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Notes and Comment

Tables of d′ for the triangular method and the 3-AFC signal detection procedure

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The triangular method (e.g., Amerine, Pangborn, & Roessler, 1965; Peryam, 1958) was developed in the field of sensory evaluation of food (Helm & Trolle, 1946), while the 3-AFC signal detection task is a special case of the family of m-AFC procedures originating from visual and auditory detection and discrimination research (Green & Swets, 1966). One common feature of the two procedures is the use of sets of three stimuli, from each of which the subject has to select one stimulus under forced-choice instructions. Only one of the three alternative responses should be given. Another communality is that no stimuli are designated as standards. However, there are two essential differences between the two forced-choice tasks.

The first difference is related to the types of tri-stimulus sets ("triangles") to be used in one experiment. When the triangular method is applied, triangles are composed from the two types of similar stimuli, A and B, such that all of the six temporal or spatial permutations AAB, ABA, BAA, BBA, BAB, and ABB are used and presented in random and balanced order. In contrast, in the 3-AFC procedure, only three permutations of one combination are used in the same experiment; *either* the set of triangles AAB, ABA, and BAA are used *or* the triangles BBA, BAB, and ABB are presented in a balanced and random order.

The second difference concerns the instructions given. In the triangular method, the subject is instructed to select the odd stimulus, without specification of the nature of oddity. In the 3-AFC procedure, the subject is instructed to select *either* the strongest stimulus (or the stimulus containing the signal) *or* the weakest stimulus (or blank) from each triangle presented.

For valid application of the 3-AFC procedure, prior evidence is required as to whether the stimuli A are sensorily more intense than the stimuli B in order to be able to specify a criterion for stimulus selection compatible with the set of triangle types to be used. In the triangular method, such information is not required, since the instruction to select the odd stimulus is undirected. Thus, as more information is needed for a valid application of the 3-AFC procedure, it can be concluded that its use is more restricted than that of the triangular method.

OTHER THREE-STIMULUS PARADIGMS

The triangular method should not be confused with one of a number of other three-stimulus procedures. These are: the ABX paradigm (Pastore, Friedman, & Buffato, 1976; Pierce & Gilbert, 1958), well known in speech research (Macmillan, Kaplan, & Creelman, 1977); the duo-trio procedure (Amerine et al., 1965; Bradley, 1963; Peryam, 1958), another method often applied in gustatory and olfactory psychophysics; and the method of triads (Richardson, 1938; cf. Torgerson, 1958), extensively used in the study of concept formation and for the scaling of multidimensional stimuli.

In the ABX paradigm, the stimuli A and B are both designated as standards and the subject is requested to state whether the third stimulus X is identical to A or to B. In the duo-trio method (which actually is an AX_1X_2 paradigm), the subject is presented first with the standard A, and then is requested to state which stimulus of the pair X_1 and X_2 is different from A. In the method of triads, the subject normally is requested to give two responses, i.e., to select the two stimuli of a triad which are most similar, and also to select the pair of most dissimilar stimuli. In contrast to all previous procedures, the responses obtained are not evaluated in terms of correct and incorrect; instead, proportions of the responses are directly converted into psychological distances.

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THE THURSTONE-URA AND 3-AFC MODELS

Recently, Frijters (1979) pointed out that one particular type of unidimensional probabilistic model for the triangular method is closely related to such a model for the 3-AFC procedure; both have d′ in common. This model was called the Thurstone-Ura model, since it was first developed by Ura (1960), who used Thurstone's conception of d′ as a parameter for psychological distance. However, its complete derivation was published later by David and Trivedi (Note 1). [Some more easily available references dealing with the Thurstone-Ura model are Bradley (1963) and David (1969).] In order to comprehend the basis of the present table of d′, the difference between the Thurstone-Ura and 3-AFC models is briefly explained.

