Psychophysical methodology: Deductions from the phi-gamma hypothesis and related hypotheses

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The phi-gamma hypothesis is a special case of the general hypothesis of a cumulative symmetrical distribution. Assuming any cumulative symmetrical distribution, with stimuli equally spaced about the axis of symmetry, (a) the descending method of limits (DML) threshold distribution is asymmetrical and is a mirror image of the ascending method of limits (AML) threshold distribution; (b) the combined method of limits (CML) threshold distribution is symmetrical; (c) with the subscripts A, D, and C referring to AML, DML, and CML distributions: $M_A < M_C < M_D$; $\sigma_A = \sigma_D$; $\sigma_C > \sigma_A$; *(d) as step size increases:* M_A *increases,* M_D *decreases,* M_C *remains constant,* σ_A *and* σ_D *increase,* σ_C *first decreases and then increases; (e) the mean threshold of the method of constant stimuli (MCS) equals* M_C*. For a particular assumption of a cumulative symmetrical distribution, statistical measures of the method of limits can be used to estimate MCS statistics. The analyses are supported by data from brightness discrimination experiments.*

The phi-gamma hypothesis and other related hypotheses are commonly used with the Method of Constant Stimuli (MCS). The purpose of this paper is to deduce, from such hypotheses, predictions of the outcome of Method of Limits (ML) experiments.

The model used in deriving the predictions was first invented by Urban (1908). Since Urban's time, the model has been independently reinvented, e.g., by Dixon and Mood (1948), by Herrick (1967), and undoubtedly by others. The model reflects the kind of data obtained in a two-category ("Yes"-"No") MCS experiment. In a word, the model assumes that the probability of a "Yes" response increases with an increase in the stimulus intensity.

PSYCHOPHYSICAL PROBABILITY MODEL

The reader is referred to two earlier publications (Herrick, 1967, 1969) for a full description of the model and for deductions from the model. In this section, only a summary description will be given.

Description of Model

Assume that equally spaced, increasing stimulus values, $S_0, S_1, S_2, \cdots, S_n$, evoke "Yes" responses with increasing probabilities, $p_0, p_1, p_2, \cdots, p_n$, respectively, and "No" responses with decreasing probabilities, $q_0, q_1, q_2, \cdots, q_n$, respectively, where $p_0 + q_0 = p_1 + q_1 = \cdots = p_n + q_n = 1.00$. Also, assume $p_0 = .00$, i.e., the probability of a "Yes" response at S_0 is zero, and $p_n = 1.00$, i.e., the probability of a "Yes" response at S_n is 1.00. The p values will be called MCS p values. (See Table 1.)

Relationships Between MCS p Values and ML Threshold Distributions

In the ascending method of limits (AML), successive stimuli, S_0, S_1, S_2, etc., are presented until a "Yes" response occurs. The probability of the "Yes" occurring at stimulus value S_i is $q_0 q_1 q_2 \cdots q_{i-1} p_i$ (see Table 1).[2] With $p_n = 1.00$, the *proportion* of ascending series terminating at a given stimulus value equals the probability of an ascending series terminating at that value. Thus, of all the ascending series, the proportion terminating at S_4 is $q_0 q_1 q_2 q_3 p_4$. For every ascending series terminating at a given stimulus value, the threshold is midway between that value and the preceding value in the series.[3] Thus, an ascending series that terminates at S_4 yields a threshold of $(S_3 + S_4)/2$ or k_4.

Similar considerations applied to the descending method of limits (DML) lead to the conclusion that the proportion of descending series terminating at, say, S_4 in the Table 1 example, is $p_8 p_7 p_6 p_5 q_4$, and for every descending series that terminates at S_4, the resulting threshold is $(S_4 + S_5)/2$ or k_5.

Combining the AML threshold distribution with the DML threshold distribution gives the threshold distribution of the combined method of limits (CML).

Summary Statistical Measures of ML Distributions

The mean threshold of the AML threshold distribution is:

$$M_A = k_0 + c(1 + q_1 + q_1 q_2 + q_1 q_2 q_3 + \cdots + q_1 q_2 \cdots q_{n-1}), \quad (1)$$

where c is the step size, i.e., $c = (S_n - S_{n-1}) = \cdots = (S_2 - S_1) = (S_1 - S_0)$, and $k_0 = (S_0 - c/2)$.

