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The F-coindex of some graph operations

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Abstract

The F-index of a graph is defined as the sum of cubes of the vertex degrees of the graph. In this paper, we introduce a new invariant which is named as F-coindex. Here, we study basic mathematical properties and the behavior of the newly introduced F-coindex under several graph operations such as union, join, Cartesian product, composition, tensor product, strong product, corona product, disjunction, symmetric difference of graphs and hence apply our results to find the F-coindex of different chemically interesting molecular graphs and nano-structures.

Keywords: Topological index, Vertex degree, First and second Zagreb indices, F-index, F-coindex, Graph operations

Mathematics Subject Classification: Primary 05C35; Secondary 05C07, 05C40

Background

Topological indices are found to be very useful in chemistry, biochemistry and nanotechnology in isomer discrimination, structure–property relationship, structure-activity relationship and pharmaceutical drug design. Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$ respectively. Let, for any vertex $v \in V(G)$, $d_G(v)$ denotes its degree, that is the number of adjacent vertices of v in G. The complement of a graph G is denoted by $\bar{G}$ and is the simple graph with the same vertex set $V(G)$ and any two vertices $uv \in E(\bar{G})$ if and only if $uv \notin E(G)$. Thus $E(G) \cup E(\bar{G}) = E(K_n)$ and $|E(\bar{G})| = \frac{|V(G)|(|V(G)|-1)}{2} - |E(G)|$. Also the degree of a vertex v in $\bar{G}$ is given by $d_{\bar{G}}(v) = |V(G)| - 1 - d_G(v)$.

The first and second Zagreb indices of a graph are among the most studied vertex-degree based topological indices. These indices were introduced by Gutman and Trinajstić (1972) to study the structure-dependency of the total π-electron energy (ε) and are denoted by $M_1(G)$ and $M_2(G)$ respectively. They are defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v).$$

Another vertex-degree based topological index was defined in the same paper where the Zagreb indices were introduced, and that was shown to influence ε. This index was not

further studied until it was studied by Furtula and Gutman (2015) in a recent article. They named this index as "forgotten topological index" or "F-index". F-index of a graph G is denoted by $F(G)$ and is defined as the sum of cubes of the vertex degrees of the graph.

$$\text{i.e., } F(G) = \sum_{v \in V(G)} d_G(v)^3.$$

It can be easily shown that the above definition is equivalent to

$$F(G) = \sum_{uv \in E(G)} \left[d_G(u)^2 + d_G(v)^2 \right].$$

Very recently the present authors have studied the F-index of different graph operations in De et al. (2016).

Doslic (2008) introduced Zagreb coindices while computing weighted Wiener polynomial of certain composite graphs. In this case the sum runs over the edges of the complement of G. Thus the Zagreb coindices of G are defined as

$$\bar{M}_1(G) = \sum_{uv \in E(\bar{G})} [d_G(u) + d_G(v)]$$

and

$$\bar{M}_2(G) = \sum_{uv \in E(\bar{G})} d_G(u) d_G(v).$$

Like Zagreb coindices, corresponding to F-index, we introduce here a new invariant, the F-coindex which is defined as follows.

$$\bar{F}(G) = \sum_{uv \in E(\bar{G})} \left[d_G(u)^2 + d_G(v)^2 \right].$$

Like Zagreb coindices, F-coindex of G is not the F-index of $\bar{G}$. Here the sum runs over $E(\bar{G})$, but the degrees are with respect to G.

Motivation

According to the *International Academy of Mathematical Chemistry*, to identify whether any topological index is useful for prediction of chemical properties, the coorelation between the values of that topological index for different octane isomers and parameter values related to certain physicochemical property of them should be considered. Generally octane isomers are convenient for such studies, because the number of the structural isomers of octane is large (18) enough to make the statistical conclusion reliable. Furtula and Gutman (2015) showed that for octane isomers both M_1 and F yield correlation coefficient greater than 0.95 in case of entropy and acentric factor. They also improved the predictive ability of these index by considering a simple linear model in the form $(M_1 + \lambda F)$, where λ varies from -20 to 20.

In this paper, we find the correlation between the logarithm of the octanol-water partition coefficient (P) and the corresponding F-coindex values of octane isomers. The dataset of octane isomers (first three columns of Table 1) are taken from www.moleculardescriptors.eu/dataset/dataset.htm and the last two columns of Table 1 are computed from the definitions of $F(G)$ and $\bar{F}(G)$. F-coindex values against $\log P$ values are plotted in Fig. 1. Here we find that the correlation coefficient between $\log P$ and $\bar{F}$ is 0.966, whereas the correlation coefficient between $\log P$ and M_1 and that between $\log P$ and F are 0.077 and 0.065 respectively. Thus using this F-coindex, we can predict the $\log P$ values with high accuracy.

