PATTERNS AND RULES FOR BEHAVIOURAL SUBTYPING

Heike Wehrheim

Universitat Oldenburg

Fachbereich Informatik

Postfach 2503, D-26111 Oldenburg, Germany

wehrheim@informatik.uni-oldenburg.de

Abstract Subtyping relations for object-oriented formalisms describe relationships between super- and subclasses which satisfy the substitutability requirement imposed on types and their subtypes. *Behavioural* subtyping is concerned with subtypes for *active* classes with an explicit dynamic behaviour, specifiable for instance by object-oriented formal methods combining state-based with behavioural formalisms.

In this paper we develop syntactic *patterns* and semantic *rules* for the state-based part of a subclass which guarantee that the subclass is a behavioural subtype of its superclass. This allows to check for subtypes *without* computing the behavioural semantics of the class at all. Our results are similar to the ones linking data refinement in state-based methods with failure-divergence refinement in CSP. In contrast to classical data refinement, subtyping has to cope with additional new operations in the subclass.

Keywords: Object-orientation, refinement, behavioural subtyping, Z, CSP.

1. INTRODUCTION

Inheritance is one of the key features for the success of object-oriented analysis and design methods as well as programming languages. Inheritance is employed to re-use already existing class descriptions and extend them with additional attributes and methods, thus creating a new subclass from an existing superclass. Subtyping for object-oriented formalisms [1, 6, 12] studies relationships between super- and subclasses with the aim of clarifying when a subclass can be said to be *type substitutable* for its superclass. Type substitutability means that “a subtype instance can be used wherever a supertype instance is expected” [17]. Subtyping is also studied in the context of *active* classes with a behavioural semantics (as for instance obtained when using LOTOS or CSP as a specification language). Active classes have an explicit

specification of the dynamic behaviour which describes synchronisation constraints of the class or certain protocols which have to be obeyed. In this paper, we use the term behavioural subtyping to refer to subtyping for this type of classes. A number of proposals for *behavioural subtyping* relations have been made (among others [16, 2, 4, 13]; see conclusion for a discussion of related work), with for instance Petri nets or labelled transition systems as semantic basis. The starting point of this paper are three subtyping relations from [9] with proven substitutability properties, two of them also adequate for shared mutable objects. The formal basis for these two relations are transition systems and their (CSP) failures and divergences.

Here we will be concerned with *checking* subtype relationships among classes specified in object-oriented formal methods combining state-based with behavioural aspects. The formal method which we have in mind (and which we use throughout our examples) is CSP-OZ [7], a combination of CSP [10] and Object-Z [15], but the theory we develop will be independent of this particular choice. Two aspects of CSP-OZ are however of importance: first, the fact that the state-based part of CSP-OZ has a *behavioural semantics* (and thus we have to use *behavioural* subtyping relations), and second, that the semantics of the combination is obtained by separately computing the semantics of the Object-Z and CSP part, and afterwards joining them by CSP parallel composition. This second point allows us to look at state-based and behaviour part in isolation when checking for subtypes.

A straightforward approach to check subtyping relationships among classes is the following: given two class descriptions, compute their behavioural semantics and compare the classes according to some chosen relation. However, since the semantics of CSP-OZ (and most other such types of integrated formal methods) has to be constructed by hand, this approach is in general impracticable. Instead we are interested in developing simple, at the best *syntactic, patterns*, which allow to avoid the explicit check and instead guarantee subtypes *by construction*. Such patterns or rules have to give conditions on attributes and operations in subclasses, which, when followed, immediately lead to subtypes. In this paper we will solely be interested in subtyping patterns for the state-based part (which, due to the compositional semantics, can without change be combined with patterns for the behaviour part). Our investigations are similar to the work about the relationship between data refinement rules in state-based formalisms and failure-divergence refinement [11, 18]. There, it has been studied whether data refinement induces failure-divergence refinement when the state-based formalism is given a behavioural semantics. The basic approach taken here is the same: we assume to have a behavioural semantics for a state-based formalism, have a set of conditions on two class specifications and show that the conditions induce a behavioural subtyping relationship between the classes (soundness of rules).

We illustrate our approach by a small example of a buffer which is extended by means of inheritance in three different ways yielding three different kinds of subtypes.

2. CLASSES

We start with the specification of the buffer. We use CSP-OZ [7] syntax in the specifications. The following class specifications will *not* contain any CSP behaviour descriptions since we are only interested in patterns for the state-based part. Thus the classes almost look like Object-Z classes. The specified buffer can hold elements (in a finite bag) of a basic type *Element* up to some arbitrary number n . Elements may be put into the buffer and retrieved by means of the methods *put* and *get*, and an attribute *alive* is used to indicate whether some specific instance of class *Buffer* is alive or has already been deleted.