The mean threshold of the DML threshold distribution is:

Table 1
Derivation of Method of Limits Thresholds from Method of Constant Stimuli (MCS) p Values on Assumption that MCS Distribution is a Cumulative Symmetrical Distribution

Stimulus Value	Method of Constant Stimuli: Proportion of Responses "Yes"	"No"	Ascending Method of Limits: Probability of series terminating at given stimulus value		Descending Method of Limits: Probability of series terminating at given stimulus value	
S_0	$p_0 = .00$	q_0	p_0	$= p_0$	$p_8 p_7 \cdots p_1 q_0$	$= p_8 p_7 \cdots p_1 (p_8)$
S_1	p_1	q_1	$q_0 p_1$	$= p_8 (p_1)$	$p_8 p_7 \cdots p_2 q_1$	$= p_8 p_7 \cdots p_2 (p_7)$
S_2	p_2	q_2	$q_0 q_1 p_2$	$= p_8 p_7 (p_2)$	$p_8 p_7 \cdots p_3 q_2$	$= p_8 p_7 \cdots p_3 (p_6)$
S_3	p_3	q_3	$q_0 q_1 q_2 p_3$	$= p_8 p_7 p_6 (p_3)$	$p_8 p_7 \cdots p_4 q_3$	$= p_8 p_7 \cdots p_4 (p_5)$
S_4	p_4	q_4	$q_0 q_1 q_2 q_3 p_4$	$= p_8 p_7 p_6 p_5 (p_4)$	$p_8 p_7 p_6 p_5 q_4$	$= p_8 p_7 \cdots p_5 (p_4)$
S_5	p_5	q_5	$q_0 q_1 \cdots q_4 p_5$	$= p_8 p_7 \cdots p_4 (p_5)$	$p_8 p_7 p_6 q_5$	$= p_8 p_7 p_6 (p_3)$
S_6	p_6	q_6	$q_0 q_1 \cdots q_5 p_6$	$= p_8 p_7 \cdots p_3 (p_6)$	$p_8 p_7 q_6$	$= p_8 p_7 (p_2)$
S_7	p_7	q_7	$q_0 q_1 \cdots q_6 p_7$	$= p_8 p_7 \cdots p_2 (p_7)$	$p_8 q_7$	$= p_8 (p_1)$
S_8	$p_8 = 1.00$	q_8	$q_0 q_1 \cdots q_7 p_8$	$= p_8 p_7 \cdots p_1 (p_8)$	q_8	$= p_0$

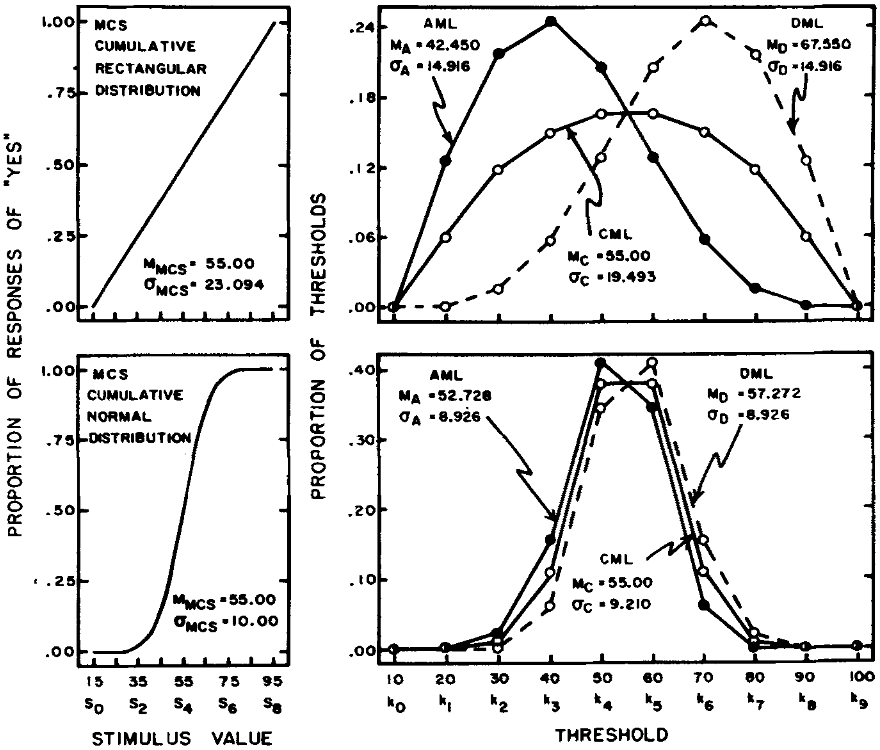

Fig. 1. Derivation of ascending method of limits (AML), descending method of limits (DML), and combined method of limits (CML) threshold distributions from p values representing the quantal hypothesis (upper) and the phi-gamma hypothesis (lower). Hypothetical data.