Graph operations play an important role in chemical graph theory. Different chemically important graphs can be obtained by applying graph operations on some general or particular graphs. For example, the linear polynomial chain (or the ladder graph L_n) is the molecular graph related to the polynomial structure obtained by the Cartesian product of P_2 and P_{n+1}.

Table 1 Experimental values of the logarithm of the octanol–water partition coefficient and the corresponding values of different topological indices of octane isomers

Molecules	Log *P*	$M_1(G)$	$F(G)$	$\bar{F}(G)$
Octane	3.67	26	50	132
2-Methyl-heptane	3.61	28	62	134
3-Methyl-heptane	3.61	28	62	134
4-Methyl-heptane	3.61	28	62	134
3-Ethyl-hexane	3.61	28	62	134
2,2-Dimethyl-hexane	3.65	32	92	132
2,3-Dimethyl-hexane	3.54	30	74	136
2,4-Dimethyl-hexane	3.54	30	74	136
2,5-Dimethyl-hexane	3.54	30	74	136
3,3-Dimethyl-hexane	3.65	32	92	132
3,4-Dimethyl-hexane	3.54	30	74	136
2-Methyl-3-ethyl-pentane	3.54	30	74	136
3-Methyl-3-ethyl-pentane	3.65	32	92	132
2,2,3-Trimethyl-pentane	3.58	34	104	134
2,2,4-Trimethyl-pentane	3.58	34	104	134
2,3,3-Trimethyl-pentane	3.58	34	104	134
2,3,4-Trimethyl-pentane	3.48	32	86	138
2,2,3,3-Tetramethyl-butane	3.62	38	134	132

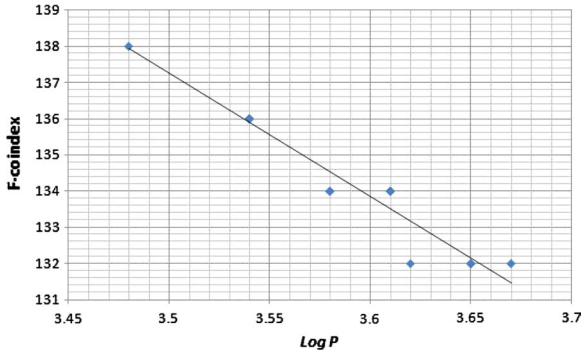

Fig. 1 Experimental values of log *P* versus calculated values of *F*-coindices of octane isomers

The C_4 nanotube $TUC_4(m,n)$ is the Cartesian product of P_n and P_m and the C_4 nanotorus $TC_4(m,n)$ is the Cartesian product of C_n and C_m. For a given graph G, one of the hydrogen suppressed molecular graph is the bottleneck graph, which is the corona product of K_2 and G. There are several studies on various topological indices under different graph operations available in the literature. Khalifeh et al. (2009) derived some exact formulae for computing first and second Zagreb indices under some graph operations. Das et al. (2013), derived some upper bounds for multiplicative Zagreb indices for different graph operations. Veylaki et al. (2015), computed third and hyper-Zagreb coindices of some graph operations. In De et al. (2014), the present authors computed some bounds and exact formulae of the connective eccentric index under different graph operations. Azari and Iranmanesh (2013) presented explicit formulas for computing the eccentric-distance sum of different graph operations. Interested readers are referred to Ashrafi et al. (2010), Khalifeh et al. (2008), Tavakoli et al. (2014), De et al. (2015a, b, c, d, Eskender and Vumar (2013) for other studies in this regard.

In this paper, we first derive some basic properties of F-coindex and hence present some exact expressions for the F-coindex of different graph operations such as union, join, Cartesian product, composition, tensor product, strong product, corona product, disjunction, symmetric difference of graphs. Also we apply our results to compute the F-coindex for some important classes of molecular graphs and nano-structures.

Basic properties of F-coindex

From definition, the F-coindex for some special graphs such as complete graph, empty graph, path, cycle and complete bipartite graph on n vertices can be easily obtained as follows.

(i) $\bar{F}(K_n) = \bar{F}(\bar{K}_n) = 0$,
(ii) $\bar{F}(C_n) = 4n(n-3)$,
(iii) $\bar{F}(P_n) = 4n^2 - 18n + 20$,
(iv) $\bar{F}(K_{m,n}) = mn(2mn - m - n)$.

Let for the graph G we use the notation $|V(G)| = n$ and $|E(G)| = m$. Also let $|E(\bar{G})| = \bar{m}$. Now first we explore some basic properties of F-coindex.

