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Controlling the optical properties of a laser pulse at $\lambda = 1.55\mu\text{m}$ in InGaAs/InP double coupled quantum well nanostructure

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Abstract

Background: The transient and steady-state behaviour of the absorption and the dispersion of a probe field propagating at $\lambda = 1.55\mu\text{m}$ through an InGaAs/InP double coupled quantum well are studied. The effect of terahertz signal excitation, electron tunnelling and incoherent pumping on the optical properties of the probe field is discussed.

Methods: The linear dynamical properties of the double coupled quantum well by means of perturbation theory and density matrix method are discussed.

Results: We show that the group velocity of a light pulse can be controlled from superluminal to subluminal or vice versa by controlling the rates of incoherent pumping field, terahertz signal and tunnelling between the quantum wells. The required switching time is calculated and we find it between 3 to 15 ps.

Conclusions: In the terahertz (30 ~ 300 μm or 1 ~ 10THz) intersubband transition, the incoming photon energy is (4 ~ 41meV) and maybe in the order of electron thermal broadening ($KT \sim 6\text{ meV}-25\text{ meV}$ for 77 K -300 K). Therefore in the conventional structure, the incoming photon can directly excite the ground state electrons to higher energy levels. It is shown that the absorption and the dispersion of the probe field can be controlled by the intensity of terahertz signal and incoherent pumping field.

Keywords: Electro-optical switching, Dispersion and absorption, Group velocity, Terahertz signal, Tunnelling effects, Incoherent pumping field

Background

It is known that the absorption and the dispersion properties of a weak probe field can be modified effectively by atomic coherence and quantum interference [1–5]. Atomic coherence can be achieved by the strong coupling fields, the spontaneous emission and incoherent pumping fields. It is known that atomic coherence due to the coherent laser field has essential roles for modifying the optical properties of atomic systems such as spontaneously generated coherence (SGC) [1], lasing without inversion [2], modifying spontaneous emission [3], coherent population trapping (CPT) [5], optical bistability [6–10] and so on [11–16]. Furthermore, it

has been shown that quantum interference arising from SGC [8] and incoherent pumping field [17] can be used for analyse of some interesting phenomena such as lasing without population inversion [4], optical bistability [17], and superluminal/subluminal light propagation [18]. Similar phenomena involving quantum coherence in solid state systems such as semiconductor quantum wells (QWs) and quantum dots (QDs) [19], can also be occurred [20, 21]. In the past decade, there has been an increasing interest in optical properties of quantum dot molecules (QDMs) and quantum wells (QWs), due to important role in optoelectronic devices. Recently, investigators have examined the effects of an external field and inter-dot tunnel coupling on the optical properties of QDs and QWs [22–28]. Quantum well semiconductors were

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chosen because of their advantage in flexible design, controllable interference strength, long dephasing times [29, 30], large dephasing rates [~ 10 ps $^{-1}$] [31] and large electric dipole moment which make them suitable for application in the optoelectronic devices. Quantum coherence in a QW structure can be induced by electron tunnelling or applying a laser field [32, 33]. Coherence induced by incoherent field and tunnel coupling in the QW system plays an important role in light-matter interaction and has found numerous implementations in semiconductor optics. On the other hand, coherent control [34–37] over the dispersive and absorptive properties of solid-state media such as photonic crystals and semiconductors has recently attracted a lot of attention [38–42]. Several proposals for quantum coherence and interference in QWs have been performed and analysed. To utilize the tunnelling effect, an electron is excited by a laser field, then tunnels to the second QW by controlling the external voltage between the wells [43]. An interesting application of QWs is modification of light pulse to make a fast electro-optical switch by controlling the propagation of a weak light pulse in a semiconductor system, which depends on the dispersive properties of the medium.

In this paper, we introduce a compact four level quantum wells system composed of two QWs. Then, we investigate effect of terahertz signal, incoherent pumping field and tunnelling between QWs on the absorption, dispersion and the group velocity of a weak probe field. The required switching time when propagation of light changes from subluminal to superluminal and vice versa is also discussed. We find that the dispersion/absorption spectra of the probe pulse can be changed via the effect of terahertz signal, incoherent pumping field and tunnelling effect.

