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Analyzing the effects of the choice of model in the context of marginal changes in final demand

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Abstract

Literature on the choice of model for deriving an input–output table (IOT) from a pair of supply–use tables (SUTs) has focused on the consequences for the IOT and the Leontief inverse. Analyzing the technology and fixed sales structure transformation models and their applications involving impact analysis and multipliers of factor inputs or environmental extensions, we prove that the product technology and fixed sales structure assumption models are effectively identical and so are the industry technology and fixed product sales structure models. A dimensional analysis shows that the product technology and fixed sales structure assumption models maintain consistency in accounting units, while the industry technology and fixed product sales structure models do not. Comparison with selected topics in environmental life cycle assessment (LCA) shows that the commodity technology and fixed industry sales structure models yield results that are compatible with mainstream LCA. We conclude these models are "correct" in the context of impact analysis and multipliers of the satellite of a SUT/IOT system, despite the fact that they may result in "negatives." We propose a new quantity, the intensity matrix, and highlight its benefits in terms of the consistency of dimension and ease of interpretation. We illustrate our findings with examples of a SUT/IOT for several EU countries. We finally discuss briefly the possibility of calculating contributions to multipliers, where it is shown that models that are equivalent in terms of observable results (multipliers) disagree on unobservable quantities (contributions to multipliers).

Keywords: Supply–use table (SUT), Input–output table (IOT), Choice of model, Multiplier, Life cycle assessment (LCA)

1 Introduction

Input–output analysis (IOA) as a technique for investigating the economy-wide effect of changes in demand on the basis of the input–output table (IOT) was introduced by Leontief (1936). Stone (1961) incorporated the IOT in the system of national accounts (SNA). One of the crucial elements in this was the introduction of the framework of supply–use tables (SUT). Nowadays, the situation is such that national statistical offices, such as BEA, compile SUTs, and that policy researchers, involved in planning and forecasting, use IOTs to do IOA. In this storyline, the compiled SUTs need to be transformed into IOTs. However, there are several transformation approaches, and these approaches yield different results. Thus, one concrete policy question may have a number of different answers, not for reasons of incomplete or imprecise data, but due to lack of a unique methodological approach. The debate has become known under the name "choice of model." Key contributions in this debate include those by Stone (1961) himself, Ten Raa et al. (1984), Kop Jansen and Ten Raa (1990), Konijn (1994), SNA (United Nations 2009), Miller and Blair (2009), Rueda-Cantuche et al. (2009), Rueda-Cantuche and Ten Raa (2009) and Majeau-Bettez et al. (2014). Eurostat (2008) describes the most often used transformation models (labeled A through D) and some other transformation models, with illustrations on small numerical hypothetical tables. An overview of these main transformation models is given in Fig. 1. For convenience, we will follow the naming convention (A–D) of Eurostat as shown in this figure.

The main issue in the choice of model is the treatment of co-products. BEA's supply table of the USA in 2010 in 61 × 61 resolution shows that the average industry produces more than ten products, which demonstrates that co-production is ubiquitous. Even though there may typically be a single primary product for each industry, the supply of many co-products by many industries does create a multifunctionality problem. When an industry has co-products, it is not clear which part of the labor inputs (and emissions) of that industry is related to the output of a particular (co-)product. Quoting Sraffa (1960) "For in the case of joint-products there is no obvious criterion for apportioning the labor among individual products" (p. 56). The different SUT-to-IOT transformation model essentially creates single-output systems, solving the multifunctionality problem using different assumptions on the allocation of co-products in a mechanistic manner. With the increase of the extent and detail of the SUTs, as witnessed by the advent of large databases, such as GTAP (http://www.gtap.org/), WIOD (http://www.wiod.org/), EORA (http://www.worldmrio.com/) and EXIOBASE (http://www.exiobase.eu/), this