$$M_D = k_0 + c(n - p_{n-1} - p_{n-1}p_{n-2} - p_{n-1}p_{n-2}p_{n-3} - \cdots - p_{n-1}p_{n-2}p_{n-3}\cdots p_2p_1). \quad (2)$$

The mean threshold for the CML threshold distribution, when the number of thresholds of the AML distribution, namely, N_A, equals the number of thresholds of the DML distribution, namely, N_D, is:

$$M_C = (M_A + M_D)/2. \quad (3)$$

No matter what set of MCS p values are assumed,

$$M_D > M_C > M_A. \quad (4)$$

The standard deviation of the AML threshold distribution is:

$$\sigma_A = \left\{c^2[1 + 3q_1 + 5q_1q_2 + 7q_1q_2q_3 + \cdots + (2n-1)q_1q_2\cdots q_{n-1}] - (M_A - k_0)^2\right\}^{1/2}. \quad (5)$$

The standard deviation of the DML threshold distribution is:

$$\sigma_D = \left\{c^2[n^2 - 3p_{n-1}p_{n-2}\cdots p_1 - 5p_{n-1}p_{n-2}\cdots p_2 - 7p_{n-1}p_{n-2}\cdots p_3 - \cdots - (2n-1)p_{n-1}] - (M_D - k_0)^2\right\}^{1/2}. \quad (6)$$

When $N_A = N_D$, the standard deviation for the CML threshold distribution is:

$$\sigma_C = \left\{[(M_A - M_D)/2]^2 + (\sigma_A{}^2 + \sigma_D{}^2)/2\right\}^{1/2} \quad (7)$$

CUMULATIVE SYMMETRICAL DISTRIBUTIONS

Definition of Cumulative Symmetrical Distribution

Consider a distribution, $y = f(x)$. If, with respect to some x value, x_0, $f(x_0 - \Delta x) = f(x_0 + \Delta x)$ for any value of Δx, the equation is an equation of a symmetrical distribution. When the ordinates of such a distribution are cumulated, the resulting distribution is a cumulative symmetrical distribution. The left half of Fig. 1 illustrates two cumulative symmetrical distributions, each symmetrical with respect to the stimulus value, 55.

If the ordinate of a cumulative symmetrical distribution at $(x_0 - \Delta x)$ is p_i, the ordinate at $(x_0 + \Delta x)$ is $(1 - p_i)$ or q_i. Now, with respect to the probability model described above, if stimulus values for the ML are selected with respect to the axis of symmetry, $p_0 = q_n$, $p_1 = q_{n-1}$, $p_2 = q_{n-2}$, ···, $p_{n-1} = q_1$, $p_n = q_0$.

Summary Statistics of Cumulative Symmetrical Distribution

With MCS p values selected with respect to the axis of symmetry, the middlemost stimulus must have an associated $p = 0.50$. In the left half of Fig. 1, for example, the middlemost stimulus is S_4, so p_4 must equal 0.50. With an even number of stimuli, the two middlemost stimuli must have associated p values that fall equally above and below 0.50. Since the stimulus value associated with $p = 0.50$ is the mean (or median) threshold of the MCS, the mean (or median) threshold of the MCS is always midway between S_0 and S_n. That is:

$$M_{MCS} = (S_0 + S_n)/2. \quad (8)$$

The variability of a cumulative symmetrical distribution may be specified by the standard deviation of the distribution. The symbol for this measure will be σ_{MCS}.