[*Element*]

The class is parameterised in the length of the buffer. The first two lines define the *interface* of the class: it has methods *put* and *get*. Next the attributes and their initial values are defined. The application of methods *put* and *get* is described by enable and effect predicates, the enabling predicate specifying a *guard* for the application of the method, the effect predicate the possible results of applying the method.

In general we represent a class C by a *state schema*, an *initialisation schema* and a number of *operation schemas*: $C = (State, Init, (enable_Op_j)_{j \in J}, (effect_Op_j)_{j \in J})$. The components have the following meaning: *State* describes the state space of the class. It is determined by giving a set of *attributes* $Attr$ and their types together with an *invariant* Inv further restricting the allowed values. *Init* describes the set of initial states determined by a predicate over $Attr$. $(Op_j)_{j \in J}$ gives a family of methods or operations on the state space. Every operation Op_j has a number of input variables $(in_{j,i}?)_{1 \leq i \leq k_j}$ of type $T_{in_{j,i}}$ and a number of output variables $(out_{j,i}!)_{1 \leq i \leq l_j}$ of type $T_{out_{j,i}}$. Two schemas are given for every operation (an omitted schema in the CSP-OZ specification corresponds to *true*):

- 1 an enabling schema $enable_op_j$ whose predicate ranges over the current state $Attr$ and input variables $in_{j,i}?$. It acts as a *guard* to the operation execution, and
- 2 an effect schema $effect_op_j$ with predicate ranging over current state $Attr$, next state $Attr'$, input variables $in_{j,i}?$ and output variables $out_{j,i}!$ which describes the postcondition of the operation (where $Attr'$ refers to the new state of attributes after execution of the operation).

We use the following abbreviation for input and output variables:

$$\begin{aligned} In_j &\hat{=} [in_{j,1}?: T_{in_{j,1}}; \dots; in_{j,k_j}?: T_{in_{j,k_j}}] \\ Out_j &\hat{=} [out_{j,1}!: T_{out_{j,1}}; \dots; out_{j,l_j}!: T_{out_{j,l_j}}] \end{aligned}$$

Additionally we define the precondition of an operation Op_j to be $pre_Op_j := \exists State', Out_j : effect_Op_j$.

Next we provide the class description with a behavioural semantics in terms of labelled transition systems. The labels are taken from an alphabet Σ which contains all events $op.i_1 \dots i_k.o_1 \dots o_l$ where op is the name of a method and the rest are possible values of input and output parameters. For an event $ev = op.i_1 \dots i_k.o_1 \dots o_l$ we write $ch(ev) = op$ to refer to the method or operation name of an event. Σ_τ stands for $\Sigma \cup \{\tau\}$. A *labelled transition system* (LTS) is a tuple $T = (Q, \rightarrow, Q^{in})$ with Q a set of states, $\rightarrow$ a transition relation and Q^{in} the set of initial states. A state q is said to be *stable* if it has no outgoing τ transitions, q *diverges* ($q \uparrow$) if an infinite number of τ steps is possible from q , and $next(q)$ is the set of all events which are executable from q .

The basic idea of the semantics is to view a class as a *process* which is continuously executing enabled operations and thus changes its state. Initially it is in one of the states of *Init*. The symbol $\perp$ is used to indicate one particular *divergent* state: The class diverges if an enabled operation is applied outside its precondition. In the following we take a schema to stand for a set of *bindings* and for a binding $q = \langle a_1 \Rightarrow v_1, \dots, a_n \Rightarrow v_n \rangle$ and a schema S we write $S(q)$

if q is a valid binding of schema S . The operator $\cup$ is used to join bindings: $\langle a_1 \Rightarrow v_1, \dots, a_n \Rightarrow v_n \rangle \cup \langle b_1 \Rightarrow w_1, \dots, b_m \Rightarrow w_m \rangle = \langle a_1 \Rightarrow v_1, \dots, a_n \Rightarrow v_n, b_1 \Rightarrow w_1, \dots, b_m \Rightarrow w_m \rangle$ assuming disjointness of the sets $\{a_1, \dots, a_n\}$ and $\{b_1, \dots, b_m\}$. For a state q , the notation q' is used to describe the same binding but with components decorated with ', the notation $q|_{State}$ is a binding which only gives values to variables in q which are declared in $State$.

Definition 2.1

The operational semantics of a class C is defined as $\llbracket C \rrbracket = (Q_C, \rightarrow_C, Q_C^{in})$, where

- $Q_C = State \cup \{\perp\}$, $Q_C^{in} = Init$ and
- $\rightarrow_C$ is defined as $\perp \xrightarrow{\tau} \perp$ and
 - $q_1 \xrightarrow{ev} q_2$ if $ev = op_j.i_1 \dots i_{k_j}.out_1 \dots out_{l_j}$ and $enable_op_j(q_1 \cup \langle in_n \Rightarrow i_n, 1 \leq n \leq k_j \rangle)$ and $effect_op_j(q_1 \cup q_2' \cup \langle in_n \Rightarrow i_n, out_m \Rightarrow o_m, 1 \leq n \leq k_j, 1 \leq m \leq l_j \rangle)$,
 - $q \xrightarrow{ev} \perp$ if $ev = op_j.i_1 \dots i_{k_j}.out_1 \dots out_{l_j}$ and $enable_op_j(q_1 \cup \langle in_n \Rightarrow i_n, 1 \leq n \leq k_j \rangle)$ but not $pre_op_j(q_1 \cup \langle in_n \Rightarrow i_n, 1 \leq n \leq k_j \rangle)$.