Consider the triangle composed of the stimuli A, A′, and B, where A and A′ are physicochemically identical and A and B differ with respect to only one sensory attribute, x. The stimuli A and B give rise to the sensory distributions $f(x_a)$ and $f(x_b)$, respectively, with means μ_a and μ_b, and equal variances σ^2. Let $\mu_a < \mu_b$, and let $d' = (\mu_b - \mu_a)/\sigma$. When the stimuli A, A′, and B are presented in a particular trial, they give rise to the corresponding momentary sensory values x_a, x_a', and x_b. As a result of the triangular instructions to select the odd stimulus, it is assumed that a correct response will be produced if:

$$|x_a - x_a'| < |x_b - x_a| \; \textit{and} \; |x_a - x_a'| < |x_b - x_a'|$$

Alternatively, under the appropriate 3-AFC instruction to select the strongest stimulus with respect to the attribute x, a correct response is assumed to be obtained if:

$$x_b > x_a \; \textit{and} \; x_b > x_a'.$$

Thus, in the triangular method, differences between *distances* are compared, while, in the 3-AFC procedure, differences between *absolute* momentary sensory values are the basis for response selection.

Additional assumptions of both models are that the subject has no response preferences with respect to the temporal or spatial intratriangle positions of the stimuli, and that no such bias results from processes as sensory adaptation and short-term memory.

Depending on the assumed type of distributions, $f(x_a)$ and $f(x_b)$, a normal and logistic model for the triangular method and the 3-AFC procedure can be specified. These models result in functions relating the probability of a correct response (P) to the unidimensional sensory distance d′ between the triangle stimuli. Frijters (1979) found great differences between the functions resulting from the two procedures, but not between the normal and logistic variants of the same model. Since these four functions have been extensively discussed in the Frijters paper, here only the expressions are given.

Normal model for the triangular method:

$$P_\Delta = 2\int_0^\infty \left[\Phi\left(-u\sqrt{3} + d'\sqrt{\tfrac{2}{3}}\right) + \Phi\left(-u\sqrt{3} - d'\sqrt{\tfrac{2}{3}}\right)\right] \frac{e^{-\frac{1}{2}u^2}}{\sqrt{2\pi}} du. \tag{1}$$

Logistic model for the triangular method:

$$P_{\Delta l} = -1 + 2\gamma\left[\frac{\gamma^2 - 2\gamma \ln\gamma - 1}{(\gamma-1)^3}\right] - 2\gamma\int_0^\infty \frac{1}{(1+\gamma u)^2}\left[\frac{u}{(u+1)^2}\ln\frac{u}{(u+1)^2} - \frac{u(u-1)}{(u+1)^2} - \frac{(u-1)\sqrt{u}}{(u+1)^2} \times (\pi/2 - 2\arctan\sqrt{u})\right] du. \tag{2}$$

Normal model for 3-AFC procedure:

$$\tilde{P}_\Delta = \int_0^\infty [\Phi^2(u+d') + \Phi^2(-u+d')]\frac{e^{-\frac{1}{2}u^2}}{\sqrt{2\pi}} du. \tag{3}$$

Logistic model for 3-AFC procedure

$$\tilde{P}_{\Delta l} = \gamma\left[\frac{\gamma^2 - 2\gamma\ln\gamma - 1}{(\gamma-1)^3}\right]. \tag{4}$$

In Expressions 1 and 3, $\Phi(u)$ is the cumulative distribution of the standard normal distribution. In Expressions 2 and 4, $\gamma = \exp[d'(\pi/\sqrt{3})]$.

TABLES OF d′

Both Ura (1960) and David and Trivedi (Note 1) have produced tables for the normal Thurstone-Ura model containing values of d′ = 0(.2)5. Bradley (1963) produced another table of d′ resulting from his own triangular method model.

To facilitate experimental comparison of the triangular method and the 3-AFC procedure, the equations for all four conditions have been solved numerically. Table 1 contains the values of d′ for the probabilities ranging between .3333 and .99.