Methods

In Fig. 1 we consider a compact double coupled quantum well nanostructure which is fabricated using InGaAs/InP nanostructures in material grown by an attractive growth technique i.e. organometallic vapor phase epitaxy (OMVPE). The QWs consist of two periods of alternating 10 nm InGaAs and 10 nm InP layers. The sample can be grown in a horizontal OMVPE system at atmospheric pressure. The growth chamber should contain a system that allows the growth of a QW as narrow as 10 Å with an average roughness of half the lattice constant of InGaAs. Typical growth rates are 10 Å/s for InGaAs and 5 Å/s for InP [44]. An incoherent pumping field and weak probe field are applied to first quantum well (QW₁). According to the band gap difference between In_{0.47}Ga_{0.53}As (0.7 eV) and InP (1.35 eV), the wave length of the incoherent

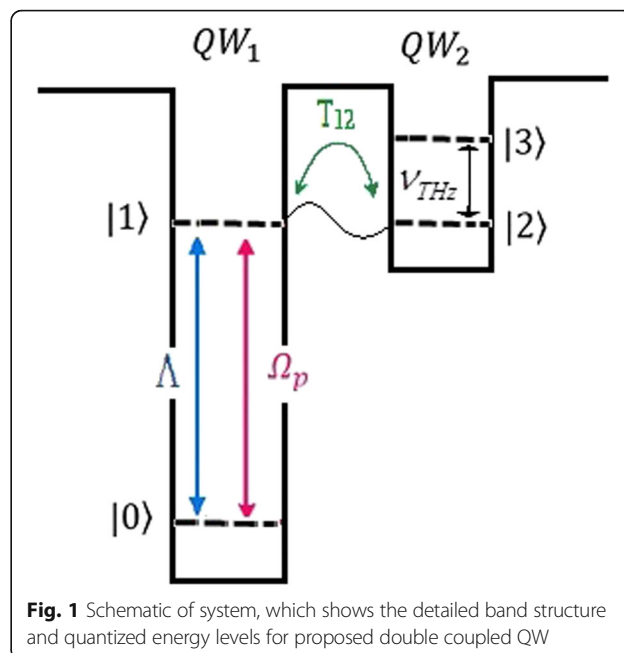


Fig. 1 Schematic of system, which shows the detailed band structure and quantized energy levels for proposed double coupled QW

pump field can be in the range of $\lambda = 1 - 2 \mu\text{m}$ [44]. For controlling of the tunnelling rate between QWs, the system is placed between two connected electrodes, as electrodes are in contact to this system. By applying independently tuneable gate voltages, electron tunnelling can easily be accomplished between QW₁ and QW₂. The range of the applied voltage to the electrodes varies as $V \approx 0 - 30$ mV. For more details we refer to [44–46]. Figure 1 shows the detailed band structure and energy levels of the system. Lower level |0> and upper level |1> are conducting band levels of QW₁. Level |2> and level |3> are the excited conducting levels of the QW₂ of the right of QW₁. It is assumed that the energy difference of three excited levels and the lower level is large, so their tunnelling couplings can be ignored. By applying a gate voltage the level |2> and the level |3> get closer to the level |1>. A weak tuneable probe field of the frequency ω_p with Rabi frequency $\Omega_p = \frac{\vec{E} \cdot \vec{\phi}}{2\hbar}$ and an incoherent pumping field Λ are applied to the transition |0> \rightarrow |1>. Here, $\vec{\phi}$ is electric dipole moment and E is amplitude of the probe field laser. By forming the resonant coupling of the probe field with the QW₁, an electron is excited from the |0> band to the |1> band of the QW₁. By providing the tunnelling conditions the electron can be transferred to level |2> in QW₂ and the Ω_{THz} (terahertz signal) prompt the electron from the level |2> to the level |3>. The total Hamiltonian in the rotating-wave approximation method [47, 48], which represents the interaction of the probe laser field, terahertz signal and incoherent pumping

field with the double coupled QWs system, can be expressed in the form of

$$H = \sum_{j=0}^3 E_j |j\rangle\langle j| + \left[(\Omega_p e^{-i\omega_p t} |0\rangle\langle 1| + \wp \varepsilon |0\rangle\langle 1| + T_{12} |1\rangle\langle 2| + \Omega_{THz} e^{-i\omega_{THz} t} |2\rangle\langle 3| + H.C. \right] \quad (1)$$

Where $E_j = \hbar\omega_j$ denotes the energy of state $|i\rangle$. \wp is the dipole moment of the atomic transition corresponding to the pumping of the electrons from level $|0\rangle$ to level $|1\rangle$, and the electric field ε implies the electrical amplitude of the incoherent pumping field. T_{12} correspond to tunnelling between “QW₁” and “QW₂”. The tunnelling can be described by perturbation theory which can be given by Bardeen’s approach [49]. According to this approach, the tunnelling probability of an electron in state Ψ with energy E_Ψ from the first QW to state Φ with energy E_Φ in the second QW is given by Fermi’s golden rule [50]

$$W = \frac{2\pi}{\hbar} |T_e|^2 \delta(E_\Psi - E_\Phi). \quad (2)$$

The tunnelling matrix elements can then be acquired by an integral over a surface in the barrier region lying between the QWs