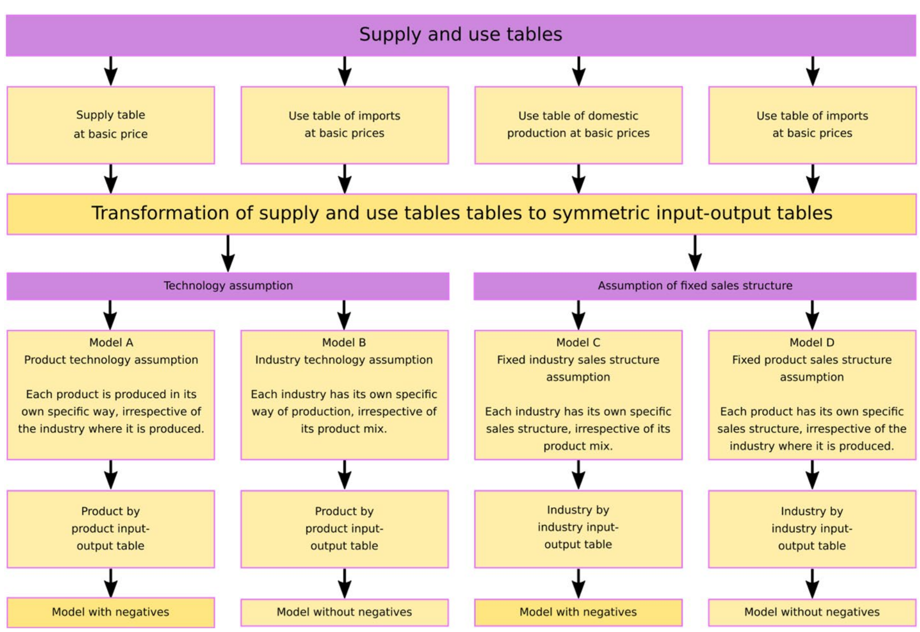

Fig. 1 Transformation of supply and use tables to symmetric input–output tables. Simplified after Eurostat (2008, p. 296)

issue shows up more pervasively than before and the only practical solution is the application of the transformation model. The different transformation models yield different results, and these results are used for impact studies for policy on employment, innovation, climate change and so on. In addition, IOTs from consecutive years are used for econometric estimation, which again provide a basis for decision-making. Given the differences and the possible real-world implications, this paper revisits the choice of model problem, bringing in three new elements in the discussion.

In the first place, it argues that the data in an IOT are not empirically observable, but that only some of the results of calculations made with an IOT are empirically observable, for instance, the total input of labor or the total CO_2 emission. Thus, disputes on the "true" IOT are not amenable to scientific discourse, while disputes on results obtained with them are. In this positivist emphasis on the empirically observable results, we to some extent follow Rueda-Cantuche and Ten Raa (2013). We also build on a paper by Suh et al. (2010), developed in the context of environmental IOA (EIOA), where it is recognized that adding a satellite matrix containing environmental emission and resource consumption data shifts the attention from the total output vector to the vector with environmental extensions. We extend their argument to the traditional value-added vector, effectively bringing the discussion back to the economic domain via the detour of environmental accounting. Finally, we connect to Eurostat's observation (2008, p. 310) that "it remains to be seen in empirical research which type of tables is the better option."

Secondly, we argue that a change in accounting units should in the end not affect the final empirically observable result, although non-empirically observable intermediate results may be affected by such changes. Here, we build on Dietzenbacher et al. (2009) and Weisz and Duchin (2006) who study the difference between physical and monetary IOT, i.e., IOTs that are expressed in physical accounting units (such as kg) and IOTs that are expressed in monetary accounting units (such as USD).

A third line that enters our argument is that of environmental science, where scientists have developed their own analytical approaches [notably life cycle assessment (LCA); see Heijungs and Suh (2002)] and where a close comparison with IOA reveals interesting points of correspondence and divergence (Suh et al. 2010; Majeau-Bettez et al. 2014, 2016, 2018).

We demonstrate that some of the established models A–D, described by Eurostat (2008), that have been understood as distinctively different approaches to convert SUT into IOT are identical when it comes to calculating empirically observable results (argument 1). We also demonstrate that some of the models are consistent with the requirements from dimensional analysis, but some others are not (argument 2). Finally, we strengthen the conclusion by Suh et al. (2010) that the models in LCA are comparable to the models in IOA and EIOA.

