*Bulletin of Mathematical Biology* (1999) **61**, 0–0
Article No. bulm.1999.0124
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# Erratum*

## Probability Density Functions Useful for Parametrization of Heterogeneity in Growth and Allometry Data

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*Bulletin of Mathematical Biology* (1998) **60**, 1099–1121.

The author of the above paper would like to point out that the following sections should read as follows,

**4.2.** ***Stochastic versions of the Gompertz model.*** The growth models in the form p.d.fs. for size-increment and size-at-age data with Weibull, gamma and log-normal distributed Gompertz parameter $L_\infty$ are presented.

Consider $\Delta l$ as a function of $L_\infty$, then from (18) the inverse function is

$$L_\infty = [(l^1 + \Delta l) l^{1^{(-\exp(-g\Delta t))}}]^{\frac{1}{1-\exp(-g\Delta t)}}, \qquad l^1 \leq L_\infty.$$

Let $L_\infty$ be a positively distributed random variable with Weibull, gamma or log-normal distribution with domain $[l^1, \infty)$. Using calculation (1) with $\xi = \Delta l$, $\psi = L_\infty$ with corresponding Jacobian of the transformation

$$J(\Delta t) = \frac{1}{1 - \exp(-g\Delta t)} \left(\frac{l^1 + y}{l^1}\right)^{\frac{1}{\exp(g\Delta t)-1}}$$

we have

$$f_{\Delta l}(y, \Delta t, g, \eta, \alpha, l^1) = [\eta((x/\alpha)^\eta \exp(-x/\alpha)^\eta)/x] J(\Delta t), \tag{19}$$

where $\eta$ and $\alpha$ are parameters of the Weibull distribution and $x = L_\infty - l^1$. Model (19) is the probability density function of length increments $\Delta l$, where $l^1$ and $\Delta t$ are parameters of the distribution. If $l^1$ is a random variable, then (19) is the p.d.f

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0092-8240/99/050000 + 01 $30.00/0

conditional on size $l$. The following models are constructed using gamma and log-normal p.d.fs.:

$$f_{\Delta l}(y, \Delta t, g, \lambda, \rho, l^1) = [(\lambda x)^\rho \exp(-\lambda x)/(\Gamma(\rho)x)]J(\Delta t), \qquad (20)$$

where $\lambda$ and $\rho$ are parameters of the gamma distribution, $\Gamma$ is gamma function,

$$f_{\Delta l}(y, \Delta t, g, \mu, \sigma, l^1) = [\exp(-(\log(x-\mu)^2/(2\sigma^2))/(\sigma\sqrt{2\pi}x)]J(\Delta t), \qquad (21)$$

where $\mu$ and $\sigma$ are parameters of the log-normal distribution.

To calculate the p.d.fs. for size-at-age I use model (15) with Weibull, gamma or log-normal distributed parameter $L_\infty$. Consider $l$ as a function of $L_\infty$, then from (15) the inverse function is

$$L_\infty = [lL_0^{-\exp(-gt)}]^{\frac{1}{1-\exp(-gt)}} \text{ and } J(t) = \frac{1}{1-\exp(-gt)}\left(\frac{y}{L_0}\right)^{\frac{1}{\exp(gt)-1}}.$$

Using calculation (1) with $\xi = l$ and $\psi = L_\infty$ we have the Gompertz growth models for size-at-age with Weibull, gamma and log-normal distributed parameter $L_\infty$:

$$f_l(y, t, g, L_0, \eta, \alpha),\ f_l(y, t, g, L_0, \lambda, \rho) \text{ and } f_l(y, t, g, L_0, \mu, \sigma). \qquad (22)$$

The models (22) are probability density functions, with age $t$ as a parameter of the distributions. From construction of the models (22) follow that asymptotic distributions ($t = \infty$) of size $l$ are, accordingly, Weibull, gamma and log-normal with domain $[L_0, \infty)$. Biological meaning of the parameter $L_0$ is the size at birth. In some cases, this parameter can be observed. In the general case, models (19), (20) and (21) have three parameters and models (22) have four parameters which should be estimated using likelihood function method. So, the stochastic models have only one more parameter than deterministic models for size-increment and size-at-age data.

*The following lines should read*:

**Page 1101, Line 34;**
$x_1, \ldots, x_k$ and $t$ is time. If $n = k$, and inverse transformation $\Psi = S^{-1}(\xi, t)$ exist.

**Page 1104, Formula (9);**

$$\Delta\omega^j = S^{-1}(\xi^{j-1}, \xi^j) = S^{-1}(\xi^{j-1}, \xi^{j-1} + \Delta\xi^j). \qquad (9)$$

**Page 1109, Subsection 4.3, Lines 2 and 3 under formula (24);**

$$\lim_{t\to\infty} f_l(y, t, \rho, \lambda, t_0, L_\infty) = \delta(y - l_\infty)$$

and therefore $\lim_{t\to\infty} E[l, t] = L_\infty$, $\lim_{t\to\infty} V[l, t] = 0$, where $\delta$ is the ‘delta function’.