Note that we use state q_2' here since the effect predicate uses primed variables to refer to the state after execution.

The dynamic behaviour of class *Buffer*(2) with an instantiation of type *Element* with the set $\{1,2\}$ is depicted by the transition system in Figure 1.

Next we present three extensions of the basic class *Buffer* by inheritance: the first adds a possibility of explicitly deleting the buffer, the second adds a new method checking for emptiness and the third a method which allows to insert two elements at once.

Figure 1. Dynamic behaviour of superclass *Buffer*(2)

Conceptually inheritance is defined as conjunction on schemas: given an abstract class $A = (AState, AInit, (enable_AOp_j)_{j \in J}, (effect_AOp_j)_{j \in J})$ and specialised class $C = (CState, CInit, (enable_COp_i)_{i \in I}, (effect_COp_j)_{j \in J})$ assuming C is constructed from A by means of inheritance, then $J \subseteq I$ and

$$\begin{aligned}
 CState &= AState \wedge ExtState \\
 CInit &= AInit \wedge ExtInit,
 \end{aligned}$$

$$\begin{aligned}
\text{enable_COp}_j &= \text{enable_AOp}_j \wedge \text{enable_ExtOp}_j & \forall j \in J, \\
\text{effect_COp}_j &= \text{effect_AOp}_j \wedge \text{effect_ExtOp}_j & \forall j \in J, \\
\text{enable_COp}_i &= \text{enable_ExtOp}_i & \forall i \in I \setminus J, \\
\text{effect_COp}_i &= \text{effect_ExtOp}_i & \forall i \in I \setminus J.
\end{aligned}$$

ExtState (*ExtInit* etc.) is the schema which is explicitly written in the subclass and *AState* (etc.) is inherited.

3. SUBTYPING

Subtyping on ordinary types is usually demanded to fulfill the requirement of *type substitutability*: subtypes should be usable wherever supertypes are expected without any noticeable differences [17]. When lifting this concept to classes, the first idea is to use the inheritance hierarchy for defining subtyping relations. This however does not fit the requirement of substitutivity anymore: inheritance is first of all concerned with *re-use* of class descriptions and allows fundamental changes to the behaviour of superclasses in a subclass. The requirement of substitutivity suggests that instead some sort of refinement should be used as a subtyping relation,

Since we are working in the context of CSP, our notion of refinement is failure-divergence refinement. In the process algebra CSP process refinement is used to compare an implementation against a specification with respect to their failures and divergences: the implementation has to be more deterministic than the specification, i.e. has less failures and divergences. Failures are sets of pairs $(\sigma, X) \in \Sigma^* \times 2^\Sigma$ with the meaning that a process has such a pair in its failure set if it can execute the trace σ and may thereby reach a stable state (no τ -transitions possible) in which none of the events from X are possible. Divergences are sets of traces σ such that a process may diverge after the trace, i.e. execute an infinite number of invisible actions. Failures and divergences can for instance be computed from the transition system of a process, but due to lack of space we refrain from giving a formal definition here. *Buffer(2)* for instance has failures (with maximal refusals) $(\epsilon, \{get.1, get.2\}), (put.1, \{get.2\}), (put.1 \cap put.2, \{put.1, put.2\}), \dots$ and an empty set of divergences.

Refinement then simply is inclusion of failure and divergence sets: A labelled transition system I is a *failure-divergence refinement* of an lts S ($S \sqsubseteq_{\mathcal{FD}} I$) iff $failures(I) \subseteq failures(S)$ and $divergences(I) \subseteq divergences(S)$.

According to this definition none of the subclasses of *Buffer* are refinements (e.g. *FinBuffer* has a failure (del, Σ) which is not in *Buffer*'s failure set). Nevertheless refinement will be the basis for our three behavioural subtyping relations. The relations additionally have to cope with the fact that the subclass may introduce new methods. The comparison of sub- with superclass has to

“ignore” these methods or at least relate new methods to old ones. For this, we use three different operators: restriction, concealment (next) and substitution (below), and define them by their effect on failures and divergences.

Definition 3.1 Let $T = (Q, \rightarrow, Q^{in})$ be a labelled transition system and $N \subseteq \Sigma$.