The paper is structured as follows. Section 2 briefly summarizes the status quo of the SUT–IOT debate and introduces notation. Section 3 discusses the different types of use of IOA. Section 4 repeats the argument from Suh et al. (2010), but with the focus shifted to economic analysis instead of environmental analysis, and adding more types of IOT transformation models into the discussion. Section 5 analyzes the effects of changing the accounting units. Section 6 analyzes to what extent use of transformation models A–D can be circumvented, thus connecting to the practice in LCA, where a consistent calculation procedure has been developed which not necessarily involves the detour to coefficient matrices. Section 7 provides an empirical case study as an illustration of the mathematical theory, including an analysis of the way sectors or products contribute to a result. Section 8 addresses the relevance of the result and discusses the issue of negatives, one of the traditional key ingredients of the debate on the choice of models.

2 The transformation from SUT to IOT

This section recaps the mainstream literature on the derivation of IOTs from SUTs. It primarily builds on Eurostat's manual (2008), but goes on the one hand further in preparing for the critical arguments of Sects. 4 until 6, and on the other hand skips many points (such as valuation layers and international trade) that are irrelevant given our focus.

A SUT is a pair of tables $\left(\mathbf{V}^{\mathrm{T}}, \mathbf{U}\right)$, the supply table $\mathbf{V}^{\mathrm{T}}$ (T denotes transposition) and the use table $\mathbf{U}$, both of which are in product-by-industry format. The use table $\mathbf{U}$ represents the inputs of a set of industries within a given system boundary (e.g., one country) in terms of the products needed. An element of the use table u_{ij} represents the input of product i into industry j, in a certain year. The supply table $\mathbf{V}^{\mathrm{T}}$ represents the outputs of a set of industries within the same system boundary in terms of the products produced. An element of the supply table $v_{ij}^{\mathrm{T}} \left(= v_{ji}\right)$ represents the output of product i by industry j, in the same year. Note that we use Eurostat's (2008) convention that the matrix $\mathbf{V}^{\mathrm{T}}$ has rows for products and columns for industries; the matrix $\mathbf{V}$ is then occasionally referred to as the make matrix [other conventions also show up in the literature (see, e.g., Konijn and Steenge (1995)]. Important additional elements are the final demand vector $\mathbf{d}$, the value-added matrix $\mathbf{W}$ and the total vectors for products $\mathbf{q}$, for industries $\mathbf{g}$ and for value added $\mathbf{w}$. Total final demand is a scalar, d. See Fig. 2. We write consistently column vectors with bold lowercase font, like $\mathbf{x}$; row vectors are then written as transposed column vectors, e.g., $\mathbf{x}^{\mathrm{T}}$.

An IOT has merged $\mathbf{V}^{\mathrm{T}}$ and $\mathbf{U}$ into one table, which is "symmetric" in the sense that rows and columns follow the same classification (Eurostat 2008, p. 24). This can be either products, in which case we have a product-by-product table, or it can be industries, in which case we have an industry-by-industry table. In Fig. 3, both forms are shown, with the symbol $\mathbf{S}$ used for the product-by-product form and $\mathbf{B}$ for the industry-by-industry form.

Supply table

	Industries	Supply
Products	$\mathbf{V}^{\mathrm{T}}$	$\mathbf{q}$
Output	$\mathbf{g}^{\mathrm{T}}$	

Use table

	Industries	Final demand	Use
Products	$\mathbf{U}$	$\mathbf{d}$	$\mathbf{q}$
Value added	$\mathbf{W}$		$\mathbf{w}$
Output	$\mathbf{g}^{\mathrm{T}}$	d	

Fig. 2 Format of the SUT with the definition of the symbols. Based on Eurostat (2008, p. 348)

Input-output table – product-by-product format (models A and B)

	Products	Final demand	Output
Products	$\mathbf{S}$	$\mathbf{d}$	$\mathbf{q}$
Value added	$\mathbf{E}$		$\mathbf{w}$
Input	$\mathbf{q}^{\mathrm{T}}$	d	

Input-output table – industry-by-industry format (models C and D)

	Industries	Final demand	Output
Industries	$\mathbf{B}$	$\mathbf{h}$	$\mathbf{g}$
Value added	$\mathbf{W}$		$\mathbf{w}$
Input	$\mathbf{g}^{\mathrm{T}}$	d	

Fig. 3 Format of the two forms of IOT with the definition of the symbols. Based on Eurostat (2008, p. 348)

The Eurostat manual describes the equations to calculate the matrices **S** and **B**, as well as the matrices **E** and **F**, given the matrices of the supply–use tables (**U**, **V**, **q**, **g**, as well as **d** and **W**). It does so by means of four basic transformations; see Table 1 for the naming and Table 2 for the transformation formulas.

