d}$, $\dot{Z}_\Delta = e_\Delta$ in the above formula. The control input u_Δ designed must satisfy sliding condition: $S \dot{S} \leq -\eta |S|$ (13)

In the above formula, η is a positive definite constant. We may design control law u_Δ of compensatory controller from invariable structure control law. Define Lyapunov function as $V = 0.5S^2$. Thus

$$\begin{aligned} \dot{V} &= S \dot{S} \\ &= S \cdot (e_\Delta^{(n)} + a_1 e_\Delta^{(n-1)} + \dots + a_{n-1} \dot{e}_\Delta + K_I e_\Delta) \\ &= S \cdot \{f_\Delta(x_\Delta) + b u_\Delta - x_{\Delta d}^{(n)} + a_1 e_\Delta^{(n-1)} + \dots + a_{n-1} \dot{e}_\Delta + K_I e_\Delta\} \\ &= S \cdot \{f_\Delta(x_\Delta) + b u_{\Delta eq} + u_{\Delta s}\} \end{aligned} \quad (14)$$

Suppose $u_{\Delta eq}$ is equivalent control and $u_{\Delta s}$ is striking control. While $V = 0$, we may get a optimal equivalent control $u_{\Delta eq}$:

$$u_{\Delta eq}^* = b^{-1}(\hat{u} - f_\Delta(x_\Delta)) \quad (15)$$

Because b is unknown, we use b_m as an approximate of b and get an equal control $u_{\Delta eq}$:

$$u_{\Delta eq} = b_m^{-1}(\hat{u} - f_\Delta(x_\Delta)) \quad (16)$$

$$\text{Thereinto, } \hat{u} = -(a_1 e_\Delta^{(n-1)} + a_2 e_\Delta^{(n-2)} + \dots + a_{n-1} \dot{e}_\Delta + K_I e_\Delta)$$

Determinate striking control $u_{\Delta s}$ through Lyapunov theory and method as follows:

$$\begin{aligned} V &= S \dot{S} \\ &= S \cdot \{f_\Delta(x_\Delta) + b u_\Delta - \hat{u}\} \\ &= S \cdot \{f_\Delta(x_\Delta) + b u_{\Delta eq} + b u_{\Delta s} - \hat{u}\} \\ &= S \cdot \{f_\Delta(x_\Delta) + (b b_m^{-1} - 1) \hat{u} - b b_m^{-1} f_\Delta(x_\Delta) + b u_{\Delta s}\} \end{aligned} \quad (17)$$

Let $u_{\Delta s} = b_m^{-1}(f_\Delta(x_\Delta) - G^* \text{sgn}(S))$ and expression of G^* is as follows:

$$G^* = b_m b^{-1} \eta + |b_m b^{-1} f_\Delta(x_\Delta) + (1 - b_m b^{-1}) \hat{u}| \quad (18)$$

We may confirm $\dot{V} = S \dot{S} \leq -\eta|S| \leq 0$ if we induce $u_{\Delta s}$ and G^* into the formula $\dot{V} = S \dot{S}$.

We find that $u_{\Delta s} = b_m^{-1}(f_{\Delta}(x_{\Delta}) - G^* \text{sgn}(S))$ can satisfy sliding condition. So synthesis control law from equivalent control and striking control is as follows:

$$\begin{aligned} u_{\Delta} &= u_{\Delta eq} + u_{\Delta s} \\ &= b_m^{-1}(\hat{u} - f_{\Delta}(x_{\Delta})) + b_m^{-1}(f_{\Delta}(x_{\Delta}) - G^* \text{sgn}(S)) \\ &= b_m^{-1}(\hat{u} - G^* \text{sgn}(S)) \end{aligned} \quad (19)$$

Because b is unknown and f_{Δ} is completely uncertain, G^* cannot implement. We use CMAC neural network and make it full approach G^* through study and training.

Suppose that input of CMAC network is as follows [5]: $G_N(e_{\Delta} | \underline{w}) = \underline{w}^T \underline{\mu}(e_{\Delta})$. In above formula, $\underline{w}$ is a weighted vector and $\underline{\mu}(e_{\Delta}) \in R^{m \times 1}$ is a basis function of network. Suppose there is an optimal weight value $\underline{w}^*$ that makes $\varepsilon = G_N(e_{\Delta} | \underline{w}^*) - G^*$ minimum. A renewed law of weighted vector may be determined by Lyapunov theory for guaranteeing stability of system.