- **Restriction:** $failures(T \setminus_r N) = \{(\sigma, X) \mid \exists Y \bullet (\sigma, Y) \in failures(T), \sigma \in (\Sigma \setminus N)^*, X \subseteq Y \cup N\}$,
- **Concealment:** $failures(T \setminus_c N) = \{(\sigma, X) \mid \exists (\sigma', Y) \in failures(T), \sigma = \sigma' \upharpoonright (\Sigma \setminus N) \wedge X \subseteq Y \cup N\}$.

Divergences are unchanged by restriction and concealment.

Restriction forbids the execution of events in N , concealment makes N -events invisible. Note that concealment differs from classical hiding in process algebras. Hiding may usually introduce new non-stable states, whereas concealment does not.

The basic idea of substitution is to *map* new methods onto sequences of old methods: the state changes achieved by new method should be the same as that achieved by a consecutive execution of the old methods. This is the “dynamic behaviour” counterpart of Liskov and Wing’s extension maps [12].

Definition 3.2 Let $(\sigma, X) \subseteq \Sigma \times 2^\Sigma$ be a pair of trace and refusal, and $N \subseteq \Sigma$ a set of events (the new events to be mapped onto old ones). Furthermore let $f_N : N \rightarrow \Sigma^*$ be a function, the substitution function. We define $f_N(\sigma, X)$ (by overloading the function f_N) to be $(f_N(\sigma), X \cup N)$ where $f_N(\sigma)$ is inductively defined by $f_N(\varepsilon) = \varepsilon$ and

$$f_N(a \frown \sigma) = \begin{cases} a \frown f_N(\sigma) & \text{if } a \notin N \\ f_N(a) \frown f_N(\sigma) & \text{else} \end{cases}$$

Superclass and subclass are then compared by applying concealment, restriction or substitution to the new methods before checking for failure and divergence inclusion (see [9]).

Definition 3.3 Let A, C be labelled transition systems, A standing for the super- and C for the subclass, and let N be the set of events over new methods.

- C is a weak subtype of A (denoted $A \sqsubseteq_{wst}^N C$) iff $A \sqsubseteq_{\mathcal{FD}} C \setminus_r N$.
- C is a safe subtype of A according to a substitution function f_N (denoted $A \sqsubseteq_{sst}^{f_N} C$) iff $\forall (\sigma, X) \in failures(C) : f_N(\sigma, X) \in failures(A)$ and $\forall \sigma \in divergences(C) : f_N(\sigma) \in divergences(A)$.
- C is an optimal subtype of A (denoted $A \sqsubseteq_{ost}^N C$) iff $A \sqsubseteq_{\mathcal{FD}} C \setminus_c N$.

Coming back to our four buffers: *FinBuffer* is a weak subtype of *Buffer* and *EmptyCheckBuffer* is a weak as well as an optimal subtype. *FinBuffer* is not an optimal subtype of *Buffer* since $\llbracket \text{FinBuffer} \rrbracket \setminus_c \{\text{del}\}$ has a failure (ε, Σ) which *Buffer* does not have. Finally *2Buffer* is a safe subtype (mapping *put2* to *put* $\wedge$ *put*) as well as a weak subtype of *Buffer* but no optimal subtype.

In order to check for subtypes we first had to construct the transition systems, compute failures/divergences for the classes and afterwards apply the subtyping definition. Next we look at *patterns* which give us the possibility of obtaining a subtype *by construction* and *rules* which allow to *locally* check for subtypes without computing the semantics of classes at all.

4. PATTERNS AND RULES

We refer to *patterns* as conditions on the specification of a subclass which can be checked (almost) completely syntactically, without looking at the semantics at all. *Rules* will be conditions on classes which can be checked by looking at the semantics of methods in isolation, i.e. without computing the transition system semantics of a class but by comparing the meaning of methods of sub- and superclass.

Besides the patterns and rules discussed here, it is always possible to employ *data refinement rules* (specific to CSP-OZ [8]): any subclass which is a refinement of its superclass is a subtype in the sense of all three relations. All of the following patterns and rules can thus be combined with a valid data refinement.

Patterns. We present two patterns here, one for weak and one for optimal subtyping. The patterns are conditions on the changes and extensions allowed in the subclass. We again assume $A = (AState, AInit, (AOp_j)_{j \in J})$ to be the super- and, letting $J \subseteq I$, $C = (CState, CInit, (COp_i)_{i \in I})$ to be the subclass.

P1 $\forall j \in J : \text{enable_}COp_j = \text{enable_}AOp_j$ and $\text{effect_}COp_j = \text{effect_}AOp_j$, i.e. the specialised subclass does not change the old methods at all. There are no restrictions on new methods or attributes.

P2 **P2.A** $\forall j \in J : \text{enable_}COp_j = \text{enable_}AOp_j$ and $\text{effect_}COp_j = \text{effect_}AOp_j$,

P2.B $\forall n \in I \setminus J$: the predicate of schema $\text{effect_}COp_n$ ranges solely over *Attr*, *ExtAttr* and *ExtAttr'*, i.e. no changes to old attributes are allowed,

P2.C $\forall n \in I \setminus J : \text{enable_}COp_n \Rightarrow \text{pre_}COp_n$.