To distinguish the coefficient tables made by the four models, we have added superscripts (A), (B), (C) and (D) to the different symbols in Table 2, where needed. Some of the formulas in Table 2 assume, with many other texts (like Eurostat 2008), that the matrices **V** and **U** are square, i.e., that the number of products is equal to the number of industries. In the discussion, we will return to the case of non-square ("rectangular") matrices. It should be noted that origin and naming of these transformation models at least partly resides in the problem of treating industries that produce more than one product and products that are produced by more than one industry. Eurostat (2008, p. 327) writes that with "supply and use tables, it is no longer difficult to describe by-products properly in the system… However, they still create a problem for symmetric input–output tables." Preprocessing steps may be needed to fix this. Eurostat (2008, p. 325) recommends that "before applying the product technology, each product should be assigned to a primary producer." As we will see in our final discussion, this step can be omitted. In fact, we believe that omitting it will avoid several complications.

Using the same initial data (**U**, **V**, **q**, **g**, **d** and **W**), the four basic models yield different IOTs (**S** or **B**) with different extra tables (**d** or **h** and **E** or **W**). Some of these forms (namely models A and C) may result in so-called negatives, which is deemed as problematic (Konijn 1994; Ten Raa and Rueda-Cantuche 2003; Eurostat 2008); see also the discussion.

Table 1 Basic modeling principles to derive an IOT from a SUT. *Source*: Eurostat (2008, p. 296)

Model	Format	Assumption
A (PTA-p*p)	Product-by-product	Product technology assumption
B (ITA-p*p)	Product-by-product	Industry technology assumption
C (ISA-i*i)	Industry-by-industry	Fixed industry sales structure assumption
D (PSA-i*i)	Industry-by-industry	Fixed product sales structure assumption

Table 2 Basic transformation models to derive an IOT (S and E or B and h) from a SUT (V and U). ***Source*****: Eurostat (2008, p. 349)**

Model	Formulas for S or B	Formulas for E	Formulas for h
A (PTA-p*p)	$\mathbf{S}^{(A)} = \mathbf{U}\mathbf{V}^{-T}\hat{\mathbf{q}}$	$\mathbf{E}^{(A)} = \mathbf{W}\mathbf{V}^{-T}\hat{\mathbf{q}}$	–
B (ITA-p*p)	$\mathbf{S}^{(B)} = \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{V}$	$\mathbf{E}^{(B)} = \mathbf{W}\hat{\mathbf{g}}^{-1}\mathbf{V}$	–
C (ISA-i*i)	$\mathbf{B}^{(C)} = \hat{\mathbf{g}}\mathbf{V}^{-T}\mathbf{U}$	–	$\mathbf{h}^{(C)} = \hat{\mathbf{g}}\mathbf{V}^{-T}\mathbf{d}$
D (PSA-i*i)	$\mathbf{B}^{(D)} = \mathbf{V}\hat{\mathbf{q}}^{-1}\mathbf{U}$	–	$\mathbf{h}^{(D)} = \mathbf{V}\hat{\mathbf{q}}^{-1}\mathbf{d}$

The superscript −1 indicates inversion; the superscript −T indicates transposition and inversion; the hat ($\hat{\mathbf{x}}$) on top of a vector indicates diagonalization into a square matrix