$$T_e = \frac{\hbar}{2m} \int_{z=z_0} \left(\Phi^* \frac{\partial \Psi}{\partial z} - \Psi \frac{\partial \Phi^*}{\partial z} \right) dS, \quad (3)$$

where z_0 lies in the barrier, and m is the effective mass of the electron. Applying a bias voltage V , the current is

$$I = \frac{4\pi e}{\hbar} \int_0^{eV} \rho_1(E_F - eV + \varepsilon_0) \rho_2(E_F + \varepsilon_0) |T_e|^2 d\varepsilon_0, \quad (4)$$

where ε_0 is the energy difference between two discrete states in two wells. The current corresponds to the local density state of each QW (ρ_0, ρ_1) of the Fermi energy (E_F). The magnitude of coupling between two QWs can be adjusted by the bias voltage applied to the wells. Note that T_e is relevant to the applied bias to the molecule. For $T_e = T_{12} \neq 0$ some interaction terms should be appeared in total Hamiltonian as depicted in Eq. (1).

The density-matrix approach given by

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho], \quad (5)$$

Can be used to obtaining the density operator in an arbitrary multilevel QWs system. Substituting Eq. (1) in Eq. (2), the density matrix equations of motion can be expressed as

$$\begin{aligned} \dot{\rho}_{01} &= (i\delta_p - \Gamma_{01} - \Lambda) \rho_{01} + iT_{12} \rho_{02} - i\Omega_p (\rho_{00} - \rho_{11}), \\ \dot{\rho}_{02} &= iT_{12} \rho_{01} + (i(\delta_p + \omega_{12}) - \Gamma_{02} - \Lambda/2) \rho_{02} + i\Omega_{THz} \rho_{03} + i\Omega_p \rho_{12}, \\ \dot{\rho}_{03} &= i\Omega_{THz} \rho_{02} + (i(\delta_p + \omega_{12} + \omega_{23}) - \Gamma_{03} - \Lambda/2) \rho_{03} + i\Omega_p \rho_{13}, \\ \dot{\rho}_{12} &= i\Omega_p \rho_{02} - (i\omega_{12} + \Gamma_{12} + \Lambda/2) \rho_{12} + i\Omega_{THz} \rho_{13} + iT_{12} (\rho_{11} - \rho_{22}), \\ \dot{\rho}_{13} &= i\Omega_p \rho_{03} + i\Omega_{THz} \rho_{12} - (i(\omega_{12} + \omega_{23}) + \Gamma_{13} + \Lambda/2) \rho_{13} - iT_{12} \rho_{23}, \\ \dot{\rho}_{23} &= -iT_{12} \rho_{13} - i(\omega_{23} + \Omega_{THz}) \rho_{23} + i\Omega_{THz} (\rho_{22} - \rho_{33}), \\ \dot{\rho}_{00} &= -i\Omega_p (\rho_{01} - \rho_{10}) - \Lambda \rho_{00} + (\gamma_{10} + \Lambda) \rho_{11} + \gamma_{20} \rho_{22} + \gamma_{30} \rho_{33}, \\ \dot{\rho}_{11} &= i\Omega_p (\rho_{01} - \rho_{10}) + \Lambda \rho_{00} - (\gamma_{10} + \Lambda) \rho_{11} + iT_{12} (\rho_{12} - \rho_{21}), \\ \dot{\rho}_{22} &= -iT_{12} (\rho_{12} - \rho_{21}) + i\Omega_{THz} (\rho_{23} - \rho_{32}) - \gamma_{20} \rho_{22}, \\ \dot{\rho}_{33} &= -i\Omega_{THz} (\rho_{23} - \rho_{32}) - \gamma_{30} \rho_{33}, \end{aligned} \quad (6)$$

