Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Technical note:

A simple model for the hysteretic elastic shear modulus of unsaturated soils*

Jie XU[1], Chao ZHOU[†‡2]

([1]Key Laboratory of Ministry of Education for Geomechanics and Embankment Engineering, Hohai University, Nanjing 210098, China)

([2]Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China)

[†]E-mail: czhou@connect.ust.hk

Received Apr. 7, 2016; Revision accepted June 27, 2016; Crosschecked June 28, 2016

Abstract: Considering the great importance of the elastic shear modulus G_0 of unsaturated soils to the serviceability of many geo-structures in geo-energy and geo-environmental engineering, some semi-empirical models have been reported for the G_0 of unsaturated soils. Existing models require at least three parameters and the calibration of the model parameters requires extensive time-consuming unsaturated soil tests. In this study, a simple semi-empirical model is proposed for the hysteretic G_0 of unsaturated soils, requiring only two parameters. The constitutive variables of the mean Bishop's stress and a bonding variable are adopted for considering the average stress between soil particles and the additional normal forces between soil particles provided by water menisci. The derived equation is applied to simulate the G_0 of unsaturated silts and sands. Comparisons between the measured and calculated results demonstrate that the proposed equation is able to describe the influences of various factors on G_0, including mean net stress, suction, wetting-drying, and void ratio.

Key words: Unsaturated soil, Modeling, Elastic shear modulus, Small strain, Suction, Degree of saturation
http://dx.doi.org/10.1631/jzus.A1600300 **CLC number:** TU433

1 Introduction

The elastic shear modulus G_0 (also referred as very small strain (less than 0.001%) shear modulus) of soil is an important parameter to predict the serviceability of many earth structures in geo-energy and geo-environmental engineering, such as landfill covers and energy foundations (Atkinson and Sallfors, 1991). Furthermore, soils are often unsaturated and subjected to wetting-drying cycles in the field in geo-energy and geo-environmental engineering. An investigation of G_0 of unsaturated soils is therefore of great significance.

G_0 of soils can be measured by bender element or resonant column tests (Zhou *et al.*, 2008; Jardine, 2011). By performing suction-controlled resonant column and bender element tests, it was found that the G_0 of unsaturated soils increases with an increase of suction (Mancuso *et al.*, 2002; Ng and Yung, 2008; Dong *et al.*, 2016). At a given suction, the G_0 along the wetting path is consistently larger than that along the drying path (Ng *et al.*, 2009; Khosravi and McCartney, 2012).

To describe the G_0 of unsaturated soils, Mancuso *et al.* (2002) and Ng and Yung (2008) proposed two semi-empirical models. It was assumed that G_0 is a function of net stress, suction, and void ratio. The

---

[‡] Corresponding author

[*] Project supported by the National Natural Science Foundation of China (Nos. 51308190 and 51509041), the Jiangsu Provincial Natural Science Foundation of China (No. BK20130848), and the General Research Fund of the Research Grants Council of the Hong Kong Special Administrative Region (No. 16209415), China

ORCID: Jie XU, http://orcid.org/0000-0003-3841-7242; Chao ZHOU, http://orcid.org/0000-0002-9443-6707

© Zhejiang University and Springer-Verlag Berlin Heidelberg 2016

influences of the wetting-drying history (hydraulic hysteresis) were not considered. To consider the influences of the hydraulic hysteresis, some recent studies incorporated the degree of saturation (S_r) in their semi-empirical models (Sawangsuriya *et al.*, 2009; Biglari *et al.*, 2011; Khosravi and McCartney, 2012; Oh and Vanapalli, 2014; Wong *et al.*, 2014). Wong *et al.* (2014) found that by incorporating S_r, the models are able to simulate the variation of G_0 during compression, drying, and wetting. It should be pointed out that the existing models require at least three parameters. Extensive time-consuming unsaturated soil tests are needed for parameter calibration. Recently, Dong *et al.* (2016) and Dong and Lu (2016) developed a new G_0 model with three model parameters by using the suction stress-based effective stress principle. Based on the experimental data from about 30 types of soils, the relationship between two model parameters and the soil water retention curve was proposed.