So far, all tables refer to the transactions of industries and/or products in a certain period of time. For analytical purposes, a conversion into coefficient form is made (Miller and Blair 2009; Eurostat 2008). In particular, the IO-matrices ($\mathbf{S}^{(A)}$, $\mathbf{S}^{(B)}$, $\mathbf{B}^{(C)}$ and $\mathbf{B}^{(D)}$) are converted into an "input coefficient intermediates" matrix (Eurostat 2008, p. 349), usually indicated by $\mathbf{A}$. Because we will distinguish four models, we will add a superscript to $\mathbf{A}$, so we will write $\mathbf{A}^{(A)}$ until $\mathbf{A}^{(D)}$. In addition, the satellite matrices ($\mathbf{E}^{(A)}$, $\mathbf{E}^{(B)}$ and $\mathbf{W}$) are converted into an "input coefficient value-added" matrix $\mathbf{R}$ ($\mathbf{R}^{(A)}$ until $\mathbf{R}^{(D)}$). These coefficient matrices are derived from the transaction matrices by dividing by a properly selected output vector; see Table 3.

3 The use of IOT for IOA

In general, a SUT is considered to be the preferred system of compiling data (Eurostat 2008; United Nations 2009). But SUTs are only "seen as a necessary first step for the calculation of input–output tables" (Eurostat 2008, p. 51) and they are considered to be a first step in a "three-step approach," of which the third step is the "calculation of standard analytical results" (Eurostat 2008, p. 368). So, when it comes to applications, IOT seems to be the preferred way. We will criticize this logic later on, but for now accept it and present the mainstream ideas. These include the use of IOT:

- to calculate the consequences (GDP, value added, emissions, etc.) of an exogenous final demand scenario;
- to calculate multipliers, which represent the consequences (GDP, value added, emissions, etc.) of a marginal change in the final demand;
- to perform analytical calculations without a change, e.g., the contributions made by different final demand categories, structural decomposition analysis and structural path analysis.

Table 3 Basic transformation models to derive IO coefficient matrices A and R from a SUT. After Eurostat (2008, p. 349)

Model	Formulas for A	Formulas for R
A (PTA-p*p)	$\mathbf{A}^{(A)} = \mathbf{S}^{(A)}\hat{\mathbf{q}}^{-1}$	$\mathbf{R}^{(A)} = \mathbf{E}^{(A)}\hat{\mathbf{q}}^{-1}$
B (ITA-p*p)	$\mathbf{A}^{(B)} = \mathbf{S}^{(B)}\hat{\mathbf{q}}^{-1}$	$\mathbf{R}^{(B)} = \mathbf{E}^{(B)}\hat{\mathbf{q}}^{-1}$
C (ISA-i*i)	$\mathbf{A}^{(C)} = \mathbf{B}^{(C)}\hat{\mathbf{g}}^{-1}$	$\mathbf{R}^{(C)} = \mathbf{W}\hat{\mathbf{g}}^{-1}$
D (PSA-i*i)	$\mathbf{A}^{(D)} = \mathbf{B}^{(D)}\hat{\mathbf{g}}^{-1}$	$\mathbf{R}^{(D)} = \mathbf{W}\hat{\mathbf{g}}^{-1}$

In this paper, the emphasis will be on the second group of application: marginal changes. In the discussion, we will briefly consider the other two types of application. Note that we write superscripts (A/B), (A/B/C/D), etc. whenever we write one equation that holds for several models.

4 Use of IOT for impact analysis

In the SUT scheme and in the product-by-product IOT (models A and B), a marginal change in final demand for products will be written as a change from $\mathbf{d}$ to $\mathbf{d} + \Delta\mathbf{d}$. For the industry-by-industry IOT (models C and D), things are more complicated. A marginal change in product demand $\Delta\mathbf{d}$ transforms into a marginal change in industry output demand given by $\Delta\mathbf{h}^{(\mathrm{C})} = \hat{\mathbf{g}}\mathbf{V}^{-\mathrm{T}}\Delta\mathbf{d}$ or $\Delta\mathbf{h}^{(\mathrm{D})} = \mathbf{V}\hat{\mathbf{q}}^{-1}\Delta\mathbf{d}$ according to Table 2.