Thus, by taking an experimentally obtained proportion of correct responses as an estimate of the probability of a correct response, Table 1 can be used to obtain the corresponding value of d′ using whichever of the models is appropriate.

Tables for a number of other m-AFC procedures (m = 2, 4, 8, 16, 32, 256, and 1,000) (Elliot, 1964)

Table 1
Values of d′ Corresponding to the Probabilities of a Correct Response (P) Ranging Between .3333 and .99

Proportion Correct Responses	Triangular Normal	Triangular Logistic	3-AFC Normal	3-AFC Logistic
.3333	.00	.00	.00	.00
.34	.27	.25	.02	.02
.35	.43	.39	.06	.05
36	.55	.50	.09	.09
.37	.64	.59	.13	.12
.38	.73	.67	.16	.15
.39	.81	.74	.20	.18
.40	.88	.81	.23	.21
.41	.95	.87	.26	.24
.42	1.01	.93	.30	.28
.43	1.07	.99	.33	.31
.44	1.13	1.05	.36	.34
.45	1.19	1.10	.39	.37
.46	1.25	1.16	.43	.40
.47	1.31	1.21	.46	.43
.48	1.36	1.26	.49	.46
.49	1.41	1.31	.52	.49
.50	1.47	1.36	.56	.52
.51	1.52	1.41	.59	.55
.52	1.57	1.46	.62	.58
53	1.62	1.51	.65	.61
.54	1.67	1.56	.69	.64
.55	1.72	1.61	.72	.67
.56	1.77	1.66	.75	.70
.57	1.82	1.71	.79	.74
.58	1.87	1.76	.82	.77
.59	1.92	1.80	.85	.80
.60	1.98	1.85	.89	.83
.61	2.03	1.90	.92	.86
.62	2.08	1.95	.95	.89
.63	2.13	2.01	.99	.93
.64	2.18	2.06	1.02	.96
.65	2.23	2.11	1.06	.99
.66	2.29	2.16	1.09	1.03
.67	2.34	2.21	1.13	1.06
.68	2.39	2.27	1.16	1.10
.69	2.45	2.32	1.20	1.13
.70	2.50	2.38	1.24	1.17
.71	2.56	2.44	1.28	1.21
.72	2.62	2.49	1.31	1.25
.73	2.68	2.55	1.35	1.28
.74	2.74	2.62	1.39	1.32
.75	2.80	2.68	1.43	1.36
.76	2.86	2.74	1.48	1.40
.77	2.92	2.81	1.52	1.45
.78	2.99	2.88	1.56	1.49
.79	3.06	2.95	1.61	1.54
.80	3.13	3.02	1.65	1.58
.81	3.20	3.10	1.70	1.63
.82	3.28	3.18	1.75	1.68
.83	3.35	3.26	1.80	1.73
.84	3.44	3.35	1.85	1.79
.85	3.52	3.44	1.91	1.85
.86	3.61	3.53	1.97	1.91
.87	3.71	3.64	2.03	1.97
.88	3.81	3.75	2.09	2.04
.89	3.91	3.86	2.16	2.11
.90	4.03	3.99	2.23	2.19
.91	4.15	4.13	2.31	2.28
.92	4.29	4.28	2.39	2.37
.93	4.44	4.45	2.49	2.47
.94	4.61	4.65	2.59	2.59
.95	4.80	4.87	2.71	2.73
.96	5.04	5.15	2.85	2.90

Table 1 Continued

Proportion Correct Responses	Triangular Normal	Triangular Logistic	3-AFC Normal	3-AFC Logistic
.97	5.34	5.50	3.02	3.10
.98	5.73	5.98	3.25	3.39
.99	6.34	6.81	3.62	3.86

Note—Column heads refer to the types and variants of the models.

and for a number of other signal detection paradigms (Kaplan, Macmillan, & Creelman, 1978) are also available.

Since the present normal 3-AFC model is consistent with Elliot's general expression for m-AFC procedures, the present values of d′ are directly comparable to those from Elliot's table.

REFERENCE NOTE

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