Where $\rho_{mn} = |m\rangle\langle n|$ ($m, n = 0, 1, 2, 3$) and $\rho_{mm} = |m\rangle\langle m|$ ($m = 0, 1, 2, 3$) [51] represent the coherent terms and the population operators for the QWs, respectively. We get $\omega_{12} = \omega_{10} - \omega_{20}$ and $\omega_{23} = \omega_{20} - \omega_{30}$. The probe field detuning with respect to the QW transition frequencies is $\delta_p = \omega_{10} - \omega_p$. The term $\Lambda = 2(\wp^2/\hbar^2)\Gamma_p$ is the incoherent pumping rate. Note that the incoherent pumping process can also take place in unspecified auxiliary levels. So we assume that the electric field has a broad frequency spectrum or effectively δ -like correlation, i.e., $\langle \varepsilon^*(t)\varepsilon(t') \rangle = \Gamma_p \delta(t - t')$. The spontaneous emission rates for sub band $|i\rangle$, denoted by γ_{10} , are due primarily to longitudinal optical (LO) phonon emission events at low temperature. The total decay rates Γ_{ij} ($i \neq j$) are given by $\Gamma_{0n} = \gamma_{n0}/2 + \gamma_{n0}^{dph}$, $\Gamma_{mn} = (\gamma_{n0} + \gamma_{m0})/2 + \gamma_{mn}^{dph}$, $m, n = 1, 2, 3$ and $m \neq n$, here γ_{mn}^{dph} , are the dephasing rates of the quantum coherence of the $|i\rangle \leftrightarrow |j\rangle$ pathway and determined by electron–electron, interface roughness, and phonon scattering processes. Usually, γ_{mn}^{dph} is the dominant mechanism in a semiconductor solid-state system. Equation (3) can be solved to obtain the steady state response of the medium. The susceptibility of the compact double QWs to the weak probe field is determined by coherence term ρ_{01}

$$\chi = \frac{2N\wp}{E\varepsilon_0} \rho_{01}, \quad (7)$$

Where N is the carrier density in the proposed QWs system. Susceptibility comprise two parts, real and imaginary ($\chi = \chi' + i\chi''$). Note that the real part of the susceptibility χ' correspond to the dispersion and imaginary part χ'' correspond to absorption. The dispersion slope of the probe field has a major role in the group velocity. The group velocity v_g of the light, which is propagates in the medium, given by [52]:

$$v_g = \frac{c}{1 + 2\pi\chi'(\omega_p) + 2\pi\omega_p(\partial\chi'(\omega_p)/\partial\omega_p)}, \quad (8)$$

Equation (8) implies that for a negligible real part of susceptibility, the light propagation can be superluminal

as a negative slope of dispersion, on the other hand, for positive dispersion slope, the light propagation in the medium can be subluminal.

Results and discussion

Now, we analyse the numerical results of the above equations and discuss the transient and the steady-state behaviour of the absorption and the dispersion. It is assumed that the system is initially in the ground state, i.e. $\rho_{00}(0) = 1$ and $\rho_{ij}(0) = 0$ ($i, j = 0, 1, 2, 3$). We take typically spontaneous emission $\gamma_{10} = 1\text{THz}$ [45] and other relevant parameters by the factor of these rates. Introduced rates are equivalent to dephasing times in the order of picoseconds. Here the according to the Eq. (8), the positive and negative dispersion slope are representing the propagation of light subluminal and superluminal respectively.

Figure 2(a) shows the dispersion (dashed) and absorption (solid) properties of a probe field versus wavelength, in the absence of incoherent pumping field Λ and tunnelling effect. We observe that the absorption peak accompanies by a negative dispersion. Thus, superluminal light with large absorption propagates through the medium. In Fig. 2(b) we show at the steady state behaviour of the probe dispersion (dashed) and absorption (solid) in the presence of the first tunnelling effect T_{12} and absence of an incoherent pumping field. It can easily be seen that the absorption of the probe field reduced around $\lambda = 1.55\mu\text{m}$ just by applying tunnelling rate $T_{12} = 1\gamma$. We find that the slope of dispersion is very sensitive to the tunnelling effect. When we increase T_{12} from 0 to 1γ , slope of dispersion changes from negative to positive. Figure 2(c) Shows the effect of the terahertz signal on the system, when keeping all other parameters fixed in Fig. 2(b), and with applying the terahertz signal, the dispersion slope does not change, but tow windows transparency created that is accompanied by three absorption peaks.

In Fig. 3. We investigate the incoherent pumping field effect on the system. In Fig. 3(a) we applied an incoherent pumping field to condition of Fig. 2(a) with values $\Lambda = 8\gamma$. By increasing the incoherent pumping field Λ from 0 to 8γ , the absorption peak of the probe field becomes broaden, while the slope of the dispersion is still negative around the $\lambda = 1.55\mu\text{m}$. Physically, by increasing the incoherent pump rate, the upper levels are populated, for this condition the probe absorption will reduce in transition $|0\rangle \rightarrow |1\rangle$, thus the peak maximum of the probe field absorption gets reduce. In Fig. 3(b), by applying the tunnelling rate T_{12} mid incoherent pumping field, one absorption peak of the probe field is crated at $\lambda = 1.55\mu\text{m}$. By applying the incoherent pumping field, when keeping all other parameters fixed in Fig. 2(b), the slope of dispersion changes from positive to negative that is