Here = stands for syntactical equality. Pattern P1 is a pattern for a weak subtype, e.g. *FinBuffer* may be constructed from *Buffer* using pattern P1, it leaves

all old methods unchanged. Pattern P2 is valid for *EmptyCheckBuffer*: the new method *empty* does not change old attributes and its precondition holds whenever the method is enabled. We however cannot construct *FinBuffer* from *Buffer* using pattern P2 since *FinBuffer*'s effect predicate for operation *del* changes a value of an old attribute (*alive*). Condition P2.C is essentially used to avoid new divergences in the subclass. It is the only part of the patterns which is not purely syntactic, but this is necessary since we want to take divergence into account.

The following main theorem now states the soundness of the patterns for the subtyping relations.

Theorem 1

If C is a subclass of A obeying pattern P1 then $A \sqsubseteq_{wst} C$.

If C is a subclass of A obeying pattern P2 then $A \sqsubseteq_{ost} C$.

Note that such a relationship cannot be guaranteed by inheritance alone. Inheritance allows all kinds of changes in the subclass. Due to lack of space we only show the proof for the second part of the theorem. For the proof we show that, when constructed by a pattern, subclass and superclass are even more close to each other than defined by subtyping: the superclass *simulates* the subclass in a certain sense. Hence in the following we define a simulation relation inducing optimal subtyping. The relation is in spirit a forward simulation. We do not consider backward simulations here since we are not interested in completeness. In fact, it is immediately clear that the patterns are not complete: there are as well valid subtypes which cannot be constructed according to the patterns.

For the simulation definition we assume that the semantics of Definition 2.1 is used for classes, with the consequence that there are no τ -transitions except from $\perp$ to $\perp$.

Definition 4.1 *Class A simulates C concealing N , $A \preceq_{(N)} C$, if there is a relation $\rho \subseteq Q_A \times Q_C$ such that*

- 1 $\forall q_C \in Q_C^{in} \exists q_A \in Q_A^{in} : (q_A, q_C) \in \rho$,
- 2 $\forall (q_A, q_C) \in \rho$ the following holds:
 - $stable(q_C) \Rightarrow stable(q_A)$, $next(q_C) \setminus N = next(q_A)$, $q_C \uparrow \Rightarrow q_A \uparrow$,
 - $\forall a \notin N$:

$$q_C \xrightarrow{a}_C q'_C \Rightarrow \exists q'_A : q_A \xrightarrow{a}_A q'_A \wedge (q'_A, q'_C) \in \rho,$$
 - $\forall a \in N$:

$$q_C \xrightarrow{a}_C q'_C \Rightarrow (q_A, q'_C) \in \rho.$$

The idea behind simulation for optimal subtypes is that for the N -steps of the subclass we do not require a matching step of the superclass, instead states q'

reached after executing new methods from some state q must be simulated by the matching partner of q .

Theorem 2 $A \preceq_{(N)} C \Rightarrow A \sqsubseteq_{ost}^N C$.

The remaining part for proving (the second part of) Theorem 1 is to show that subclasses constructed by patterns can be simulated by the superclasses. We again use N to refer to the set of all events over new methods.

Theorem 3 If C is a subclass of A obeying pattern $P2$ then $A \preceq_{(N)} C$.

Rules. We proceed with giving rules for the last subtyping relation, safe subtyping. Here, we actually have to compare the semantics of new methods with that of (sequences of) old methods. This requires a little more technical overhead. We explain this by means of *2Buffer*. *2Buffer* is a safe subtype of *Buffer* since the new method *put2* can be explained in terms of the old method *put*: the effect of *put2* is the same as that of two *put*'s. However, the substitution function f_N that we need does not map a new method/operation to a sequence of operations but *events* to *sequences of events*. Besides transformation of operations we also need some means for transforming inputs and outputs of a new operation onto the inputs and outputs of the old operations. For this, we use (an adaption of) the technique of input and output transformers as for instance advocated in [5]. An input or output transformer simply is a schema that relates inputs (or outputs) of operations. Looking at *2Buffer* the input transformer should relate *el1?* and *el2?* with the two inputs *el?* of two *put*'s, the output transformer is empty. In order to be able to distinguish the inputs of the two *put*'s we define another decoration on schemas, that decorate input and output variables with numbers.