The principal objective of this study is to develop a simple model to capture the hysteretic G_0 of unsaturated soils. To meet this objective, a simple semi-empirical model with only two parameters is newly proposed by adopting the mean Bishop's stress and a bonding variable. Then, the derived model is applied to simulate the hysteretic G_0 of various soils. The calculated and measured results are compared and analyzed.

2 Mathematical formulations

Based on the results of suction-controlled bender element tests, Ng and Yung (2008) proposed a semi-empirical equation to describe the G_0 of unsaturated soils using two independent stress state variables (i.e., net stress and matric suction):

$$G_{0(ij)} = C_{ij}^2 f(e)\left(\frac{p}{p_{\mathrm{ref}}}\right)^{2n}\left(1+\frac{s}{p_{\mathrm{ref}}}\right)^{2k}, \tag{1}$$

where C_{ij} is a constant reflecting the inherent soil structure in the *ij* plane (i.e., plane of shear), $f(e)$ is a void ratio function relating shear modulus to void ratio, p and s are the mean net stress and matric suction, respectively, p_{ref} is a reference pressure for normalizing p, and n and k are regression parameters. The net stress and matric suction are defined as $(\sigma-u_a)$ and (u_a-u_w), where σ, u_a, and u_w are the total stress, pore air pressure, and pore water pressure, respectively. By comparing measured and calculated results, Ng and Yung (2008) found that Eq. (1) is able to capture the influences of p and s on G_0.

On the other hand, Ng *et al.* (2009) measured the G_0 of compacted clayey silt specimens along a drying and wetting cycle. They found that at a given suction, measured G_0 was consistently larger along the wetting path than that along the drying path. Similar findings were reported by some other researchers, such as Khosravi and McCartney (2012). The observed hysteretic effects are due to at least two reasons. First, an increase in suction induces the shrinkage and densification of the soil specimen. Therefore, the soil specimen along the wetting path has a higher density and hence a larger G_0 than that subjected to drying. Second, at a given suction, the S_r of the soil specimen along the wetting path is lower than that along the drying path. At a lower S_r, the number of water menisci per unit soil volume would be larger while the amount of bulk water would be smaller. As a result, the additional inter-particle normal forces provided by more water menisci tend to stiffen the soil skeleton. Both mechanisms are taken into account in this study to make the proposed model theoretically and physically sound. It should be pointed out that the volumetric strain of the soil specimen induced by the drying and wetting cycle is not significant (less than ±0.3%) according to the experimental data of Ng *et al.* (2009). This suggests that the effects of the soil density on hysteretic G_0 behavior are very minor for the soil tested. Therefore, the effects of drying and wetting on G_0 reported by Ng *et al.* (2009) cannot be captured well by Eq. (1), which assumes that G_0 is affected by net stress, suction, and void ratio only.

The limitation of Eq. (1) is primarily due to the fact that the net stress and suction are not sufficient to satisfactorily describe the complicated water distribution within unsaturated soils. Wheeler and Karube (1996) postulated that the soil water in unsaturated soils may be classified into two different types: namely bulk water and meniscus water. Bulk water affects the tangent and normal forces between soil particles, whereas a change of meniscus water alters the normal force only. The presence of meniscus water would stabilize the soil skeleton by reducing the mobilized ratio of the tangent force and normal force. Considering the different roles of bulk water and meniscus water, the degree of saturation of the soil specimen is expected to impose a significant influence on pore water distribution and hence the G_0 of the unsaturated soil. To fully capture the hysteretic G_0 behavior of unsaturated soil, not only soil suction but also the degree of saturation should be incorporated.