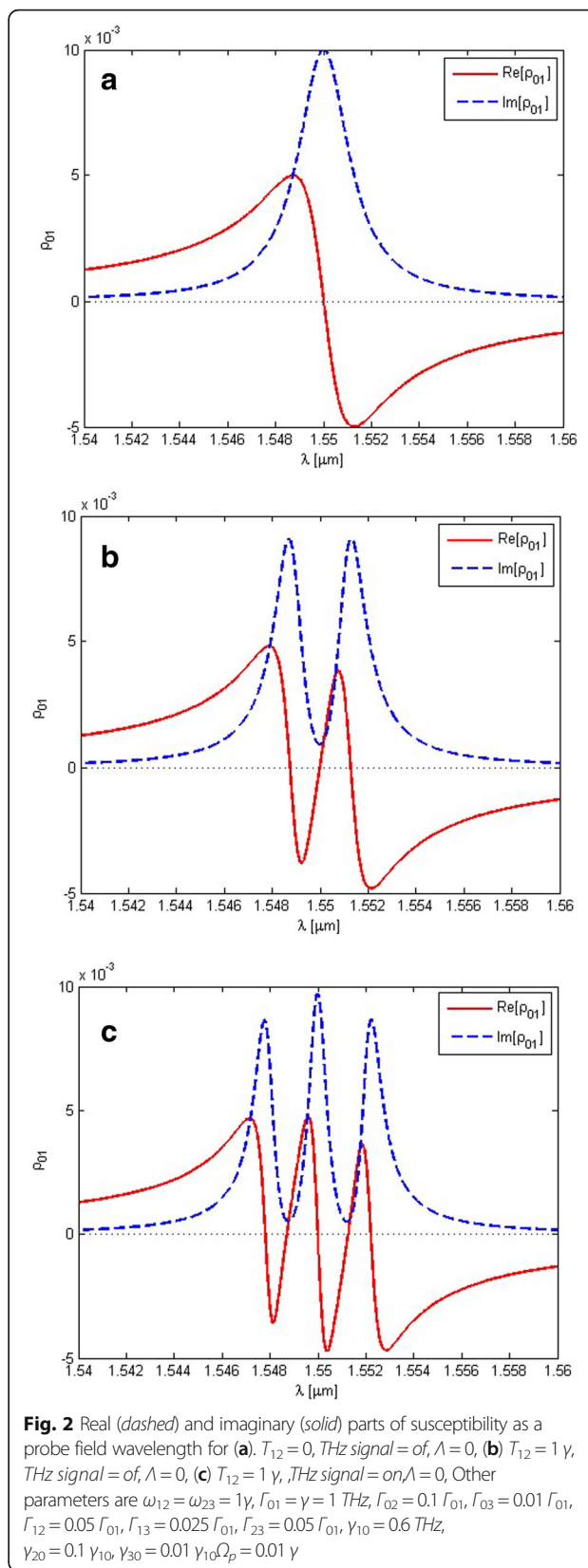


Fig. 2 Real (dashed) and imaginary (solid) parts of susceptibility as a probe field wavelength for (a). $T_{12} = 0$, THz signal = of, $\Lambda = 0$, (b) $T_{12} = 1\gamma$, THz signal = of, $\Lambda = 0$, (c) $T_{12} = 1\gamma$, THz signal = on, $\Lambda = 0$. Other parameters are $\omega_{12} = \omega_{23} = 1\gamma$, $\Gamma_{01} = \gamma = 1\text{THz}$, $\Gamma_{02} = 0.1\Gamma_{01}$, $\Gamma_{03} = 0.01\Gamma_{01}$, $\Gamma_{12} = 0.05\Gamma_{01}$, $\Gamma_{13} = 0.025\Gamma_{01}$, $\Gamma_{23} = 0.05\Gamma_{01}$, $\gamma_{10} = 0.6\text{THz}$, $\gamma_{20} = 0.1\gamma_{10}$, $\gamma_{30} = 0.01\gamma_{10}$, $\Omega_p = 0.01\gamma$