For an operation schema $S = [a_1 : T_1; \dots; a_n : T_n, in? : T_{in}, out! : T_{out} \mid p]$ we define S^k , $k \in \mathbb{N}$, to be $[a_1 : T_1; \dots; a_n : T_n, in^k? : T_{in}, out^k! : T_{out} \mid p[in?/in^k?, out!/out^k!]]$ (and similar for more than one input or output variable). The decoration is used to distinguish variables with the same name coming from operations at different positions in the substituting sequence. The input transformer from *put2* to $put^1 \circ put^2$ can thus be formulated as:

<i>IT</i>
$el1? : Elwmmwt; el2? : Element$
$el^1! : Element; el^2! : Element$
$el1? = el^1! \wedge el2? = el^2!$

It transforms inputs of *put2* into outputs which can than be fed into $put^1 \circ put^2$ (using the Z pipe operator $>>$). Conversely, an output transformer from a sequence of old operations $AOp_1^1 \circ AOp_2^2$ to a new operation *COp* would take the

outputs of $AOp_1^1 \S AOp_2^2$ as inputs and transform them into outputs of COp . The output transformer from $puts^1 \S put^2$ to put is an empty schema: no declarations and the predicate is true.

To simplify matters, we only consider substitutions which map one operation to a sequence of two operations in the sequel. This can however easily be generalised to an arbitrary number of operations.

Definition 4.2 *An input transformer from COp to $AOp_1^1 \S AOp_2^2$ is a schema IT s. t.*

- *all input variables from COp are declared in IT ,*
- *for every input variable $x^i?$ of type T from operation AOp_i^i , $i = 1, 2$, there is one output variable $x^i! : T$ in IT ,*
- *no other variables are declared in IT and*
- *the predicate of IT is a conjunction of equalities of the form $x? = y!$ such that every output variable occurs exactly once.*

Conversely, an output transformer from $AOp_1^1 \S AOp_2^2$ to COp is a schema OT in which all output variables from COp are declared, for every output variable $x^i! : T$ from AOp_i^i , $i = 1, 2$, there is one input variable $x^i? : T$, no other variables are declared in OT , and the predicate of IT is a conjunction of equalities of the form $x? = y!$ such that every output variable occurs exactly once.

Figure 2 illustrates the use of input and output transformers: execution of COp should be the same as that of $AOp_1 \S AOp_2$, and the input and output transformers define the valid relationships between the input and outputs.

We now again assume A to be the super- and C to be the subclass. The following rule formalises Liskov and Wings's extension maps [12] in our setting.

Extension Rule: The extension rule R holds between A and C if all old methods remain unchanged in the subclass (or are changed according to data refinement rules) and there is an *extension map* $E : N_{op} \rightarrow O_{op}^*$ such that the following holds: for every $COp \in N_{op}$, either COp does not change attributes at all and $E(COp) = \epsilon$, or else COp does change attribute values, $E(COp) = AOp_1 \wedge AOp_2$ and there is an input transformer IT from COp to $AOp_1 \S AOp_2$, an output transformer OT from $AOp_1 \S AOp_2$ to COp and the following conditions are furthermore met.

$$\mathbf{Rz.A} \quad \forall CState, \forall In \bullet \text{enable_}COp \Rightarrow IT \gg (\text{enable_}AOp_1^1 \wedge \text{pre_}AOp_1^1),$$

$$\mathbf{R.B} \quad \forall CState, \forall In \bullet \text{enable_}COp \Rightarrow \text{pre_}COp,$$

$$\mathbf{R.C} \quad \forall CState, CState', \forall In \bullet \text{enable_}COp \wedge (IT \gg \text{effect_}AOp_1^1) \\ \Rightarrow IT \gg (\text{enable_}AOp_2^{2'} \wedge \text{pre_}AOp_2^{2'}),$$

Figure 2. Input and output transformer

$$\mathbf{R.D} \quad \forall CState, CState', \forall In, Out \bullet \text{enable_COp} \wedge \text{effect_COp} \\ \Rightarrow IT \gg (\text{effect_AOp}_1^1 \circ \text{effect_AOp}_2^2) \gg OT.$$

The conditions have the following meaning: R.A states that whenever COp is enabled so is AOp_1 and its precondition is also fulfilled; R.B guarantees that COp does not introduce new divergences; R.C states that after execution of AOp_1 the second operation of the sequence AOp_2 is enabled and defined. Finally R.D guarantees that the effect of executing COp is the same as that of $AOp_1 \circ AOp_2$.

An extension map and the corresponding input and output transformers IT , OT induce a substitution function $f_{IT,OT}^E : N \rightarrow \Sigma^*$. This function has to transform events into sequences of events; the extension map tells us which operation to map onto which sequence of operations and the input and output transformers tell us the values of parameters. Additionally we need the *ordering* among inputs and outputs as defined in the interface declaration of the class. Let COp and AOp_1 , AOp_2 be defined by

$$\begin{aligned} \text{method } COp : [& cin_1? : T_1, \dots, cin_k : T_k, \\ & cout_1 : T_{k+1}, \dots, cout_l : T_{k+l}] \\ \text{method } AOp_1 : [& ain_{1,1}? : T_{1,1}, \dots, ain_{1,k_1} : T_{1,k_1}, \\ & aout_{1,1} : T_{1,k_1+1}, \dots, aout_{1,l_1} : T_{1,k_1+l_1}] \\ \text{method } AOp_2 : [& ain_{2,1}? : T_{2,1}, \dots, ain_{2,k_2} : T_{2,k_2}, \\ & aout_{2,2} : T_{2,k_2+1}, \dots, aout_{2,l_2} : T_{2,k_2+l_2}] \end{aligned}$$