Suppose Lyapunov function as $V = 0.5(S^2 + b b^{-1} \lambda^{-1} \underline{\xi}^T \underline{\xi})$, $\underline{\xi} = \underline{w} - \underline{w}^*$.

$$\begin{aligned} \dot{V} &= S \dot{S} + b b^{-1} \lambda^{-1} \underline{\xi}^T \dot{\underline{\xi}} \\ &= S \cdot \{f_{\Delta}(x_{\Delta}) + (b b_m^{-1} - 1)\hat{u} - b b_m^{-1}(G^* \text{sgn}(S))\} + b b_m^{-1} |S| \varepsilon \\ &\quad + \underline{\xi}^T |S| \underline{\mu}(e_{\Delta}) - \lambda^{-1} \underline{\xi}^T \dot{\underline{w}} \\ &\leq -\eta |S| - b b_m^{-1} (|S| \varepsilon + \underline{\xi}^T |S| \underline{\mu}(e_{\Delta}) - \lambda^{-1} \underline{\xi}^T \dot{\underline{w}}) \\ &= -|S| \cdot \{\eta + b b_m^{-1} \varepsilon\} + b b_m^{-1} \underline{\xi}^T \{-|S| \underline{\mu}(e_{\Delta}) + \dot{\underline{w}} \lambda^{-1}\} \end{aligned} \quad (20)$$

$\dot{\underline{w}} = \lambda |S| \underline{\mu}(e_{\Delta})$ is tenable, we may get that an inequality (19) is tenable.

$$\dot{V} \leq -|S| \{\eta + b b_m^{-1} \varepsilon\} \quad (21)$$

Only if ε is full closer zero, we must get $\eta + b b_m^{-1} \varepsilon \geq 0$, thus $\dot{V} \leq 0$. According to above deduction, choose study equation of CMAC network as follow:

$$\dot{\underline{w}} = \lambda |S| \underline{\mu}(e_{\Delta}) \quad (22)$$

According to literature^[2] discussion, we divide movement space of system into eligible region and ineligible region by size of S and adopt different control in the different regions. Furthermore, we transfer sign function $\text{sgn}(\cdot)$ into saturation function $\text{sat}(\cdot)$ for reducing phenomenon of vibration. Thus, the control law of compensatory controller may modify as

$$\text{follows: } u_{\Delta} = \begin{cases} -b_m^{-1}G \cdot \text{sat}(S), S \geq 1 \\ b_m^{-1}(\hat{u} - G \cdot \text{sat}(S)), S < 1 \end{cases} \quad (23)$$

We sum formula (9) and formula (21), get a final control law of system as follows: $u = u_m + u_{\Delta}$ (24)

5 SIMULATION AND DISCUSSION

Confirming actual example is to use nonlinear system equation as follows:

$$\ddot{x} + (0.5 + 0.2\sin(t))\dot{x} + 3.2\cos(x) = (1 + 0.2\sin(t))u \quad (25)$$

Suppose that reference model built is as follows:

$$\ddot{x} + 0.5\dot{x} + 3\cos(x - 0.5) = b_m u \quad (26)$$

Figure2 output response of state vector

Evidently, $b = 1 + 0.2\sin(t)$, estimative value of b is $b_m = 0.98$. Suppose that the anticipant state of system or instruction input is as follows: $x_d = 2\sin(t + 2)$, $a_1 = 1, a_2 = 2$. In the formula (8), let $\varepsilon = 1, k = 10$. A simulative result is shown in figure 2 and figure 3. We find through simulation that inducing the project can more improve system performance of dynamic state and static state, make system steady, make system strong adaptability, make parameter definite easier, make effect of each parameter little, not demand more precise for parameter in arithmetic, make one-off determination. Also it confirms that controllable method in the paper has a good performance of track.

6 CONCLUSIONS

In this paper, we put forward an designing method of uncertain nonlinear systemic controller that can eliminate phenomenon of integral saturation, decrease overshoots, make system adaptability strong and steady, make parameter definite easier, make effect of each parameter little, make one-off determination for parameter in arithmetic. Also we find through simulation that inducing the project can more improve sy

Figure 3 the error curve of reference model

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