Due to the different effects of these two types of water, suction affects the mechanical behavior of the unsaturated soil via at least two different ways, namely modifying the average skeleton force and providing additional bonding forces at particle contacts by water menisci (stabilization effects on soil skeleton). To describe these two mechanisms explicitly, Gallipoli *et al.* (2003) proposed the following two constitutive variables:

$$p^* = p + S_r s, \qquad (2)$$

$$\xi = f(s)(1 - S_r), \qquad (3)$$

where p^* is the mean Bishop's stress, and ξ is the bonding variable. These two constitutive variables have clear physical meanings. The first one (p^*) denotes the average stress between soil particles, while the bonding variable ξ was proposed by Gallipoli *et al.* (2003) as a scalar constitutive variable for unsaturated soils. The bonding variable is related to the inter-particle normal forces exerted by water menisci, and it is a function of the soil suction and the degree of saturation.

The first term $f(s)$ on the right hand side of Eq. (3) describes the inter-particle normal forces exerted by a single water meniscus at a given suction, normalized by that at zero suction. The relationship between $f(s)$ and suction was derived by Fisher (1926) and is shown in Fig. 1. This relationship can be described by the following equation (Gallipoli *et al.*, 2003; Zhou *et al.*, 2015):

$$f(s) = \frac{3T_s}{rs} \frac{\left(\sqrt{9 + 8rs/T_s} - 3\right)\left(\sqrt{9 + 8rs/T_s} + 1\right)}{16}, \qquad (4)$$

where T_s is the surface tension coefficient of water which is equal to 72.8 mN/m at 20 °C, and r is the radius of the spherical particles. It can be seen from Fig. 1 that the value of $f(s)$ is higher with a larger r at a given suction. For simplicity, a constant r value of 1×10^{-6} m is assumed for soil specimens to calculate $f(s)$ in this study. This simplification should not significantly affect model prediction with proper model parameters. This is because according to Eq. (4) and Fig. 1, for all values of r, the value of $f(s)$ is limited to a range of 1 to 1.5 over a full suction range. The percentage difference should be therefore smaller than 50%. It should be pointed out that Eq. (4) describes the suction effects on the value of the inter-particle normal force exerted by a single water meniscus. It was first derived by Fisher (1926) based on the air-water interface between two identical spheres. Although this equation is derived based on granular material, it also works well for fine-grained soils, as reported by Gallipoli *et al.* (2003). Therefore, this variable is considered and used to model hysteretic G_0 behavior in this study.

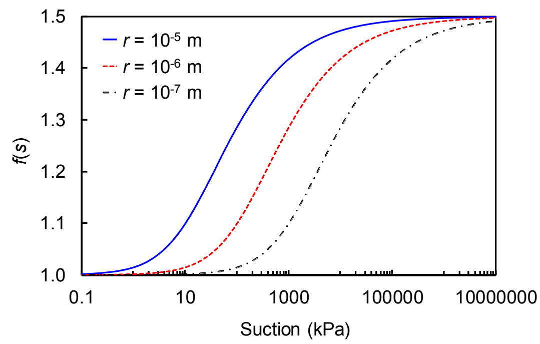

Fig. 1 Relationship between $f(s)$ and suction with different values of r

The second term $(1-S_r)$ is adopted to account for the number of water menisci per unit volume in unsaturated soil through a simplified approach. At the fully saturated state (an ideal case with S_r=100%), there should be no meniscus water within a soil specimen. Stabilization effects arising from meniscus water would be absent, and therefore $(1-S_r)$ and ξ are equal to zero. At unsaturated states, the values of $(1-S_r)$ and ξ increase as the soil specimen desaturates. This increase of ξ with increasing suction is consistent with experimental observations that stabilization effects are more significant at a lower degree of saturation.