Proposition 1 *Let G be a simple graph with n vertices and m edges, then*

$$F(\bar{G}) = 2(n-1)^2(\bar{m} - 2m) + 3(n-1)M_1(G) - F(G).$$

Proof From definition of F-index, we have

$$\begin{aligned} F(\bar{G}) &= \sum_{v \in V(\bar{G})} d_{\bar{G}}(v)^3 \\ &= \sum_{v \in V(G)} [n - 1 - d_G(v)]^3 \\ &= \sum_{v \in V(G)} \left[(n-1)^3 - 3(n-1)^2 d_G(v) + 3(n-1)d_G(v)^2 - d_G(v)^3\right] \\ &= n(n-1)^3 - 6m(n-1)^2 + 3(n-1)M_1(G) - F(G) \\ &= 2(n-1)^2(\bar{m} - 2m) + 3(n-1)M_1(G) - F(G). \end{aligned}$$

□

Proposition 2 *Let G be a simple graph with n vertices and m edges, then*

$$\bar{F}(G) = F\left(\bar{G}\right) - 2(n-1)M_1\left(\bar{G}\right) + 2\bar{m}(n-1)^2.$$

Proof From definition of F-coindex, we have

$$\begin{aligned}
\bar{F}(G) &= \sum_{uv\notin E(G)} \left[d_G(u)^2 + d_G(v)^2\right] \\
&= \sum_{uv\in E(\bar{G})} \left[\left\{n-1-d_{\bar{G}}(u)\right\}^2 + \left\{n-1-d_{\bar{G}}(v)\right\}^2\right] \\
&= \sum_{uv\in E(\bar{G})} \left[(n-1)^2 + d_{\bar{G}}(u)^2 - 2(n-1)d_{\bar{G}}(u) + (n-1)^2 + d_{\bar{G}}(v)^2 - 2(n-1)d_{\bar{G}}(v)\right] \\
&= 2\bar{m}(n-1)^2 + \sum_{uv\in E(\bar{G})} \left[d_{\bar{G}}(u)^2 + d_{\bar{G}}(v)^2\right] - 2(n-1) \sum_{uv\in E(\bar{G})} \left[d_{\bar{G}}(u) + d_{\bar{G}}(v)\right] \\
&= 2\bar{m}(n-1)^2 + F\left(\bar{G}\right) - 2(n-1)M_1\left(\bar{G}\right).
\end{aligned}$$

□

An alternative expression for $\bar{F}(G)$ can be obtained by considering sum over the edges of G and $\bar{G}$ respectively as follows.

Proposition 3 *Let G be a simple graph with n vertices and m edges, then*

$$\bar{F}(G) = (n-1)M_1(G) - F(G).$$

Proof From definition of F-index and F-coindex, it follows that

$$\begin{aligned}
F(G) + \bar{F}(G) &= \sum_{uv\in E(G)} \left[d_G(u)^2 + d_G(v)^2\right] + \sum_{uv\notin E(G)} \left[d_G(u)^2 + d_G(v)^2\right] \\
&= \sum_{u,v\in V(G)} \left[d_G(u)^2 + d_G(v)^2\right] \\
&= (n-1) \sum_{v\in V(G)} d_G(v)^2 = (n-1)M_1(G),
\end{aligned}$$

from where the desired result follows. □

Proposition 4 *Let G be a simple graph with n vertices and m edges, then*

$$\bar{F}\left(\bar{G}\right) = 2m(n-1)^2 - (n-1)M_1(G) - \bar{F}(G).$$

Proof From definition of F-coindex, we have

$$\begin{aligned}
\bar{F}(\bar{G}) &= \sum_{uv\notin E(\bar{G})} \left[d_{\bar{G}}(u)^2 + d_{\bar{G}}(v)^2\right] \\
&= \sum_{uv\in E(G)} \left[\{n-1-d_G(u)\}^2 + \{n-1-d_G(v)\}^2\right] \\
&= \sum_{uv\in E(G)} \left[(n-1)^2 + d_G(u)^2 - 2(n-1)d_G(u) + (n-1)^2 + d_G(v)^2 - 2(n-1)d_G(v)\right] \\
&= 2m(n-1)^2 + \sum_{uv\in E(G)} \left[d_G(u)^2 + d_G(v)^2\right] - 2(n-1)\sum_{uv\in E(G)} [d_G(u) + d_G(v)] \\
&= 2m(n-1)^2 + F(G) - 2(n-1)M_1(G) \\
&= 2m(n-1)^2 - (n-1)M_1(G) - \bar{F}(G).
\end{aligned}$$

□

Main results

In the following, we study F-coindex of various graph operations like union, join, Cartesian product, composition, tensor product, strong product, corona product, disjunction, symmetric difference of graphs. These operations are binary and if not indicated otherwise, we use the notation $V(G_i)$ for the vertex set, $E(G_i)$ for the edge set, n_i for the number of vertices and m_i for the number of edges of the graph G_i respectively. Also let $\bar{m}_i$ denote the number of edges of the graph $\bar{G}_i$.