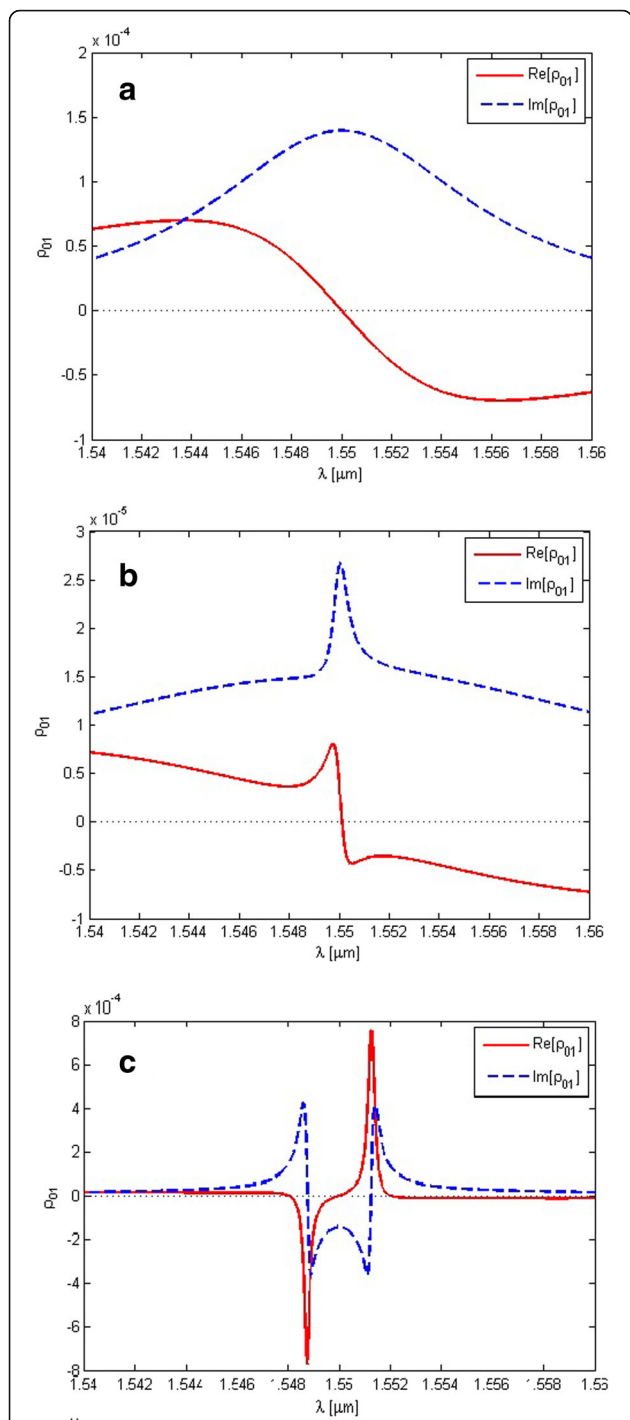


Fig. 3 Real (dashed) and imaginary (solid) parts of susceptibility as a probe field wavelength for (a) $T_{12} = 0$, THz signal = off, $\Lambda = 8\gamma$, (b) $T_{12} = 1\gamma$, THz signal = off, $\Lambda = 8\gamma$, (c) $T_{12} = 1\gamma$, THz signal = on, $\Lambda = 8\gamma$. Other parameters are as in Fig. 2

shown in Figs. 2(b) and 3(b). This indicates that group velocity of light propagates, increasing from subluminal to superluminal through the medium. In Fig. 3(c), we simultaneously apply the tunnelling rate T_{12} between QW_1 and

QW_2 and the terahertz signal, while the incoherent pump rate is still $\Lambda = 8\gamma$. Figure 3(c) show the probe dispersion (solid) and absorption (dashed) in the presence of the terahertz signal and tunnelling effect. It can easily be seen that the slope of dispersion changes from negative to positive as the presence of terahertz signal. Here, by taking into account the incoherent pumping field, tunnelling effect and terahertz signal, the probe gain is appeared around $\lambda = 1.55\mu m$ in the spectrum. Physically, by taking into account the inter-dot tunnel coupling, the coherence in the system is created. So, the electromagnetically induced transparency is established by the inter-dot tunnel coupling. But in the presence of the incoherent pump field, the upper levels are populated, for this condition the probe absorption will reduce in transition. Then, by applying the terahertz signal there will be a population inversion between level $|1\rangle$ and level $|0\rangle$, which may lead to a probe field amplification. This probe amplification (or gain) might be remarkable for laser applications. Also, it is clear that the dispersion around the probe wavelength $\lambda = 1.55\mu m$ is positive, corresponding to the superluminal light propagation. In fact, the group velocity of the probe field strongly sensitive to the rate of the incoherent pump and the tunnelling effect. This