IO impact studies use the formula

$$\Delta\mathbf{q}^{(\mathrm{A/B})} = \left(\mathbf{I} - \mathbf{A}^{(\mathrm{A/B})}\right)^{-1}\Delta\mathbf{d}^{(\mathrm{A/B})} \tag{1}$$

for product-by-product models A and B and

$$\Delta\mathbf{g}^{(\mathrm{C/D})} = \left(\mathbf{I} - \mathbf{A}^{(\mathrm{C/D})}\right)^{-1}\Delta\mathbf{h}^{(\mathrm{C/D})} \tag{2}$$

for industry-by-industry models C and D to calculate the change in output (Miller and Blair 2009). The term $\left(\mathbf{I} - \mathbf{A}^{(\mathrm{A/B/C/D})}\right)^{-1}$ is the Leontief inverse (Miller and Blair 2009); it is occasionally given a special symbol, such as $\mathbf{L}^{(\mathrm{A/B/C/D})}$. Similarly, the change in value added is calculated by

$$\Delta\mathbf{e}^{(\mathrm{A/B})} = \mathbf{R}^{(\mathrm{A/B})}\Delta\mathbf{q}^{(\mathrm{A/B})} \tag{3}$$

for models A and B and

$$\Delta\mathbf{w}^{(\mathrm{C/D})} = \mathbf{R}^{(\mathrm{C/D})}\Delta\mathbf{g}^{(\mathrm{C/D})} \tag{4}$$

for models C and D.

Traditionally, IO impact analysis calculates changed output ($\Delta\mathbf{q}^{(\mathrm{A/B})}$ or $\Delta\mathbf{g}^{(\mathrm{C/D})}$) as a function of changed demand ($\Delta\mathbf{d}^{(\mathrm{A/B})}$ or $\Delta\mathbf{h}^{(\mathrm{C/D})}$); see Eqs. (1) and (2). The result is then fed into an expression for the changed value added ($\Delta\mathbf{e}^{(\mathrm{A/B})}$ and $\Delta\mathbf{w}^{(\mathrm{C/D})}$); see Eqs. (3) and (4). In EIOA, the intermediate Eqs. (1) and (2) are often skipped (see, e.g., Suh 2009), and Eqs. (1) and (3) are combined into

$$\Delta\mathbf{e}^{(\mathrm{A/B})} = \mathbf{R}^{(\mathrm{A/B})}\left(\mathbf{I} - \mathbf{A}^{(\mathrm{A/B})}\right)^{-1}\Delta\mathbf{d}^{(\mathrm{A/B})} \tag{5}$$

for models A and B, and Eqs. (2) and (4) are combined into

$$\Delta\mathbf{w}^{(\mathrm{C/D})} = \mathbf{R}^{(\mathrm{C/D})}\left(\mathbf{I} - \mathbf{A}^{(\mathrm{C/D})}\right)^{-1}\Delta\mathbf{h}^{(\mathrm{C/D})} \tag{6}$$

for models C and D. Treatises on multipliers in traditional economic areas, so outside EIOA, also contain such formulas (Eurostat 2008, pp. 500–506).

In "Appendix 1" and Table 4, we elaborate the expressions for the coefficient matrices of Table 3. Of particular interest here is that when $\Delta\mathbf{h}^{(\mathrm{C/D})}$ is expressed in terms of $\Delta\mathbf{d}$ (Table 2), it follows that $\Delta\mathbf{e}^{(\mathrm{A})} = \Delta\mathbf{w}^{(\mathrm{C})}$ and $\Delta\mathbf{e}^{(\mathrm{B})} = \Delta\mathbf{w}^{(\mathrm{D})}$.

Table 4 Expressions for the satellite multiplier for models A–D

Model	Formula for impact analysis in terms of Δh	Formula for impact analysis in terms of Δd
A (PTA-p*p)		$\Delta\mathbf{e}^{(A)} = \mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}$
B (ITA-p*p)		$\Delta\mathbf{e}^{(B)} = \mathbf{W}\left(\hat{\mathbf{q}}\mathbf{V}^{-1}\hat{\mathbf{g}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}$
C (ISA-i*i)	$\Delta\mathbf{w}^{(C)} = \mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}\hat{\mathbf{g}}\mathbf{V}^{-\mathrm{T}}\Delta\mathbf{h}$	$\Delta\mathbf{w}^{(C)} = \mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}$
D (PSA-i*i)	$\Delta\mathbf{w}^{(D)} = \mathbf{W}\left(\hat{\mathbf{q}}\mathbf{V}^{-1}\hat{\mathbf{g}} - \mathbf{U}\right)^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1}\Delta\mathbf{h}$	$\Delta\mathbf{w}^{(D)} = \mathbf{W}\left(\hat{\mathbf{q}}\mathbf{V}^{-1}\hat{\mathbf{g}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}$