Assumptions of Cumulative Symmetrical Distributions in Psychophysics

Two assumptions commonly made in psychophysics are (a) the phi-gamma hypothesis, which states that, as a function of increasing stimulus intensity, the proportions of "Yes" responses, i.e., the MCS p values, are described by a cumulative normal distribution, and (b) the quantal hypothesis, which states that the p values form a cumulative rectangular distribution (Guilford, 1954). Both of these hypotheses may be subsumed under the hypothesis that the p values form a cumulative symmetrical distribution. Thus, any deductions from the probability model that apply to a cumulative symmetrical distribution will apply to the phi-gamma hypothesis, to the quantal hypothesis, and to any other hypothesis that assumes a cumulative symmetrical distribution.

DEDUCTIONS BASED ON THE ASSUMPTION THAT THE MCS p VALUES FORM A CUMULATIVE SYMMETRICAL DISTRIBUTION

Shapes of ML Distributions

In Table 1, the proportion of ascending series terminating at S_3 is $q_0 q_1 q_2 p_3$. In a cumulative symmetrical distribution, with nine stimuli, $q_0 = p_8$, $q_1 = p_7$, $q_2 = p_6$, etc. Therefore, $q_0 q_1 q_2 p_3 = p_8 p_7 p_6 p_3$, as indicated in Table 1. Similarly, the proportion of descending series terminating at each stimulus value may be expressed in terms containing only p values, as in Table 1. A comparison of the proportions of AML series terminating at the different stimulus values with the proportions of DML series terminating at the different stimulus values indicates correspondence. For the Table 1 data, the proportions of ascending series terminating at S_0, S_1, S_2, ···, S_8 equal the proportions of descending series terminating at S_8, S_7, S_6, ···, S_0, respectively. Thus, the proportions of AML *thresholds* of values k_1, k_2, k_3, ···, k_8 equal the proportions of DML thresholds of values k_8, k_7, k_6, ···, k_1, respectively. In short, *the DML threshold distribution is a mirror image of the AML threshold distribution.*

To be symmetrical, an AML threshold distribution must have equal ordinate values at points equally distant from an axis of symmetry. For example, with respect to Table 1, the proportion of AML thresholds of value k_1 must equal the proportion of AML thresholds of value k_8. Now, the proportion of AML thresholds of value k_1 is $q_0 p_1 = (1.00)\ p_1 = p_1$, and the proportion of AML thresholds of value k_8 is $p_1 p_2 p_3 p_4 p_5 p_6 p_7 (p_8)^2$. Thus, the proportions at k_1 and k_8 are unequal. Therefore, *an AML threshold distribution is asymmetrical.* Since a DML distribution is a mirror image of an AML distribution, it too must be asymmetrical.

The above analysis indicates that the assumption that MCS p values form a cumulative symmetrical distribution is incompatible with the assumption of normality, or of any other form of symmetry, for the AML or DML threshold distributions. Conversely, if either the AML or the DML threshold distribution is symmetrical, MCS p values cannot form points on a cumulative symmetrical distribution.[4]

By pooling the AML and DML distributions, the threshold distribution of the CML is obtained. For the CML threshold distribution, the proportion of thresholds of value k_1 (see Table 1) is $(q_0 p_1 + p_8 p_7 \cdots p_1 q_0)/2$. The proportion of thresholds of value k_8 is the same. Similarly, the proportion of thresholds of value k_2 equals the proportion of thresholds of value k_7, etc. Thus, the *CML threshold distribution is symmetrical.*

If a CML distribution, expressed in proportions of thresholds, is cumulated, is the result identical with the underlying cumulative symmetrical distribution of p values? For example, if the CML distribution of Fig. 1, with $M_C = 55.00$ and $\sigma_C = 9.210$, is cumulated, is the plot the same as the plot of the MCS p values