By applying these two constitutive variables (Eqs. (2) and (3)), a new formulation for the G_0 of unsaturated soil is proposed as follows:

$$G_0 = C_0 f(e)\left[\left(\frac{p^*}{p_{\text{ref}}}\right)^{n_\text{p}} + C_\text{s}\xi^{n_\text{s}}\right], \tag{5}$$

where C_0, C_s, n_p, and n_s are the soil parameters, and p_ref is assumed to be the atmospheric pressure (101 kPa) in the following calculations. Compared with Eq. (1), Eq. (5) explicitly incorporates S_r and considers two different suction effects. At the saturated state, the mean Bishop's stress reduces to the mean effective stress p', and ξ equals zero. Eq. (5) can be simplified as

$$G_0 = C_0 f(e)\left(\frac{p'}{p_{\text{ref}}}\right)^{n_\text{p}}. \tag{6}$$

Note that Eq. (6) was first proposed by Hardin and Black (1966). It has been widely used to estimate the G_0 of different saturated soils, including sand, silt, and clay. This suggests that Eq. (5) allows for a smooth translation between unsaturated and saturated states. Furthermore, some soil parameters (C_0 and n_p) and $f(e)$ in Eq. (5) can be calibrated by fitting measured G_0 at the saturated state. This is an effective and convenient approach, considering that unsaturated soil testing is much more time-consuming and that there is relatively less experimental data of unsaturated soil behavior in literature.

Based on extensive experimental results of saturated clay and sand, McDowell and Bolton (2001) found that G_0 varies with p' as $p'^{0.5}$ following the Hertz contact theory. This observation suggests that

$$n_\text{p} = 0.5. \tag{7}$$

For the void ratio function, various formulations have been proposed in literature. Shibuya *et al.* (1997) introduced a simplified void ratio function $f(e)=(1+e)^\alpha$, where the coefficient α is equal to −2.4 for clay based on the *in-situ* seismic survey and laboratory bender element tests. Oztoprak and Bolton (2013) used the void ratio function $f(e)=(1+e)^{-3}$ for sands based on extensive laboratory data. When the soil void ratio increases from 0.5 to 1.0, for example, the value of $f(e)$ decreases by 50% and 58% with $\alpha=-2.4$ and $\alpha=-3$, respectively. It is clear that with such a huge change in the soil void ratio, the model prediction difference is less than 10% with $(1+e)^{-2.4}$ and $(1+e)^{-3}$. Therefore, a single void ratio function $(1+e)^{-3}$ is used for both clay and sand in the proposed model for simplicity, i.e.,

$$f(e) = (1+e)^{-3}. \tag{8}$$

On the other hand, two parameters (C_s and n_s) are used in Eq. (5) to describe the effects of ξ on G_0. For simplicity, it is assumed that

$$n_\text{s} = n_\text{p}. \tag{9}$$

It is expected that this simplification does not greatly affect the capability of Eq. (5), since the effects of ξ on G_0 can be considered through the other soil parameter (i.e., C_s).

Substituting Eqs. (7)–(9) into Eq. (5), the following equation can be obtained:

$$G_0 = C_0 (1+e)^{-3}\left[\left(\frac{p^*}{p_{\text{ref}}}\right)^{0.5} + C_\text{s}\xi^{0.5}\right]. \tag{10}$$

Eq. (10) is developed to describe the hysteretic G_0 of unsaturated soils, requiring only two parameters (C_s and C_0). C_0 is a constant reflecting the effect of the inherent soil structure on G_0. C_s describes the effect of additional normal forces between soil particles provided by water menisci (ξ) on G_0.

3 Verification of the newly proposed simple model

Ng and Yung (2008) and Ng *et al.* (2009) carried out two series of suction-controlled bender element tests on compacted clayey silt specimens. G_0 was measured along two stress paths: namely isotropic compression at constant suction and drying and wetting at constant stress. Apart from G_0, e and S_r at each stress and suction condition were also monitored. Fig. 2a shows the variation of G_0 with mean net stress at different suctions, obtained from the isotropic compression tests. As expected, G_0 increases with the increasing mean net stress. At the same mean net stress, G_0 is significantly larger at a higher suction. Based on the results, the two parameters in Eq. (10) are determined by the least square method: C_0=

330 MPa and C_s=1.2. Soil properties and model parameters are summarized in Table 1. With the fitted parameters, G_0 along the two stress paths are calculated. The calculated G_0 in the compression tests is also shown in Fig. 2a for comparison. It is clearly revealed that Eq. (10) is able to capture the influences of the stress and suction on G_0.