Union

The union of two graphs G_1 and G_2 is the graph denoted by $G_1 \cup G_2$ with the vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. In this case we assume that $V(G_1)$ and $V(G_2)$ are disjoint. The degree of a vertex v of $G_1 \cup G_2$ is equal to that of the vertex in the component G_i $(i = 1, 2)$ which contains it. In the following preposition we calculate the F-coindex of $G_1 \cup G_2$.

Proposition 5 *Let G be a simple graph with n vertices and m edges, then*

$$\bar{F}(G_1 \cup G_2) = \bar{F}(G_1) + \bar{F}(G_2) + n_2M_1(G_1) + n_1M_1(G_2).$$

Proof From definition of F-coindex, it is clear that, the F-coindex of $G_1 \cup G_2$ is equal to the sum of the F-coindices of the components $G_i (i = 1, 2)$, in addition to the contributions of the missing edges between the components which form the edge set of the complete bipartite graph K_{n_1,n_2}. The contribution of these missing edges is given by

$$\sum_{u\in V(G_1)} \left[\sum_{v\in V(G_2)} \left\{d_G(u)^2 + d_G(v)^2\right\}\right] = n_2M_1(G_1) + n_1M_1(G_2),$$

from where the desired result follows. □

Join

The join of two graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ is the graph denoted by $G_1 + G_2$ with the vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$. Thus in the sum of two graphs, all the

vertices of one graph is connected with all the vertices of the other graph, keeping all the edges of both graphs. Thus the degree of the vertices of $G_1 + G_2$ is given by

$$d_{G_1+G_2}(v) = \begin{cases} d_{G_1}(v) + n_2, & v \in V(G_1) \\ d_{G_2}(v) + n_1, & v \in V(G_2). \end{cases}$$

In the following proposition the F-coindex of $G_1 + G_2$ is calculated.

Proposition 6 *Let G be a simple graph with n vertices and m edges, then*

$$\bar{F}(G_1 + G_2) = \bar{F}(G_1) + \bar{F}(G_2) + 2n_2\bar{M}_1(G_1) + 2n_1\bar{M}_1(G_2) + 2{n_2}^2\bar{m}_1 + 2{n_1}^2\bar{m}_2.$$

Proof From definition of $G_1 + G_2$, it is clear that the contribution of the edges connecting the vertices of G_1 with those of G_2 is zero. So the F-coindex of $G_1 + G_2$ is given by

$$\begin{aligned} \bar{F}(G_1 + G_2) &= \sum_{uv \notin E(G_1+G_2)} \left[d_{(G_1+G_2)}(u)^2 + d_{(G_1+G_2)}(v)^2 \right] \\ &= \sum_{uv \notin E(G_1)} \left[d_{(G_1+G_2)}(u)^2 + d_{(G_1+G_2)}(v)^2 \right] \\ &\quad + \sum_{uv \notin E(G_2)} \left[d_{(G_1+G_2)}(u)^2 + d_{(G_1+G_2)}(v)^2 \right] = J_1 + J_2. \end{aligned}$$

Now,

$$\begin{aligned} J_1 &= \sum_{uv \notin E(G_1)} \left[d_{(G_1+G_2)}(u)^2 + d_{(G_1+G_2)}(v)^2 \right] \\ &= \sum_{uv \notin E(G_1)} \left[(d_{G_1}(u) + n_2)^2 + (d_{G_1}(v) + n_2)^2 \right] \\ &= \sum_{uv \notin E(G_1)} \left[d_{G_1}(u)^2 + {n_2}^2 + 2n_2 d_{G_1}(u) + d_{G_1}(v)^2 + {n_2}^2 + 2n_2 d_{G_1}(v) \right] \\ &= \sum_{uv \notin E(G_1)} \left[d_{G_1}(u)^2 + d_{G_1}(v)^2 \right] + 2n_2 \sum_{uv \notin E(G_1)} \left[d_{G_1}(u) + d_{G_1}(v) \right] + 2{n_2}^2\bar{m}_1 \\ &= \bar{F}(G_1) + 2n_2\bar{M}_1(G_1) + 2{n_2}^2\bar{m}_1. \end{aligned}$$

Similarly, we get

$$J_2 = \bar{F}(G_2) + 2n_1\bar{M}_1(G_2) + 2{n_1}^2\bar{m}_2.$$

Combining J_1 and J_2 we get the desired result after simplification. □

Example 1 The complete bipartite graph $K_{p,q}$ can be defined as $K_{p,q} = \bar{K}_p + \bar{K}_q$. So its F-coindex can be calculated from the previous proposition as $\bar{F}(K_{p,q}) = pq(2pq - p - q)$.

The suspension of a graph G is defined as sum of G with a single vertex. So from the previous proposition the following corollary follows.

Corollary 1 *The F-coindex of suspension of G is given by*

$$\bar{F}(G + K_1) = \bar{F}(G) + 2\bar{M}_1(G) + 2\bar{m}.$$

Example 2 The star graph S_n with n vertices is the suspension of empty graph $\bar{K}_{n-1}$. So its F-coindex can be calculated from the previous corollary as $\bar{F}(S_n) = (n-1)(n-2)$.