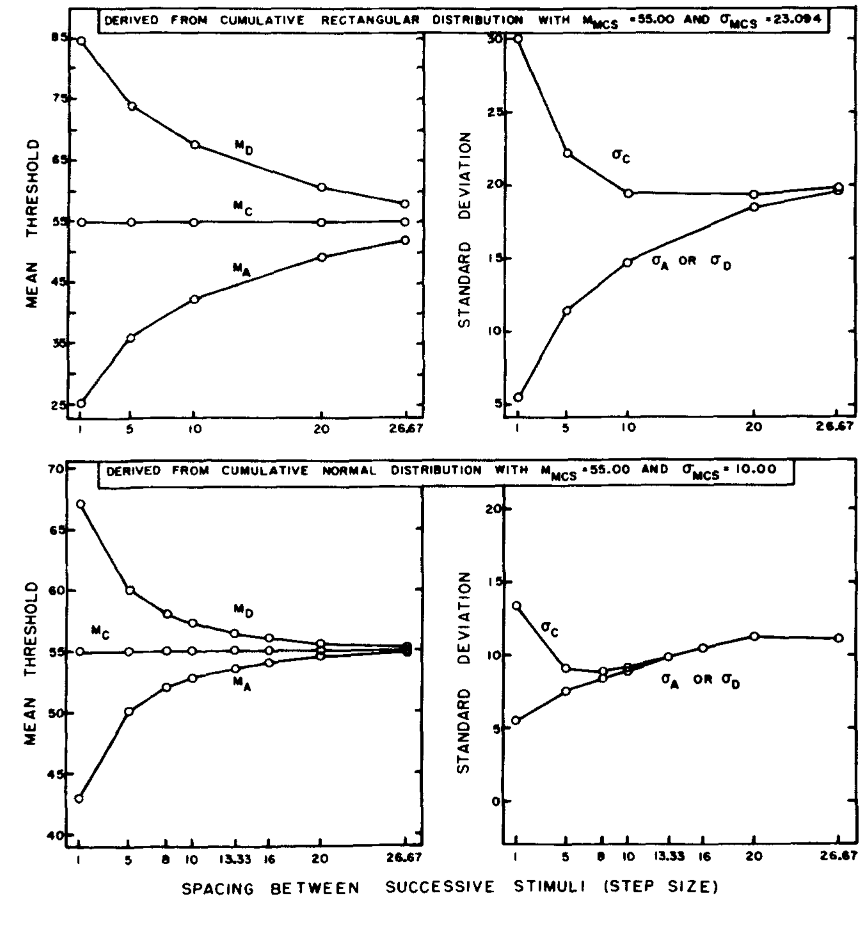

Fig. 2. Means and standard deviations of method of limits threshold distributions as a function of step size. Subscripts refer to ascending method of limits (A), descending method of limits (D), and combined method of limits (C).

with $M_{MCS} = 55.00$ and $\sigma_{MCS} = 10.00$? Obviously not, in the Fig. 1 example. Moreover, analyses given elsewhere (Herrick, 1968) indicate that, in general, a cumulated CML distribution is not equivalent to the associated MCS p values. Siegel (1962) assumed such equivalence in his comparison of psychophysical methods.

Mean Thresholds of ML Distributions

Equation 1 gave a description of M_A for any set of MCS p values. Equation 1 may also be written as

$$M_A = S_0 + cz, \quad (9)$$

where $z = .5 + q_1 + q_1 q_2 + q_1 q_2 q_3 + \cdots + q_1 q_2 \cdots q_{n-1}$. Similarly,

$$M_D = S_n - cz. \quad (10)$$

Substituting, in Eq. 3, the values for M_A and M_D given in Eqs. 9 and 10:

$$M_C = (S_0 + cz + S_n - cz)/2 = (S_0 + S_n)/2. \quad (11)$$

Equation 11 indicates that the phi-gamma hypothesis, the quantal hypothesis, or *any hypothesis of a cumulative symmetrical distribution predicts the same M_C.*

Subtracting Eq. 9 from Eq. 10 gives the amount by which M_D is greater than M_A:

$$M_D - M_A = c(n - 2z) = \text{Range} - 2cz. \quad (12)$$

Standard Deviations of ML Distributions

When distributions are mirror images, their standard deviations are equal.[5] Therefore, $\sigma_A = \sigma_D$.

Since $\sigma_A = \sigma_D$, σ_A may be substituted for σ_D in Eq. 7. When the terms are rearranged:

$$\sigma_C^2 - \sigma_A^2 = [(M_A - M_D)/2]^2. \quad (13)$$

Influence of Step Size on ML Mean Thresholds and Standard Deviations

In the examples of Fig. 1, the spacing between successive stimuli is 10 units, i.e., c = 10. The derivation of ML distributions from MCS p values was also carried out for other step sizes for the cumulative rectangular and cumulative normal distributions of Fig. 1. In all cases, $S_0 = 15$. The largest step size used, 26.667, was obtained by dividing the range by 3, i.e., $(95 - 15) \div 3 = 26.667$. This gives $S_0 = 15$, $S_1 = 41.667$, $S_2 = 68.333$, $S_4 = 95$. If the range were divided into larger steps, e.g., by dividing the range by 2, the result would be the trivial case of only three stimuli, 15, 55, and 95, with associated p values of .00, .50, and 1.00. (Note that when stimuli are selected with respect to the axis of symmetry, c can only equal the values obtained by dividing the range by integers.)

