A "C" Program to Fit Herrnstein's Hyperbola Using Wilkinson's Method

The law of effect, which is a central premise of behavioral theory and the basis for much of behavior therapy relates behavior to its reinforcing consequences. Herrnstein's (1970) mathematical statement of this law is:

$$R = kr/(r+r_e), \tag{1}$$

where R is the rate of responding for a behavior, r is the rate of contingent reinforcement for that behavior, k is the maximal rate of responding attainable by the organism, and r_e is all (extraneous) reinforcement concurrently delivered to the organism exclusive of r. Also known as the matching law, this model has received widespread support over three decades of animal research (see Davison & McCarthy, 1988) and has more recently attracted interest within applied behavior analysis (see, e.g., Martens & Houk, 1989).

Fitting Herrnstein's hyperbola to data is a focal part of the methodology in this area of research. It is generally accomplished with Wilkinson's (1961) method. This is a two-step procedure for estimating the parameters of a hyperbolic function: (1) initial estimates of k and r_e are obtained by a weighted least squares regression of $1/R$ on $1/r$, where R is response rate maintained by the reinforcement rate, r; (2) the initial estimates are then revised by fitting the bilinear equation, $R = b_1 f(r) + b_2 f'_{r_e}(r)$, where $f(r)$ is Herrnstein's hyperbola with k and r_e equal to their initial estimates, and $f'_{r_e}(r)$ is the first derivative of $f(r)$ with respect to r_e (see McDowell, 1981, for a detailed explanation).

Wilkinson's method of fitting Herrnstein's hyperbola has been available as a BASIC program (McDowell, 1981). We have written a program in C to estimate the parameters of Herrnstein's hyperbola with Wilkinson's two-step method. Unlike the earlier BASIC version, the present program does not limit the number of data pairs that may be entered. In addition, the current version provides a much better user environment. The program can also be converted into other high-level languages such as Pascal. The program outputs estimated values of k and r_e, their standard errors, associated error bars (parameter ± 1 *SE*), percentage of variance accounted for, and coordinates for plotting the fitted hyperbola. The second step of Wilkinson's method is repeated until no further increase in percentage of variance accounted for is obtained. The results are placed into a file named WILKSON.OUT, which is an appropriately delimited ASCII file. This format is imported into Borland's QuattroPro for final presentation and graphing, although it can also be imported into most spreadsheets. The presentation is handled by a macro file named QUATRO.WQ1, which should be stored with the WILKSON program. It should be noted that Herrnstein's hyperbola is constrained to pass through the origin and that any nonzero intercept will therefore diminish the hyperbolic fit. Martens and Houk (1989) have suggested transforming the data in such cases by a constant before fitting the hyperbola. This procedure can be adopted before using the present program, but with the caveat that there are theoretical ramifications for correcting nonzero y intercepts.

The source code and a compiled version of the program are available on a 5.25- or 3.5-in. diskette (along with user instructions), for $10. Correspondence should be addressed to Ephrem Fernandez, Psychology Dept., University of Queensland, Queensland, Australia 4072 (email: efenez@psych.psy.uq.oz.au).

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R2: A Computer Program for Interval Estimation, Power Calculations, Sample Size Estimation, and Hypothesis Testing in Multiple Regression

Although multiple correlation coefficients can be computed with all major statistical packages, these packages do not offer a full range of inferential procedures for multiple correlation. Most notable by its absence is a confidence interval on the population coefficient of determination, P^2.

Exact confidence intervals for P^2, though in principle simple enough to obtain, require several iterations, each in turn requiring calculations of the cumulative distribution function of the sample squared multiple correlation, R^2. This calculation is difficult to perform both quickly and reliably. Numerical routines must also be constructed very carefully so that they will remain stable under a variety of parameter choices. Perhaps this explains why this and several related procedures involving P^2 have not yet achieved popularity.

R2 is a simple menu-driven computer program that provides several useful procedures involving P^2.

Program Description. R2 has a simple Windows-like user interface. It is completely menu driven. After selecting a procedure from a menu, the user is given a series of drop-down menus of variables that can be entered from the keyboard. The requested procedure is then performed and the output is displayed on the screen or optionally written to a log file for later review.

Calculations performed by R2 depend on routines for calculating the cumulative distribution of R^2. R2 includes two different algorithms for performing this calculation. One, emphasizing speed, is accurate to about four decimal places (Lee, 1971, p. 123). The other, emphasizing accuracy, is considerably slower but achieves at least five decimal places of accuracy (Lee, 1972, p. 178). The user can change the method of approximation at any time during a session, and this choice will be recorded in the log file.

Percentage points of the distribution of R^2. R2 can calculate percentage points for the distribution of R^2 for a given P^2, sample size (N), and number of variables including the criterion k.

Exact probability integral calculations for the distribution of R^2. R2 can compute the probability of obtaining a coefficient of determination less than or equal to a given value, for any given values of k, N, R^2, and P^2 within the general program limits.

Calculation of exact confidence intervals and confidence bounds for the squared multiple correlation. A unique feature of R2 is its ability to calculate exact confidence intervals for a squared multiple correlation, using the "inversion" procedure discussed by, among others, Cox and Hinckley (1974, p. 213). This confidence interval, in our opinion, is much more informative than a shrunken estimator, because it gives an indication of precision of estimation. As an obvious specialization, one can also calculate a "statistical lower bound" for P^2.

Testing hypotheses of the form $P^2 = a$, for values of a other than zero. This is done rather routinely by calculating a confidence interval for an observed R^2 at a given N and k, and seeing whether the interval includes or excludes a. Two-tailed tests can be performed by using the upper and lower limits of the confidence interval. One-tailed tests can be performed by using the one-tailed confidence bound.

Power calculation for tests of the null hypothesis $P^2 = 0$, and calculation of the sample size necessary to achieve a desired level of power for testing $P^2 = 0$. R2 contains some powerful utilities for power calculations in multiple regression. Power for testing a hypothesis of zero squared multiple correlation, $P^2 = 0$, can be computed for any alternative. R2 can also calculate the sample size necessary to yield a given level of power for a given value of P^2.

Hardware requirements. R2 runs on any PC-compatible computer. The program requires about 350K of free memory. It uses a math coprocessor if one is present, but a coprocessor is not required. The program is not copy protected.

R2 is available as a set of EXE files. The printed user documentation includes a tutorial on program operation, as well as numerous illustrations. Please specify either a 3.5- or a 5.25-in. diskette. There is a charge of $30.00 (US or equivalent) for the program, documentation, postage, and handling. Requests should be sent with a check payable to James H. Steiger, Department of Psychology, University of British Columbia, Vancouver, BC, Canada V6T 1Z4.

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