Example 3 The wheel graph W_n on $(n+1)$ vertices is the suspension of C_n. So from the previous corollary its F-coindex is given by $\bar{F}(W_n) = 9n(n-3)$.

Example 4 The fan graph F_n on $(n+1)$ vertices is the suspension of P_n. So from the previous corollary its F-coindex is given by $\bar{F}(W_n) = 9n^2 - 37n + 38$.

We now extend the join operation to more than two graphs. Let $G_1, G_2, \ldots, G_k$ be k graphs. Then, the degree of a vertex v in $G_1 + G_2 + \cdots + G_k$ is given by $d_{G_1+G_2+\cdots+G_k}(v) = d_{G_i}(v) + n - n_i$, where v is originally a vertex of the graph G_i and $n = n_1 + n_2 + \cdots + n_k$. Also let $\bar{n}_i = n - n_i$.

Proposition 7 *The F-coindex of $G_1 + G_2 + \cdots + G_k$ is given by*

$$\bar{F}(G_1 + G_2 + \cdots + G_k) = \sum_{i=1}^{k} \bar{F}(G_i) + 2\sum_{i=1}^{k} \bar{n}_i \bar{M}_1(G_i) + 2\sum_{i=1}^{k} \bar{n}_i^2 \bar{m}_i.$$

Proof We have from definition of F-coindex

$$\begin{aligned}
\bar{F}(G_1 + G_2 + \cdots + G_k) &= \sum_{i=1}^{k} \sum_{uv \notin E(G_i)} \left[d_{(G_1+G_2+\cdots+G_k)}(u)^2 + d_{(G_1+G_2+\cdots+G_k)}(v)^2 \right] \\
&= \sum_{i=1}^{k} \sum_{uv \notin E(G_1)} \left[\left(d_{G_i}(u) + \bar{n}_i\right)^2 + \left(d_{G_i}(v) + \bar{n}_i\right)^2 \right] \\
&= \sum_{i=1}^{k} \sum_{uv \notin E(G_1)} \left[d_{G_i}(u)^2 + \bar{n}_i^2 + 2\bar{n}_i d_{G_i}(u) + d_{G_i}(v)^2 + \bar{n}_i^2 + 2\bar{n}_i d_{G_i}(v) \right] \\
&= \sum_{i=1}^{k} \sum_{uv \notin E(G_1)} \left[d_{G_i}(u)^2 + d_{G_i}(v)^2 \right] \\
&\quad + \sum_{i=1}^{k} 2\bar{n}_i \sum_{uv \notin E(G_1)} \left[d_{G_i}(u) + d_{G_i}(v) \right] + \sum_{i=1}^{k} 2\bar{n}_i^2 \bar{m}_i \\
&= \sum_{i=1}^{k} \bar{F}(G_i) + 2\sum_{i=1}^{k} \bar{n}_i \bar{M}_1(G_i) + 2\sum_{i=1}^{k} \bar{n}_i^2 \bar{m}_i,
\end{aligned}$$

which completes the proof. □

Cartesian product

The Cartesian product of G_1 and G_2, denoted by $G_1 \times G_2$, is the graph with vertex set $V(G_1) \times V(G_2)$ and any two vertices (u_p, v_r) and (u_q, v_s) are adjacent if and only if $[u_p = u_q \in V(G_1)$ and $v_r v_s \in E(G_2)]$ or $[v_r = v_s \in V(G_2)$ and $u_p u_q \in E(G_1)]$. Thus we

have, $d_{G_1\times G_2}(a,b) = d_{G_1}(a) + d_{G_2}(b)$. In the following preposition we calculate the F-coindex of $G_1 \times G_2$.

Proposition 8 *The F-coindex of* $G_1 \times G_2$ *is given by*

$$\bar{F}(G_1 \times G_2) = \{n_2(n_1n_2 - 1) - 6m_2\}M_1(G_1) + \{n_1(n_1n_2 - 1) - 6m_1\}M_1(G_2) - n_2F(G_1) - n_1F(G_2) + 8m_1m_2(n_1n_2 - 1).$$