In general, the number of stimulus values covering the range from S_0 through S_n, may be computed by:

$$\text{Number of stimulus values} = [(S_n - S_0)/c] + 1 = (\text{Range}/c) + 1. \quad (14)$$

The p values used with the cumulative rectangular distribution were derived by calculation. The p values used with the cumulative normal distribution were obtained from a normal curve table.

For the ML distributions derived with the different step sizes, the means and standard deviations computed are plotted in Fig. 2.

ESTIMATION OF M_{MCS} AND σ_{MCS} FROM ML MEASURES ON THE ASSUMPTION OF A CUMULATIVE NORMAL DISTRIBUTION

In this section, relationships concerning only the phi-gamma hypothesis will be considered.

Estimation of M_{MCS}

Dividing a ML term by σ_{MCS} expresses the ML term in units of σ_{MCS}. In what follows, when a ML term is expressed in units of σ_{MCS}, the term will carry a prime. For example, $(c/\sigma_{MCS}) = c'$; $(M_A/\sigma_{MCS}) = M'_A$.

A particular stimulus value may be expressed in terms of M_{MCS} and σ_{MCS}. For example, for the cumulative normal distribution of Fig. 1, the stimulus value S_0 may be expressed as $M_{MCS} - 4\sigma_{MCS}$.

With the above points in mind, consider Eq. 9. When $(M_{MCS} - 4\sigma_{MCS})$ is substituted for S_0, and each term is divided by σ_{MCS}:

$$M'_A = (M'_{MCS} - 4) + c'z. \quad (15)$$

Now, for the assumption of a cumulative normal distribution, the relationship between z and c′, for c′ values between 0.5 and 2.0, is approximately described by:[6]

$$z = .181 + (3.592/c'). \quad (16)$$

Substituting this value for z in Eq. 15, and multiplying by σ_{MCS} gives:

$$M_A = M_{MCS} - .408\,\sigma_{MCS} + .181\,c. \quad (17)$$

In the following section, Eq. 21 relates σ_{MCS}, σ_A, and c. Substituting in Eq. 17 the value of σ_{MCS} given in Eq. 21, and solving for M_{MCS},

$$M_{MCS} = M_A + .637\,\sigma_A - .337\,c. \quad (18)$$

Similarly,

$$M_{MCS} = M_D - .637\,\sigma_D + .337\,c. \quad (19)$$

The value of M_{MCS} may also be estimated by M_C, for $M_{MCS} = (S_n - S_0)/2 = M_C$.

Estimation of σ_{MCS}

For the assumption of a cumulative normal distribution, for c′ values between 0.5 and 2.0, the approximate relationship[7] between c′ and σ'_A is:

$$\sigma'_A = .641 + .245\,c'. \quad (20)$$

Multiplying Eq. 20 by σ_{MCS} and solving for σ_{MCS}:

$$\sigma_{MCS} = 1.56\,\sigma_A - .382\,c. \quad (21)$$

Also, since $\sigma_A = \sigma_D$,

$$\sigma_{MCS} = 1.56\,\sigma_D - .382\,c. \quad (22)$$

Substituting in Eq. 13 the values for M_A, M_D, and σ_A given above, and solving for σ_{MCS}, gives an estimate of σ_{MCS} in terms of σ_C and c:

$$\sigma_{MCS} = (-.144\,c) + (2.31\,\sigma_C^2 - .187\,c^2)^{1/2}/1.155. \quad (23)$$