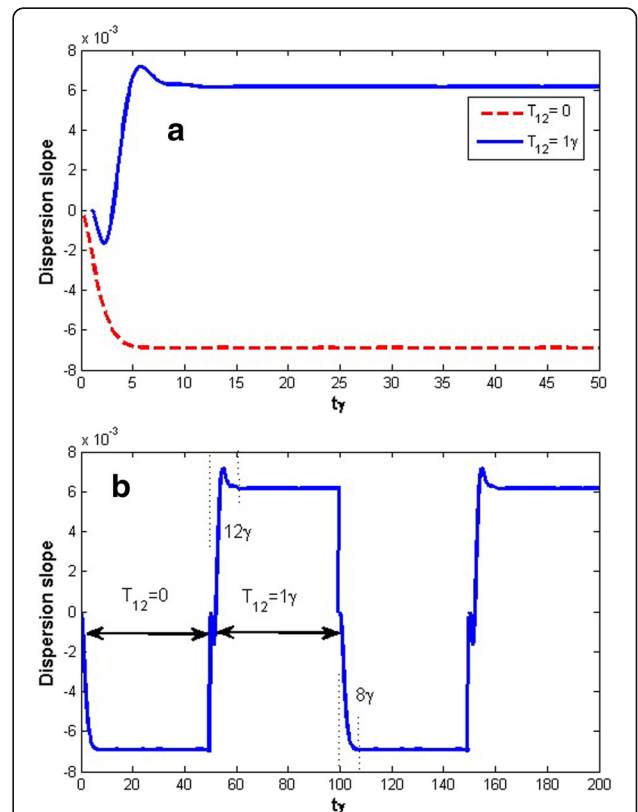
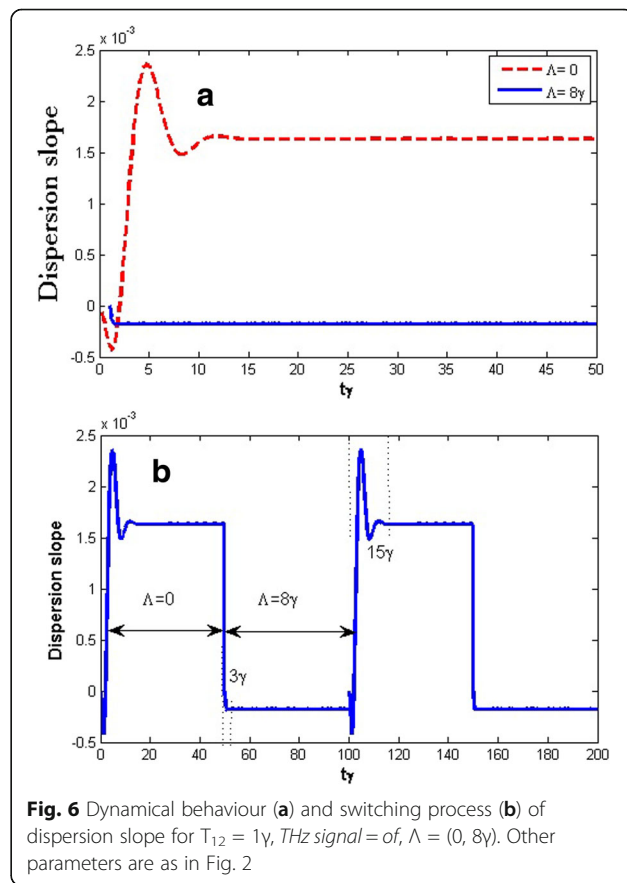
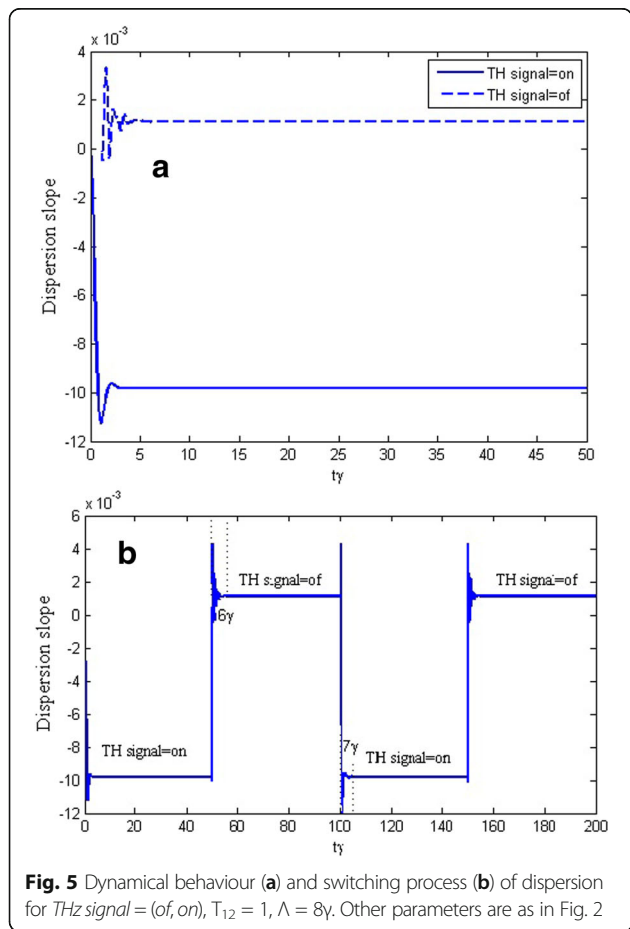


Fig. 4 Dynamical behaviour (a) and switching process (b) of dispersion slope for $T_{12} = (0, 1\gamma)$, THz signal = off, $\Lambda = 0$. Other parameters are as in Fig. 2

is an important mechanism in which the probe field absorption and dispersion can be controlled by the incoherent pump field in the quantum well.