See "Appendix 1" for the proofs

Table 4 shows that $\Delta\mathbf{e}^{(A)} = \Delta\mathbf{w}^{(C)}$ and that $\Delta\mathbf{e}^{(B)} = \Delta\mathbf{w}^{(D)}$. So, while there are four models, there are just two different answers, at least when it comes to observable change in value added. In the remaining text, we will no longer separate $\Delta\mathbf{e}$ and $\Delta\mathbf{w}$, but always write $\Delta\mathbf{w}$, also for models A and C.

The intermediate (at least considered by the authors to be intermediate) results of model A/C and B/D are the coefficient matrices $\mathbf{A}$ and $\mathbf{R}$, the Leontief inverse $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ and the output vectors $\Delta\mathbf{g}$ and $\Delta\mathbf{q}$. These are different in all four models. Most of the literature has focused on these differences [see, e.g., Kop Jansen and Ten Raa (1990), Ten Raa and Rueda-Cantuche (2003), Rueda-Cantuche and Ten Raa (2009)]. In addition, substantive arguments have been introduced on the principles behind these methods, for instance, relating to their intended use (Rueda-Cantuche and Ten Raa 2009; Oosterhaven 2012). If, by contrast, we study the observable end result of the models, namely $\Delta\mathbf{w}$, we see only two classes of results: models A and C on the one hand and models B and D on the other hand.

As far as we know, the pairwise equivalence of models A and C and of models B and D in the context of impact analysis and multipliers has never been discussed in the literature. As a matter of fact, Suh et al. (2010) do discuss the equivalence between models A and another model (the so-called by-product technology model). They write that the choice between these models "does not have significant practical meaning when it comes to actual application" and "acknowledge that these two methods have different underlying economic implications, which nevertheless does not have any effect in the results of [a] study" (Suh et al. 2010, p. 341). However, they do not fully address the equivalence of models A and C and certainly not of B and D. In a broader historical context, the pairwise similarity has been stressed before, for instance, by not distinguishing four models (A–D) but only two [UN (1999), p. 86): "There are basically two methods to combine the use and supply matrices mathematically to generate the traditional symmetric input–output matrix. These methods are based on either the industry technology assumption or the commodity technology assumption.]," but not so much in terms of an equivalence of multipliers.

Of course, strictly speaking the models are not equal. In models C and D, we have an extra transformation step for $\Delta\mathbf{d}$ to $\Delta\mathbf{h}$, which amounts to

$$\Delta\mathbf{w}^{(C)} = \mathbf{R}^{(C)}\left(\mathbf{I} - \mathbf{A}^{(C)}\right)^{-1}\hat{\mathbf{g}}\mathbf{V}^{-\mathrm{T}}\Delta\mathbf{d} \quad (7)$$

for model C and

$$\Delta\mathbf{w}^{(D)} = \mathbf{R}^{(D)}\left(\mathbf{I} - \mathbf{A}^{(D)}\right)^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1}\Delta\mathbf{d} \quad (8)$$

for model D, rather than

$$\Delta \mathbf{e}^{(A/B)} = \mathbf{R}^{(A/B)} \left(\mathbf{I} - \mathbf{A}^{(A/B)} \right)^{-1} \Delta \mathbf{d} \tag{9}$$