Given this declaration we associate with an input transformer IT two functions IT^i , $i = 1, 2$, mapping input values of COp onto input values of AOp_1 , AOp_2 ,

respectively.

$$\mathcal{IT}^i(i_1, \dots, i_k) = (w_1, \dots, w_{k_i})$$

with $w_j = i_m$, $1 \leq j \leq k_i$, $1 \leq m \leq k$, if $cin_m? = ain_j!$ is an equation in IT . For output transformers OT we have a single function that maps ordered outputs of $AOp_1 \circ AOp_2$ to outputs of COp .

$$\mathcal{OT}(v_{1,1}, \dots, v_{1,l_1}, v_{2,1}, \dots, v_{2,l_2}) = (o_1, \dots, o_l)$$

with $o_j = v_{i,m}$ if $cout_j! = aout_m?$ is an equation in OT . Then $f_{IT,OT}^E$ is defined to be

$$f_{IT,OT}^E(op.i_1 \dots i_k.o_1 \dots o_l) =$$

$$E^1(op).\mathcal{IT}^1(i_1, \dots, i_k).v_1 \dots v_{l_1} \wedge E^2(op).\mathcal{IT}^2(i_1, \dots, i_k).v_{l_1+1} \dots v_{l_1+l_2}$$

such that $\mathcal{OT}(v_1, \dots, v_{l_1}, v_{l_1+1}, \dots, v_{l_1+l_2}) = (o_1, \dots, o_l)$.

Looking at *2Buffer* we find that E defined by $E(put2) = put \wedge put$ is an extension map; for instance $\#contents < n - 1$ implies $\#contents < n$.

The following Lemma is the key to correctness of the extension rule:

Lemma 4.1

Let $A = (AState, AInit, (enable_AOp_j)_{j \in J}, (effect_AOp_j)_{j \in J})$ be the super- and $C = (CState, CInit, (enable_COp_i)_{i \in I}, (effect_COp_i)_{i \in I})$ the subclass. Let $E : N_{op} \rightarrow O_{op}^*$ be an extension map with input and output transformers IT, OT for every new operation. Let $T_C = \llbracket C \rrbracket$ and $T_A = \llbracket A \rrbracket$ be the transition systems. Then the following holds:

$$\begin{aligned} \forall q_1 \in Q_C : q_1 \xrightarrow{ev} q_2 \text{ with } ch(ev) \notin N_{op} \text{ a new operation} \\ \Rightarrow \exists q_3 \in Q_A : q_1 \mid_{AState} \xrightarrow{ev_1} q_3 \xrightarrow{ev_2} q_2 \mid_{AState} \\ \text{such that } f_{IT,OT}^E(ev) = ev_1 \wedge ev_2. \end{aligned}$$

The proof is again omitted. Analogous to the other subtyping relations, we have a simulation relation which induces safe subtyping.

Definition 4.3 Let $f_N : N \rightarrow \Sigma^*$ be a substitution function. Class A simulates C changing N by f_N , $A \preceq_{f_N} C$, if there is a relation $\rho \subseteq Q_A \times Q_C$ such that

$$1 \quad \forall q_C \in Q_C^{in} \exists q_A \in Q_A^{in} : (q_A, q_C) \in \rho,$$

$$2 \quad \forall (q_A, q_C) \in \rho \text{ the following holds:}$$

$$- \text{stable}(q_C) \Rightarrow \text{stable}(q_A), \text{next}(q_C) \setminus N = \text{next}(q_A), q_C \uparrow \Rightarrow q_A \uparrow,$$

$$- \forall a \notin N :$$

$$q_C \xrightarrow{a}_C q'_C \Rightarrow \exists q'_A : q_A \xrightarrow{a}_A q'_A \wedge (q'_A, q'_C) \in \rho.$$

$$\forall a \in N :$$

$$q_C \xrightarrow{a}_C q'_C \Rightarrow \exists q'_A : q_A \xrightarrow{f(a)} q'_A \wedge (q'_A, q'_C) \in \rho.$$

As before we get two results relating simulation with subtyping and patterns with simulation.

Theorem 4 $A \preceq_{f_N} C \Rightarrow A \sqsubseteq_{sst}^{f_N} C$.

If C is a subclass of A obeying the extension rule R then $A \preceq_{f_N} C$.

5. DISCUSSION

In this paper we presented subtyping patterns and rules for active classes with a behavioural semantics. Patterns are simple, mostly syntactic conditions which, when applied during construction of subclasses via inheritance, guarantee correct subtypes. Rules are conditions which can be locally checked on the semantics of operations.