Now, we are interested in the dynamical behaviour of the dispersion slope due to this properties can be used as an electro-optical switch of group velocity from subluminal to superluminal or vice versa. The dispersion slope is variation of the dispersion (dashed line in Figs. 2 and 3) around the $\lambda = 1.55\mu\text{m}$, which is controllable from negative to positive or vis versa by controlling of the parameters. We are interested in the required switching time for change of the light propagation from subluminal to superluminal or vice versa. In fact, this system can be used as an optical switch, in which the propagation of a laser pulse can be controlled with tunnelling effect, incoherent pumping field and terahertz signal. The transient behaviour of dispersion slope of probe field at $\lambda = 1.55\mu\text{m}$ as the controllable parameters are shown. Switching time for subluminal/superluminal light propagation is defined as the time to reach a steady-state from the superluminal state to the subluminal state and vice versa. Figure 4(a), shows the transition behaviour of the dispersion slope by consecutively switching the tunnelling rate from $T_{12} = 0$ to $T_{12} = 1\gamma$ and vice versa. By increasing the normalized time

$t\gamma$ ($\gamma = T_{10} = 1\text{ THz}$), the dispersion slope takes a steady negative values for $T_{12} = 0$ corresponding to superluminal light propagation, while it changed to negative on applying the terahertz signal which is corresponding to subluminal light propagation. In Fig. 4(b), we plot the switching diagram of the dispersion slope for two different values of T_{12} . The required switching time for propagation of the light from subluminal to superluminal is about 8 ps and from superluminal to subluminal is about 12 ps. The transient behaviour of the dispersion slope is displayed for two various rate of terahertz signal, while keeping $T_{12} = 1\gamma$ and $\Lambda = 8\gamma$ fixed in Fig. 4. We are looking for the required switching time for changing the group velocity from subluminal to superluminal or vice versa by proper manipulating the tunnelling rate T_{12} and terahertz signal. Effect of the terahertz signal leads to superluminal propagation of light in the medium (Fig. 5 (a)). Figure 5(b) shows that the required switching time from subluminal propagation light to superluminal propagation or vice versa is about 7γ . Now, we investigate the effect of the incoherent pumping rate by consecutively switching incoherent pumping rate from $\Lambda = 0$ to $\Lambda = 8\gamma$ and vice versa, while the tunnelling rate $T_{12} = 1\gamma$ is fixed in Fig. 4(a). It is clearly find that in the Fig. 6(a) the slope of the dispersion changes from positive to negative by adjusting the



incoherent field. For $\Lambda = 0$ the slope of the dispersion is positive corresponding to subluminal light propagation, while it changes to negative as the incoherent pumping changes from $\Lambda = 0$ to $\Lambda = 8\gamma$ corresponding to superluminal light propagation. The required switching time for change of the propagation light from subluminal to superluminal is about 3 ps, and vice versa is 15 ps Fig. 6(b). This approach can be utilized to produce a switch operating only by controlling the tunnelling rate T_{12} and incoherent rate Λ .

Conclusion

We investigated the transient and the steady-state behaviour of a weak probe field at $\lambda = 1.55\mu\text{m}$ in a compact double coupled QWs system with applying the tunnelling between QWs, terahertz signal and one incoherent pumping field. It is shown that the absorption and the dispersion of the probe field can be controlled by applying the tunnelling between QWs, terahertz signal and incoherent pumping fields. It has also been shown that the medium can be used as an optical switch in which the propagation of the laser pulse can be controlled with tunnelling between QWs and the incoherent pumping field. We obtained the switching time, between 3 to 15 ps as a high-speed optical switch of group velocity from subluminal to superluminal or vice versa.

Abbreviations

CPT: coherent population trapping; OMVPE: organometallic vapour phase epitaxy; QD: quantum dot; QDMs: quantum dot molecules; QW: Quantum well; SGC: spontaneously generated coherence

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Authors' contributions

JSh in the development of the mathematical model and carried out the simulation, contributed in the analysis of the results. AM conceived of the study and finalized the manuscript. Both authors helped to draft the manuscript. Both authors have read and approved the final manuscript.

Ethics approval and consent to participate

Not applicable.

Competing interests

The authors declare that they have no competing interests.

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