Fig. 2b compares the measured and calculated G_0 during the drying and wetting cycle at two isotropic net stresses (110 and 300 kPa). It can be seen that G_0 increases consistently with an increase of suction along the drying process. After drying to the maximum suction of 250 kPa, the soil suction is reduced by wetting the soil specimen. Measured G_0 decreases continuously during the wetting process. Similar to the water retention curve, there is a hysteresis between the drying and wetting stiffness curves showing variations of G_0 with suction. At the same suction, G_0 measured during wetting is consistently higher than that obtained during drying. On the other hand, the effects of drying and wetting on G_0 are generally captured by Eq. (10). Since the axial and radial strains of each soil specimen during drying and wetting are relatively small (less than ±0.3%), the changes of the void ratio and hence $f(e)$ are not significant (Ng *et al.*, 2009). According to the newly proposed simple model, shear modulus hysteresis occurs because S_r on the adsorption curve is lower than that on the desorption curve at the same suction. Eq. (2) suggests that at a given suction, the value of ξ is larger when S_r is lower. It is therefore concluded that G_0 predicted by Eq. (10) is larger along the wetting path than that along the drying path.

Similarly, Eq. (10) is used to fit the G_0 of another clayey silt (Khosravi and McCartney, 2012). Soil properties and model parameters are summarized in Table 1. Comparisons between measured and calculated results are shown in Fig. 3. It is well illustrated

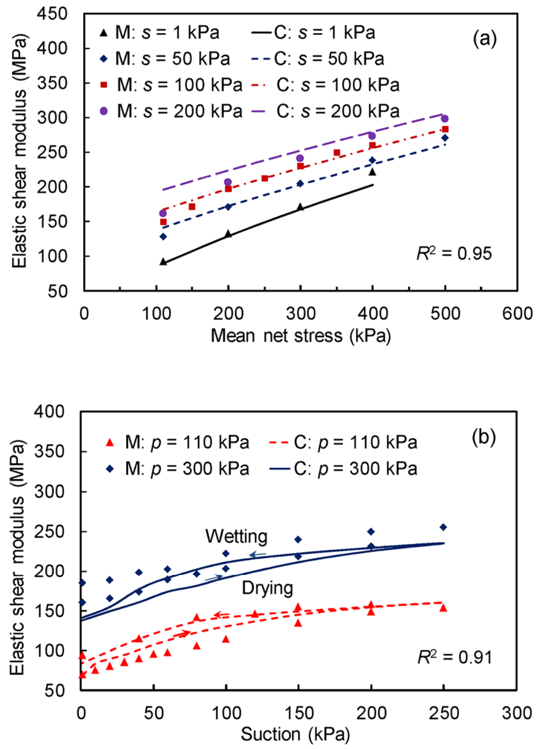

Fig. 2 Comparisons between measured and calculated G_0 of a compacted clayey silt tested by Ng and Yung (2008) (a) and Ng *et al.* (2009) (b)
M: measured; C: calculated

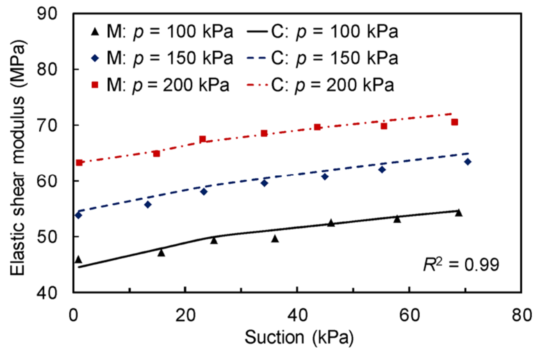

Fig. 3 Comparisons between measured and calculated G_0 of a compacted clayey silt tested by Khosravi and McCartney (2012)