Related work. First of all there is a huge amount of work on behavioural subtyping relations for object-oriented formalisms. They differ in the semantic models used as a basis and in the defined relations. A rough classification could divide them into bisimulation and refinement-based approaches. Bisimulation based approaches easily achieve some form of substitutivity since they use an *equivalence* relation for subtyping. We consider this to be too strong since subtyping in its basic nature is not symmetric. Refinement-based proposals on the other hand in most cases only discuss substitutability but do not give a formal proof. The relation most often used in this context is *extension*, developed for the process algebra LOTOS. Extension allows extension of functionality, but not only on new methods but also on old methods.

The discussion of related work concerning the specific topic of this paper can be split in two parts. The first one concerns the large amount of work on simulation relations for refinement, especially those showing the tight connections between data refinement in state-based and process refinement in behaviour-based systems [11, 18]. The ideas of these papers are clearly the basis for our simulation relations which adapt this basic concept to the case of behavioural subtyping relations. The second area are patterns for subtyping relations. The work of van der Alst and Basten [16] presents transformation rules on Petri nets which construct subtypes, where the subtyping relations are based on branching bisimilarity. Rudkin [14] studies subtyping patterns for LOTOS, i.e. specific LOTOS operators that induce subtypes. The relation chosen there is again extension.

Patterns which are similar to ours are also employed in UNITY's superposition operator [3]. This is not an object-oriented setting, nevertheless the goal is similar: extend a program in a conservative way, not disturbing any of the properties holding beforehand. In fact, the idea of conservative extension appears in many theories and behavioural subtyping is just a form of conservative extension in object-orientation.

REFERENCES

- [1] P. America. Designing an object-oriented programming language with behavioural subtyping. In J.W. de Bakker, W.P. de Roever, and G. Rozenberg, editors, *REX Workshop: Foundations of Object-Oriented Languages*, number 489 in LNCS. Springer, 1991.
- [2] H. Bowman, C. Briscoe-Smith, J. Derrick, and B. Strulo. On behavioural subtyping in LOTOS. In H. Bowman and J. Derrick, editors, *FMOODS '97*, pages 335 – 351. Chapman & Hall, 1997.
- [3] K.M. Chandy and J. Misra. *Parallel program design: a foundation*. Addison Wesley, 1988.
- [4] E. Cusack. Refinement, conformance and inheritance. *Formal Aspects of Computing*, 3:129 – 141, 1991.
- [5] J. Derrick and E. Boiten. Non-atomic refinement in Z. In J. Woodcock and J. Wing, editors, *FM'99*, number 1709 in LNCS, pages 1477–1496. Springer, 1999.
- [6] K. K. Dhara and G. T. Leavens. Forcing behavioral subtyping through specification inheritance. In *ICSE '96*, pages 258–267. IEEE Computer Society Press, March 1996.
- [7] C. Fischer. CSP-OZ: A combination of Object-Z and CSP. In H. Bowman and J. Derrick, editors, *FMOODS '97*, volume 2, pages 423–438. Chapman & Hall, 1997.
- [8] C. Fischer and S. Hallerstede. Data-Refinement in CSP-OZ. Technical Report TRCF-97-3, University of Oldenburg, June 1997.
- [9] C. Fischer and H. Wehrheim. Behavioural subtyping relations for object-oriented formalisms. In T. Rus, editor, *AMAST 2000*, number 1816 in LNCS, pages 469–483. Springer, 2000.
- [10] C. A. R. Hoare. *Communicating Sequential Processes*. Prentice-Hall, 1985.
- [11] M.B. Josephs. A state-based approach to communicating processes. *Distributed Computing*, 3:9–18, 1988.
- [12] B. Liskov and J. Wing. A behavioural notion of subtyping. *ACM Transactions on Programming Languages and Systems*, 16(6):1811 – 1841, 1994.
- [13] O. Nierstrasz. Regular types for active objects. In O. Nierstrasz and D. Tsichritzis, editors, *Object-oriented software composition*, pages 99 – 121. Prentice Hall, 1995.
- [14] S. Rudkin. Inheritance in LOTOS. In K. Parker and G. Rose, editors, *FORTE '92*, pages 409–424. Elsevier, 1992.
- [15] G. Smith. *The Object-Z Specification Language*. Kluwer Academic Publisher, 2000.
- [16] W.M.P. van der Aalst and T. Basten. Life-cycle inheritance - a Petri-net-based approach. In P. Azema and G. Balbo, editors, *ATPN '97*, number 1248 in LNCS, pages 62–81. Springer, 1997.
- [17] P. Wegner and B. Zdonik. Inheritance as an incremental modification mechanism or What like is and isn't like. In *ECOOP'88*, volume 322 of LNCS. Springer, 1988.
- [18] J. C. P. Woodcock and C. C. Morgan. Refinement of state-based concurrent systems. In *VDM Symposium 1990*, volume 428 of LNCS, pages 340–351. Springer-Verlag, 1990.