Comment

It should be noted that the differences between ML and MCS

Table 2
Summary of Experimental Procedures

Observer	Session	Procedure	Log ΔI Luminances (mL) Used
		Experiment 1	
A.L.	A.M.	AML	−.37, −.34, −.31, −.28, etc.
		DML	−.19, −.22, −.25, −.28, etc.
	P.M.	AML	−.34, −.31, −.28, −.25, etc.
		DML	−.16, −.19, −.22, −.25, etc.
J.D.	A.M.	AML	−.58, −.52, −.46, −.40, etc.
		DML	−.22, −.28, −.34, −.40, etc.
	P.M.	AML	−.76, −.70, −.64, −.58, etc.
		DML	−.04, −.10, −.16, −.22, etc.
		Experiment 2	
A.L.	A.M.	DML (large step)	−.04, −.10, −.16, −.22, etc.
		DML (small step)	−.04, −.055, −.07, −.085, etc.
	P.M.	DML (large step)	−.10, −.16, −.22, −.28, etc.
		DML (small step)	−.10, −.115, −.13, −.145, etc.
J.D.	A.M.	DML (large step)	−.04, −.16, −.28, −.40, etc.
		DML (small step)	−.04, −.07, −.10, −.13, etc.
	P.M.	DML (large step)	−.16, −.28, −.40, −.52, etc.
		DML (small step)	−.16, −.19, −.22, −.25, etc.
		Experiment 3	
A.L.	A.M. &	MCS	−.34, −.28, −.22, and −.16
	P.M.	DML	−.16, −.175, −.19, −.205, etc.
J.D.	A.M. &	MCS	−.52, −.40, −.28, and −.16
	P.M.	DML	−.16, −.19, −.22, −.25, etc.

statistical measures are quite small. For example, stating the differences in terms of σ_{MCS}, when $c' = 1.0$: $(M_{MCS} - M_A) < (.25\ \sigma_{MCS})$, $(\sigma_{MCS} - \sigma_A) < (.11\ \sigma_{MCS})$, and $(\sigma_{MCS} - \sigma_C) < (.08\ \sigma_{MCS})$. When $c' = 1.6$: $(M_{MCS} - M_A) < (.11\ \sigma_{MCS})$, $(\sigma_A - \sigma_{MCS}) < (.05\ \sigma_{MCS})$, and $(\sigma_C - \sigma_{MCS}) < (.05\ \sigma_{MCS})$.

EXPERIMENTAL EVALUATIONS

Practical Problems

Although the above analyses provide many predictions, experimental evaluation is difficult because of practical considerations. First, since only a limited number of judgments may be obtained in an experimental session, the resulting sample statistics will be somewhat unreliable. Second, a predicted difference is often quite small. Third, in selecting stimuli for a ML experiment, the axis of symmetry for the cumulative symmetrical distribution is unknown. Thus, the stimuli will not be spaced with respect to the axis of symmetry, and, consequently, the predictions given above will not hold exactly. Fourth, in an attempt to obtain many thresholds for a given number of judgments, the initial stimulus in a ML series may be selected within the appropriate range of stimuli, rather than at the extreme of the range. For example, the initial stimulus for a descending series may be one with an associated p value of, say, 0.92, rather than 1.00. Such selections would yield differences between statistical measures that were less than the differences predicted by the analyses. Keeping these problems in mind, we turn now to some brightness discrimination experiments designed to evaluate some of the gross predictions of the analyses.

Procedure

The experimental situation was as follows. Monocularly, an O centrally fixated a circular adapting field of white light, 1 deg 7 min in diam, at an apparent distance of 570 mm, with a luminance of 11.5 mL. On command, a 20-msec flash (the ΔI light) was added to the whole adapting field.

Each session consisted of 260 judgments: 120 with one procedure, 120 with another, and 20 randomly inserted "catch tests." Table 2 summarizes the procedures. In Experiment 1, the ascending and descending series were alternated randomly. In Experiment 2, 60 judgments were made with the large-step size, then 120 with the small-step size, then 60 with the large-step size, in the morning session; the order was reversed in the afternoon session. In Experiment 3, 60 judgments with the DML were followed by 120 judgments with the MCS and then by 60 judgments with the DML in the a.m. session; the order was reversed in the p.m. session.

Table 3
Experiment 1. Results of Brightness Discrimination Experiments Using the AML and the DML. [Predictions from Analyses: $M_A < M_C$; $M_C < M_D$; $\sigma_A = \sigma_D$; $\sigma_C > \sigma_A$; $\sigma_C > \sigma_D$.]

Observer	Session	Statistical Measure in Log mL M_A	M_D	M_C	σ_A	σ_D	σ_C
A.L.	A.M.	−.278	−.255	−.267	.044	.038	.042
	P.M.	−.279	−.216	−.225	.043	.038	.051
J.D.	A.M.	−.474	−.402	−.443	.073	.064	.077
	P.M.	−.526	−.268	−.399	.114	.152	.186

Table 4
Experiment 2. Results of Brightness Discrimination Experiments Using Two Step Sizes with the Descending Method of Limits. [Predictions from Analyses: M_D (large step) < M_D (small step); σ_D (large step) > σ_D (small step).]