Table 1 Summary of soil properties and regression coefficients in the newly proposed simple equation for G_0

References	Soil type	Percentage of clay (%)	Liquid limit (%)	Plastic limit (%)	Plastic index	C_0 (MPa)	C_s
Ng and Yung (2008); Ng *et al.* (2009)	Clayey silt	4	43	29	14	330	1.20
Khosravi and McCartney (2012)	Clayey silt	14	25	21	4	160	0.06
Nyunt *et al.* (2011)	Sand	0	NA	NA	NA	230	0.60

Note: compacted specimens are used in these four studies; NA means "not available" that the newly proposed simple equation is able to describe the dependency of G_0 on stress and suction for this type of soil.

Eq. (10) is then used to fit the G_0 of sand (Nyunt *et al.*, 2011). Soil properties and model parameters are summarized in Table 1. Comparisons between measured and calculated results are shown in Fig. 4. At the mean net stress of 50 and 100 kPa, the measured and calculated G_0 are quite consistent. However, at the mean net stress of 200 kPa, the measured G_0 is underestimated by the proposed model by about 25%. The discrepancies between the measured and calculated results suggest that the stress effects on the G_0 of sand are underestimated by the proposed model. The prediction errors may be reduced by adopting a larger n_p in Eq. (7). In this study, no modification is made to Eq. (7) to minimize the number of model parameters and the proposed model should be used with caution for a wide range of stresses.

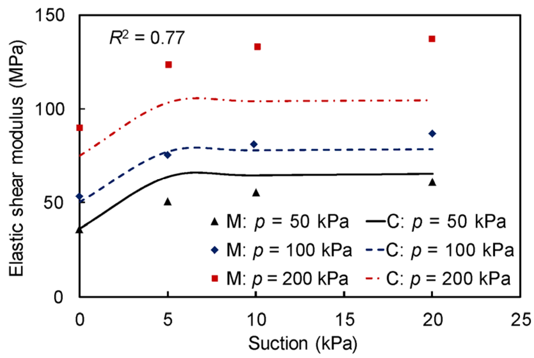

Fig. 4 Comparisons between measured and calculated G_0 of a compacted sand tested by Nyunt *et al.* (2011)

Figs. 2–4 reveal that Eq. (10) is able to describe the influences of various factors on G_0, including mean net stress, suction, drying-wetting, and void ratio. It should be pointed out that some existing semi-empirical equations for G_0 are reported to be able to simulate the G_0 of unsaturated soil along various stress paths, including drying, wetting, and isotropic compression, as reviewed by Wong *et al.* (2014). However, these existing equations require at least three parameters. Extensive data from time-consuming unsaturated soil tests is also required to calibrate them. On the other hand, the proposed model for the hysteretic G_0 of unsaturated soils only requires two parameters (i.e., C_0 and C_s). As illustrated in Section 2, C_0 is a constant reflecting the effect of the inherent soil structure on G_0, while C_s describes the effect of additional normal forces between soil particles provided by water menisci (ξ) on G_0. Compared with most existing models in literature, much fewer test results are required for the calibration of the model parameters. To use the proposed model, laboratory or filed tests should be carried out to determine the G_0 of unsaturated soil at two different suctions. Without such experimental measurements, an alternative approach is to deduce the values of C_0 and C_s from soil water retention curve, as proposed by Dong *et al.* (2016). It should be pointed out that the scope of this study is to propose a new idea for modeling the hysteretic G_0 behavior of unsaturated soil with fewer parameters. Future studies should be carried out to predict the two parameters from basic soil properties, such as the water retention curve.

4 Conclusions

A simple semi-empirical model is newly proposed for the hysteretic elastic shear modulus G_0 of unsaturated soils, requiring only two parameters. The derived equations are applied to simulate the G_0 of various soils. Comparisons between measured and calculated results demonstrate that the proposed equation is able to capture the influences of various factors on G_0, including mean net stress, suction, wetting-drying, and void ratio.