Proof Applying Theorem 1 of Khalifeh et al. (2009) and Theorem 3 of De et al. in Proposition 3 we get

$$\begin{aligned}\bar{F}(G_1 \times G_2) &= (|V(G_1 \times G_2)| - 1)M_1(G_1 \times G_2) - F(G_1 \times G_2)\\ &= (n_1n_2 - 1)[n_2M_1(G_1) + n_1M_1(G_2) + 8m_1m_2] - [n_2F(G_1) + n_1F(G_2)\\ &\quad + 6m_2M_1(G_1) + 6m_1M_1(G_2)]\\ &= (n_1n_2 - 1)n_2M_1(G_1) + (n_1n_2 - 1)n_1M_1(G_2) + 8m_1m_2(n_1n_2 - 1)\\ &\quad - n_2F(G_1) - n_1F(G_2) - 6m_2M_1(G_1) - 6m_1M_1(G_2),\end{aligned}$$

from where the desired result follows after simplification. □

Example 5 The Ladder graph L_n (linear polynomial chain) is the Cartesian product of P_2 and P_{n+1}. Thus from the last proposition the following result follows

$$\bar{F}(L_n) = 36n^2 - 40n + 20.$$

Example 6 $TUC_4(m,n)$ and $TC_4(m,n)$ denote a C_4 nanotube and nanotorus respectively. Then $TUC_4(m,n) \cong P_n \times C_m$ and $TC_4(m,n) \cong C_n \times C_m$, and so $\bar{F}(TUC_4(m,n)) = 16m^2n^2 - 14m^2n - 80mn + 88m$ and $\bar{F}(TC_4(m,n)) = 16mn(mn - 5)$.

Composition

The composition of two graphs G_1 and G_2 is denoted by $G_1[G_2]$ and any two vertices (u_1,u_2) and (v_1,v_2) are adjacent if and only if $u_1v_1 \in E(G_1)$ or $[u_1 = v_1$ and $u_2v_2 \in E(G_2)]$. The vertex set of $G_1[G_2]$ is $V(G_1) \times V(G_2)$ and the degree of a vertex (a, b) of $G_1[G_2]$ is given by $d_{G_1[G_2]}(a,b) = n_2d_{G_1}(a) + d_{G_2}(b)$. In the following proposition we compute the F-coindex of the composition of two graphs.

Proposition 9 *The F-coindex of* $G_1[G_2]$ is given by

$$\bar{F}(G_1[G_2]) = {n_2}^2\{n_2(n_1n_2 - 1) - 6m_2\}M_1(G_1) + \{n_1(n_1n_2 - 1) - 6n_2m_1\}M_1(G_2) - {n_2}^4F(G_1) - n_1F(G_2) + 8n_2m_1m_2(n_1n_2 - 1).$$

The proof of the above proposition follows from the expressions of first Zagreb index and F-index of strong product graphs given in Theorems 3 and 4 of Khalifeh et al. (2009) and De et al. respectively.

Example 7 The fence graph is the composition of P_n and P_2 and the Closed fence graph is the composition of C_n and P_2. Thus, we have

(i) $\bar{F}(P_n[P_2]) = 100n^2 - 428n + 456$,
(ii) $\bar{F}(C_n[P_2]) = 100n^2 - 300n$.

Tensor product

The tensor product of two graphs G_1 and G_2 is denoted by $G_1 \otimes G_2$ and any two vertices (u_1, v_1) and (u_2, v_2) are adjacent if and only if $u_1u_2 \in E(G_1)$ and $v_1v_2 \in E(G_2)$. The degree of a vertex (a, b) of $G_1 \otimes G_2$ is given by $d_{G_1\otimes G_2}(a, b) = d_{G_1}(a)d_{G_2}(b)$. In the following proposition, the F-coindex of the tensor product of two graphs is computed.

Proposition 10 *The F-coindex of $G_1 \otimes G_2$ is given by*

$$\bar{F}(G_1 \otimes G_2) = (n_1n_2 - 1)M_1(G_1)M_1(G_2) - F(G_1)F(G_2).$$

The proof follows from the expressions $M_1(G_1 \otimes G_2) = M_1(G_1)M_1(G_2)$ established in Theorem 2.1 of Yarahmadi (2011) and $F(G_1 \otimes G_2) = F(G_1)F(G_2)$ established in Theorem 7 of De et al.

Example 8 (i) $\bar{F}(P_n \otimes P_m) = 4(mn - 1)(2n - 3)(2m - 3) - 4(4n - 7)(4m - 7)$
(ii) $\bar{F}(C_n \otimes C_m) = 16mn(mn - 5)$
(iii) $\bar{F}(K_n \otimes K_m) = nm(n - 1)^2(m - 1)^2(m + n - 1)$
(iv) $\bar{F}(P_n \otimes C_m) = 4m(mn - 1)(2n - 3)(2m - 3) - 4(4n - 7)(4m - 7)$
(v) $\bar{F}(P_n \otimes k_m) = m(mn - 1)(4n - 6)(m - 1)^2 - m(8n - 14)(m - 1)^3$
(vi) $\bar{F}(C_n \otimes K_m) = 4nm(m - 1)^2(mn - 2m + 1)$.

