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BRIEF COMMUNICATIONS

Description of all Minimal Classes in the Partially Ordered Set $\mathcal{L}_2^3$ of Closed Classes of the Three-Valued Logic that can be Homomorphically Mapped onto the Two-Valued Logic

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Abstract—A description of all minimal classes in the partially ordered set $\mathcal{L}_2^3$ of closed classes of the three-valued logic that can be homomorphically mapped onto the two-valued logic is given.

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All necessary definitions can be found in [1, Introduction]. It was also proved in that paper that the partially ordered set (a poset in what follows) $\mathcal{L}_2^3$ of all closed classes of the three-valued logic $\mathfrak{P}_3$ which can be mapped onto the two-valued logic $\mathfrak{P}_2$ contains just a finite number of minimal elements each of which possesses a basis consisting of one function of two variables.

In this paper a description of all 15 minimal classes of the poset $\mathcal{L}_2^3$ is given; six of those classes were indicated by Maltsev in [2] and the remaining nine ones are found by the author.

Maltsev [2] found 6 classes of the three-valued logic isomorphic to the two-valued logic by means of the operation of adding a dummy variable. Two of them are generated by the following functions:

$$\begin{pmatrix} 1\,0\,1 \\ 0\,0\,0 \\ 1\,0\,1 \end{pmatrix}, \begin{pmatrix} 1\,0\,0 \\ 0\,0\,0 \\ 0\,0\,0 \end{pmatrix}.$$

Note that the matrix $\begin{pmatrix} a_{00}\,a_{01}\,a_{02} \\ a_{10}\,a_{11}\,a_{12} \\ a_{20}\,a_{21}\,a_{22} \end{pmatrix}$ generates a function $\varphi(x,y)$ by the following relation: $\varphi(x,y) = a_{xy}$.

The remaining 4 classes can be obtained by isomorphic mappings of those classes generated by permutations from the cyclic group $\mathbf{A_3}$ of order three which is generated by the permutation $\begin{pmatrix} 0\,1\,2 \\ 1\,2\,0 \end{pmatrix}$.

We indicate only 3 classes of the remaining 9. The other 6 classes can be obtained by isomorphic mappings of those three classes generated by the permutations from the group $\mathbf{A_3}$. These 3 classes are generated by the following three functions:

$$\begin{pmatrix} 1\,0\,2 \\ 0\,0\,2 \\ 2\,2\,2 \end{pmatrix}, \begin{pmatrix} 1\,0\,2 \\ 0\,0\,2 \\ 0\,0\,2 \end{pmatrix}, \begin{pmatrix} 1\,0\,0 \\ 0\,0\,0 \\ 0\,0\,2 \end{pmatrix}.$$

The class of functions generated by the first function is isomorphic to $\mathfrak{P}_2$, but without the operation of adding a dummy variable. The classes generated by the second and the third functions are not isomorphic to $\mathfrak{P}_2$. For example, 16 functions from the class generated by the third function are mapped into the Peirce's arrow (the logical NOR) $\overline{y_1 \vee y_2}$.

The methods used in the proof of the statements of this paper are based on the methods used in [1, 3].

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