It should be pointed out that the proposed simple model is intended for unsaturated sands, silts, and low-plasticity clays. For high-plasticity clays, such as expansive soils, however, it may require some modifications, such as a consideration of significant wetting-induced volume changes. On the other hand, some parameters in the model are given specified values for simplicity, which may result in slight prediction errors, particularly for sands. In addition, comparisons between measured and calculated G_0 are all limited within a low suction range (less than 500 kPa) in this study. More experimental and theoretical studies should be carried out to reveal the G_0 behavior of unsaturated soil over a wide range of stresses and suctions. Based on new evidences, the proposed model could be verified and improved if necessary in future studies.

References

Atkinson, J.H., Sallfors, G., 1991. Experimental determination of stress-strain-time characteristics in laboratory and *in-situ* tests. Proceedings of the 10th European Conference on Soil Mechanics and Foundation Engineering, Florence, Italy, **3**:915-956.

Biglari, M., Mancuso, C., d'Onofrio, A., *et al.*, 2011. Modelling the initial shear stiffness of unsaturated soils as a function of the coupled effects of the void ratio and the degree of saturation. *Computers and Geotechnics*, **38**(5): 709-720. http://dx.doi.org/10.1016/j.compgeo.2011.04.007

Dong, Y., Lu, N., 2016. Correlation between small-strain shear modulus and suction stress in capillary regime under zero total stress conditions. *Journal of Geotechnical and Geoenvironmental Engineering*, 04016056. http://dx.doi.org/10.1061/(asce)gt.1943-5606.0001531

Dong, Y., Lu, N., McCartney, J.S., 2016. Unified model for small-strain shear modulus of variably saturated soil. *Journal of Geotechnical and Geoenvironmental Engineering*, 04016039. http://dx.doi.org/10.1061/(asce)gt.1943-5606.0001506

Fisher, R.A., 1926. On the capillary forces in an ideal soil; correction of formulae given by W.B. Haines. *The Journal of Agricultural Science*, **16**(03):492-505. http://dx.doi.org/10.1017/S0021859600007838

Gallipoli, D., Gens, A., Sharma, R., *et al.*, 2003. An elastoplastic model for unsaturated soil incorporating the effects of suction and degree of saturation on mechanical behaviour. *Géotechnique*, **53**(1):123-135. http://dx.doi.org/10.1680/geot.2003.53.1.123

Hardin, B., Black, W., 1966. Sand stiffness under various triaxial stresses. *Journal of the Soil Mechanics and Foundations Division*, **92**(SM2):27-42.

Jardine, R.J., 2011. Characterization of mudrocks: a practical application of advanced laboratory testing. *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)*, **12**(1):1-14. http://dx.doi.org/10.1631/jzus.A1000420

Khosravi, A., McCartney, J.S., 2012. Impact of hydraulic hysteresis on the small-strain shear modulus of low plasticity soils. *Journal of Geotechnical and Geoenvironmental Engineering*, **138**(11):1326-1333. http://dx.doi.org/10.1061/(asce)gt.1943-5606.0000713

Mancuso, C., Vassallo, R., d'Onofrio, A., 2002. Small strain behavior of a silty sand in controlled-suction resonant column torsional shear tests. *Canadian Geotechnical Journal*, **39**(1):22-31. http://dx.doi.org/10.1139/t01-076

McDowell, G., Bolton, M., 2001. Micro mechanics of elastic soil. *Soils and Foundations*, **41**(6):147-152. http://dx.doi.org/10.3208/sandf.41.6_147

Ng, C.W.W., Yung, S.Y., 2008. Determination of the anisotropic shear stiffness of an unsaturated decomposed soil. *Géotechnique*, **58**(1):23-35. http://dx.doi.org/10.1680/geot.2008.58.1.23

Ng, C.W.W., Xu, J., Yung, S.Y., 2009. Effects of wetting-drying and stress ratio on anisotropic stiffness of an unsaturated soil at very small strains. *Canadian Geotechnical Journal*, **46**(9):1062-1076. http://dx.doi.org/10.1139/T09-043

Nyunt, T., Leong, E., Rahardjo, H., 2011. Strength and small-strain stiffness characteristics of unsaturated sand. *Geotechnical Testing Journal*, **34**(5):551-561.