Strong product graphs

The strong product of two graphs G_1 and G_2 is denoted by $G_1 \boxtimes G_2$. It has the vertex set $V(G_1) \times V(G_2)$ and any two vertices (u_p, v_r) and (u_q, v_s) are adjacent if and only if $[u_p = u_q \in V(G_1)$ and $v_rv_s \in E(G_2)]$ or $[v_r = v_s \in V(G_2)$ and $u_pu_q \in E(G_1)]$ or $[u_pu_q \in E(G_1)$ and $v_rv_s \in E(G_2)]$. Note that if both G_1 and G_2 are connected then $G_1 \boxtimes G_2$ is also connected. The degree of a vertex (a, b) of $G_1 \boxtimes G_2$ is given by

$$d_{G_1\boxtimes G_2}(a, b) = d_{G_1}(a) + d_{G_2}(b) + d_{G_1}(a)d_{G_2}(b).$$

In the following proposition we compute the F-coindex of the strong product of two graphs.

Proposition 11 *The F-coindex of $G_1 \boxtimes G_2$ is given by*

$$\begin{aligned}\bar{F}(G_1 \boxtimes G_2) = {} & \{(n_1n_2 - 1)(n_2 + 4m_2) - 6m_2\}M_1(G_1) + \{(n_1n_2 - 1)(n_1 + 4m_1) - 6m_1\}M_1(G_2) \\ & + (n_1n_2 - 7)M_1(G_1)M_1(G_2) - (n_2 + 6m_2)F(G_2) - 3F(G_2)M_1(G_1) \\ & - 3F(G_1)M_1(G_2) - F(G_1)F(G_2) - 8m_1m_2(n_1n_2 - 1).\end{aligned}$$

The proof follows from the expressions of first Zagreb index and F-index of strong product graphs from Theorems 2.6 and 6 of Tavakoli et al. (2013) and De et al. respectively.

Corona product

The corona product $G_1 \circ G_2$ of two graphs G_1 and G_2 is obtained by taking one copy of G_1 and n_1 copies of G_2 and by joining each vertex of the ith copy of G_2 to the ith vertex of G_1,

where $1 \leq i \leq n_1$. The corona product of G_1 and G_2 has total $(n_1n_2 + n_1)$ number of vertices and $(m_1 + n_1m_2 + n_1n_2)$ number of edges. Different topological indices under the corona product of graphs have already been studied (Yarahmadi and Ashrafi 2012; De et al. 2015e; Pattabiraman and Kandan 2014). It is easy to see that the degree of a vertex v of $G_1 \circ G_2$ is given by

$$d_{G_1 \circ G_2}(v) = \begin{cases} d_{G_1}(v) + n_2, & v \in V(G_1) \\ d_{G_{2,i}}(v) + 1, & v \in V(G_{2,i}), \quad i = 1, 2, \ldots, n_1. \end{cases}$$

In the following proposition, the F-coindex of the corona product of two graphs is computed.

Proposition 12 *The F-coindex of $G_1 \circ G_2$ is given by*

$$\begin{aligned} \bar{F}(G_1 \circ G_2) = {} & (n_1n_2 + n_1 - 3n_2 - 1)M_1(G_1) + n_1(n_1n_2 + n_1 - 4)M_1(G_2) \\ & - F(G_1) - n_1F(G_2) + 4(n_1n_2 + n_1 - 1)(n_1m_2 + n_2m_1) + n_1n_2(n_1n_2 \\ & + n_1 - 1)(n_2 + 1) - 6n_1m_2 - 6{n_2}^2m_1 - n_1n_2\left({n_2}^2 + 1\right). \end{aligned}$$

The proof of the above proposition follows from the relations

$$M_1(G_1 \circ G_2) = M_1(G_1) + n_1M_1(G_2) + 4(n_2m_1 + n_1m_2) + n_1n_2(n_2 + 1)$$

given in Theorem 2.8 of Yarahmadi and Ashrafi (2012) and

$$\begin{aligned} F(G_1 \circ G_2) = {} & F(G_1) + n_1F(G_2) + 3n_2M_1(G_1) + 3n_1M_1(G_2) \\ & + 6{n_2}^2m_1 + 6n_1m_2 + n_1n_2({n_2}^2 + 1) \end{aligned}$$

given in Theorem 7 of De et al.

Example 9 One of the hydrogen suppressed molecular graph is the bottleneck graph of a graph G, is the corona product of K_2 and G, where G is a given graph. F-coindex of bottleneck graph of G is given by

$$\bar{F}(K_2 \circ G) = 2F(G) + 6M_1(G) + 2n^3 + 6n^2 + 8n + 12m + 2,$$

where n is the number of vertices of G.

A t-thorny graph is obtained by joining t-number of thorns (pendent edges) to each vertex of a given graph G. A variety of topological indices of thorn graphs have been studied by a number of researchers (De 2012a, b; Alizadeh et al. 2014). It is well known that, the t-thorny graph of G is defined as the corona product of G and complement of complete graph with t vertices $\bar{K_t}$. Thus from the previous theorem the following corollary follows.