		Statistical Measure in Log mL			
Observer	Session	M_D (large step)	M_D (small step)	σ_D (large step)	σ_D (small step)
A.L.	A.M.	−.200	−.168	.060	.033
	P.M.	−.217	−.156	.047	.031
J.D.	A.M.	−.379	−.344	.093	.093
	P.M.	−.359	−.240	.129	.076

Table 5
Experiment 3. Results of Brightness Discrimination Experiments Using the Method of Constant Stimuli and the Descending Method of Limits. [Predictions from Analyses: $M_{MCS} < M_D$; $\sigma_{MCS} > \sigma_D$ when c is small.]

		Statistical Measure in Log mL			
Observer	Session	M_{MCS}	M_D	σ_{MCS}	σ_D
A.L.	A.M.	−.281	−.207	.060	.027
	P.M.	−.251	−.192	.087	.021
J.D.	A.M.	−.347	−.298	.098	.093
	P.M.	−.328	−.223	.164	.059

Results

The results of the experiments, along with predictions appropriate to each experiment, are presented in Tables 3, 4, and 5. Except for the MCS part of Experiment 3, the predictions are based on the assumption that the p values form an unspecified cumulative symmetrical distribution. The means and standard deviations for the ML procedures were computed in the usual way. In Experiment 3, M_{MCS} and σ_{MCS} were estimated from a straight line fitted, by the method of least squares, to a plot of log ΔIs vs proportion of "Yes" responses (a MCS plot) plotted on a cumulative normal scale. In all "catch tests," the Os responded correctly.

In Experiment 1, excluding the prediction $\sigma_A = \sigma_D$, 15 of the 16 evaluations support the predictions. In Experiment 2, 7 of the 8 evaluations support the predictions. In Experiment 3, all 8 evaluations support the predictions. In short, in spite of all the practical limitations, the data support the predictive value of the analyses.

If the cumulative normal distribution is assumed to be the underlying distribution, Eqs. 18, 19, 21, 22, and 23 may be used to estimate M_{MCS} and σ_{MCS} from the data given in Tables 3, 4, and 5. For example, for O J.D., for the morning session of Experiment 2 (see Tables 2 and 4) for the large-step size: c = .12, $M_D = -.379$, and $\sigma_D = .093$. Applying Eq. 19:

$$M_{MCS} = -.379 - .637(.093) + .337(.12) = -.398.$$

Also, using Eq. 22:

$$\sigma_{MCS} = 1.56(.093) - .382(.12) = .099.$$

An alternative method[8] for estimating M_{MCS} and σ_{MCS} is to treat the ML data like MCS data, i.e., calculate the proportion of "Yes" responses at each stimulus value, plot the proportion of "Yes" responses vs stimulus values (a MCS plot), and estimate M_{MCS} and σ_{MCS} from the plot. This procedure was followed for the example just discussed. The data points, plotted on a cumulative normal scale, were fitted, by the method of least squares, with a straight line. The estimates obtained were: $M_{MCS} = -.420$, and $\sigma_{MCS} = .132$.

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NOTES

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2. Some discussion of nonprobability influences may be found in an earlier publication (Herrick, 1969).

3. Other definitions of the threshold have been considered elsewhere (Herrick, 1969).

4. The points discussed in this paragraph are considered more fully elsewhere (Herrick, 1968).

5. Following the ideas presented in the first paper of this series (Herrick, 1967), Pollack (1968) performed Monte Carlo simulations of various psychophysical procedures. The results of his simulations differ slightly from the results obtained with Eqs. 9-13. The discrepancies undoubtedly result from some minor errors in the computer simulation process, noted by Pollack.

6. Over a wider range, from $c' = 0.10$ to $c' = 2.667$, the relationship may be described by the more complex equation, $z = -.28 + 4.04(c')^{-.84}$.

7. Over the range from $c' = 0.10$ to $c' = 2.667$, the equation, $\sigma'_A = .92(c')^{.22}$, gives a good approximation.

8. This method is discussed and illustrated in an earlier paper (Herrick, 1969).

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