Oh, W.T., Vanapalli, S.K., 2014. Semi-empirical model for estimating the small-strain shear modulus of unsaturated non-plastic sandy soils. *Geotechnical and Geological Engineering*, **32**(2):259-271. http://dx.doi.org/10.1007/s10706-013-9708-5

Oztoprak, S., Bolton, M.D., 2013. Stiffness of sands through a laboratory test database. *Géotechnique*, **63**(1):54-70. http://dx.doi.org/10.1680/geot.10.P.078

Sawangsuriya, A., Edil, T.B., Bosscher, P.J., 2009. Modulus-suction-moisture relationship for compacted soils in postcompaction state. *Journal of Geotechnical and Geoenvironmental Engineering*, **135**(10):1390-1403. http://dx.doi.org/10.1061/(asce)gt.1943-5606.0000108

Shibuya, S., Hwang, S., Mitachi, T., 1997. Elastic shear modulus of soft clays from shear wave velocity measurement. *Géotechnique*, **47**(3):593-601. http://dx.doi.org/10.1680/geot.1997.47.3.593

Wheeler, S.J., Karube, D., 1996. Constitutive modelling. Proceedings of the First International Conference on Unsaturated Soil, Paris, France, **3**:1323-1356.

Wong, K.S., Mašín, D., Ng, C.W.W., 2014. Modelling of shear stiffness of unsaturated fine grained soils at very small strains. *Computers and Geotechnics*, **56**:28-39. http://dx.doi.org/10.1016/j.compgeo.2013.10.005

Zhou, C., Ng, C.W.W., Chen, R., 2015. A bounding surface plasticity model for unsaturated soil at small strains. *International Journal for Numerical and Analytical Methods in Geomechanics*, **39**(11):1141-1164. http://dx.doi.org/10.1002/nag.2355

Zhou, Y.G., Chen, Y.M., Asaka, Y., *et al.*, 2008. Surface-mounted bender elements for measuring horizontal shear wave velocity of soils. *Journal of Zhejiang University-SCIENCE A*, **9**(11):1490-1496. http://dx.doi.org/10.1631/jzus.A0820323

中文概要

题　目：考虑滞回效应的非饱和土弹性剪切模量简化模型

目　的：非饱和土的弹性剪切模量是预测土体变形和土工

建筑物正常使用服役性能的重要参数之一。本文旨在提出一个描述非饱和土弹性剪切模量的简化模型，减少标定模型参数所需要的耗时非饱和土试验，并考虑吸力、应力、干湿循环及孔隙比对弹性剪切模量特性的影响。

创新点： 1. 提出考虑滞回效应的非饱和土弹性剪切模量简化模型；2. 减少标定模型参数所需要的试验。

方　法： 1. 基于前人非饱和土弹性剪切模量试验结果，考虑非饱和土中土颗粒间的平均骨架应力及毛细水提供的法向应力作用，通过理论推导建立非饱和土弹性剪切模量的半经验简化模型；2. 通过文献中不同非饱和粉土及砂土的弹性剪切模量试验结果验证简化模型的适用性。

结　论： 1. 得到一个描述非饱和土弹性剪切模量的简化模型，该模型仅需两个模型参数，减少了标定模型参数所需要的耗时非饱和土试验；2. 通过四组不同非饱和土弹性剪切模量试验数据验证了简化模型的适用性，表明该模型能考虑吸力、应力、干湿循环以及孔隙比对弹性剪切模量特性的影响；3. 由于进行了简化，该模型可能存在少量预测误差，在宽应力和吸力范围内运用该模型时需要谨慎。

关键词： 非饱和土；理论模型；弹性剪切模量；小应变；吸力；饱和度