Corollary 2 *The F-coindex of t-thorny graph of G is given by*

$$\begin{aligned} \bar{F}\left(G^t\right) = {} & (nt + n - 3t - 1)M_1(G) - F(G) + 4mt(nt + n - 1) - 6mt^2 \\ & + nt(t + 1)(nt + n - 1) - nt(t^2 + 1). \end{aligned}$$

Example 10 The F-coindex of t-thorny graph of C_n and P_n are given by

(i) $\bar{F}\left(C_n{}^t\right) = n^2t^3 - nt^3 + 6n^2t^2 - 7nt^2 + 9n^2t - 18nt + 4n^2 - 12n$
(ii) $\bar{F}\left(P_n{}^t\right) = n^2t^3 - nt^3 + 6n^2t^2 - 11nt^2 + 9n^2t - 28nt + 4n^2 + 6t^2 - 18n + 22t + 20.$

Disjunction

The disjunction of two graphs G_1 and G_2, denoted by $G_1 \wedge G_2$, consists of the vertex set $V(G_1) \times V(G_2)$ and two vertices (u_1, v_1) and (u_2, v_2) are adjacent whenever $u_1u_2 \in E(G_1)$ or $v_1v_2 \in E(G_2)$. Clearly, the degree of a vertex (u_1, u_2) of $G_1 \wedge G_2$ is given by

$$d_{G_1\wedge G_2}(u_1, u_2) = n_1 d_{G_1}(u_1) + n_2 d_{G_2}(u_2) - d_{G_1}(u_1)d_{G_2}(u_2).$$

In the following theorem we obtain the F-coindex of the disjunction of two graphs.

Proposition 13 *The F-coindex of $G_1 \wedge G_2$ is given by*

$$\begin{aligned}\bar{F}(G_1 \wedge G_2) = {} & {n_2}^2\left(6m_2 - {n_2}^2\right)F(G_1) + {n_1}^2\left(6m_1 - {n_1}^2\right)F(G_2) + F(G_1)F(G_2)\\ & - 3n_2F(G_1)M_1(G_2) + n_2\{(n_1n_2 - 1)(n_1n_2 - 4m_2) - 6n_1n_2m_2\}M_1(G_1)\\ & + n_1\{(n_1n_2 - 1)(n_1n_2 - 4m_1) - 6n_1n_2m_1\}M_1(G_2) - 3n_1F(G_2)M_1(G_1)\\ & + (7n_1n_2 - 1)M_1(G_1)M_1(G_2) + 8n_1n_2m_1m_2(n_1n_2 - 1).\end{aligned}$$

The proof of the above proposition follows from Proposition 3 with the relevant results from Khalifeh et al. (2009) and De et al.

Symmetric difference

The symmetric difference of two graphs G_1 and G_2 is denoted by $G_1 \oplus G_2$, so that $|V(G_1 \oplus G_2)| = |V(G_1)| \times |V(G_2)|$ and

$$E(G_1 \oplus G_2) = \left\{(u_1, u_2)(v_1, v_2) : u_1v_1 \in E(G_1) \quad \text{or} \quad u_2v_2 \in E(G_2) \ \text{ but not both}\right\}.$$

From definition of symmetric difference it is clear that

$$d_{G_1\oplus G_2}(v_1, v_2) = n_2 d_{G_1}(v_1) + n_1 d_{G_2}(v_2) - 2d_{G_1}(v_1)d_{G_2}(v_2).$$

In the following proposition we obtain the F-coindex of the symmetric difference of two graphs.

Proposition 14 *The F-coindex of $G_1 \oplus G_2$ is given by*

$$\begin{aligned}\bar{F}(G_1 \oplus G_2) = {} & {n_2}^2\left(12m_2 - {n_2}^2\right)F(G_1) + {n_1}^2\left(12m_1 - {n_1}^2\right)F(G_2) + 8F(G_1)F(G_2)\\ & - 12n_2F(G_1)M_1(G_2) + n_2\{(n_1n_2 - 1)(n_1n_2 - 8m_2) - 6n_1n_2m_2\}M_1(G_1)\\ & + n_1\{(n_1n_2 - 1)(n_1n_2 - 8m_1) - 6n_1n_2m_1\}M_1(G_2) - 12n_1F(G_2)M_1(G_1)\\ & + 4(4n_1n_2 - 1)M_1(G_1)M_1(G_2) + 8n_1n_2m_1m_2(n_1n_2 - 1).\end{aligned}$$

Conclusion

In this paper, we have studied the F-coindex of different graph operations and also apply our results to find F-coindex of some special and chemically interesting graphs.

However, there are still many other graph operations and special classes of graphs which are not covered here. So, for further research, F-coindex of various other graph operations and composite graphs can be considered.

Authors' contributions

All of the authors have significant contributions to this paper and the final form of this paper is approved by all of them. All authors read and approved the final manuscript.

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Competing interests

The authors declare that they have no competing interests.

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