for models A and B. So taking $\mathbf{R}(\mathbf{I}-\mathbf{A})^{-1}$ is sufficient for models A and B, but not for models C and D, because an extra transformation step from $\Delta\mathbf{d}$ to $\Delta\mathbf{h}$ is needed. Clearly, $\mathbf{R}^{(A)}\left(\mathbf{I}-\mathbf{A}^{(A)}\right)^{-1} \neq \mathbf{R}^{(C)}\left(\mathbf{I}-\mathbf{A}^{(C)}\right)^{-1}$ and likewise $\mathbf{R}^{(B)}\left(\mathbf{I}-\mathbf{A}^{(B)}\right)^{-1} \neq \mathbf{R}^{(D)}\left(\mathbf{I}-\mathbf{A}^{(D)}\right)^{-1}$, so a comparison of matrices $\mathbf{A}$, $\mathbf{R}$, $(\mathbf{I}-\mathbf{A})^{-1}$ or $\mathbf{R}(\mathbf{I}-\mathbf{A})^{-1}$ will suggest that the models A–D are all different, which they of course are, but not when we focus on the observable result $\Delta\mathbf{w}(=\Delta\mathbf{e})$. Probably it is the focus on the coefficient form with $\mathbf{R}$ and $(\mathbf{I}-\mathbf{A})^{-1}$ that has obscured the pairwise equivalence of models A/C and B/D so far.

5 Change in accounting units in impact analysis

The consideration of accounting units adds another insight. Products can be expressed in different accounting units: monetary (dollar, yen, etc.) or physical (kg, MJ, etc.). Although there is a "natural" accounting unit for some products, many products can be and are expressed in several alternative accounting units. For instance, gasoline may be expressed in liter, in kg, in MJ, in dollar, etc.

In this section, we will explore to what extent conclusions drawn from IO impact analysis depend on a change in accounting units. The general idea is that numbers will change when accounting units are changed, but only in a very specific way. For instance, if an extra demand of 1 L of gasoline leads to an extra value added of 0.1 dollar, an extra demand of 1 gallon of gasoline should lead to an extra value added of approximately 0.4 dollar, because 1 gallon corresponds to approximately 4 L, and because the multiplier model of IOA is a linear model. We implement changes in accounting unit in the product part (the rows of $\mathbf{U}$, $\mathbf{V}^{\mathrm{T}}$, $\mathbf{d}$ and $\mathbf{q}$) and in the satellite part (the rows of $\mathbf{W}$ and $\mathbf{w}$) and will develop formulas for the effects of such changes in models A–D.

In general, the topic of accounting units is underemphasized in the field of economics. While textbooks in physics and chemistry mention the importance of units and dimensional analysis (Bridgman 1922), textbooks in economics hardly even mention the existence of units, let alone the way to properly do the algebra of units (Barnett 2004). It is no wonder that critical remarks on the failure to address units in a proper way have been made mostly within the more physical areas of economics, such as ecological economics (Mayumi and Giampietro 2010) and physical input–output analysis (Weisz and Duchin 2006). Few treatments of this topic within economics are available but De Jong (1967) is a good source. A further complication is that the term unit is in the SUT- and IOT-literature typically used for something else, namely the statistical unit, which is the "entity for which the required statistics are compiled" (Eurostat 2017a) which may be an enterprise, a kind-of-activity unit, etc. Finally, the literature is not always sharp on the distinction between units and dimensions. Dimensions represent generic classes of measurement quantities, such as length and time. A unit is a specific choice for operationalizing a dimension, such as meter or mile within the dimension length or second and hour within the dimension time. We will primarily study changes in unit within the same dimension. But as one of the principles of SUT and IOT is that products within one row are homogeneous, we might also make a change from mass to price or from price to volume. As a final remark, Rueda-Cantuche and Ten Raa (2009) study the degree of "scale invariance" of the models C and D and conclude that model C is "superior from an axiomatic point of view." Basically, we re-interpret their analysis in terms of unit invariance and also broaden their argument by including models A and B.

To study to what extent the principle of unit invariance holds for all models A–D, a vector $\boldsymbol{\alpha}$ is introduced for changing the units of the products and a vector $\boldsymbol{\beta}$ to change the units of satellite categories. These vectors contain elements 1 if no change is made, but, e.g., a value of 3.6 if the unit of a product is changed from kWh to MJ. Thus, using the tilde to denote a quantity in the new unit system, the following transformation rules are introduced:

$$\tilde{\mathbf{U}} = \hat{\boldsymbol{\alpha}}\mathbf{U} \text{ and } \tilde{\mathbf{V}}^{\mathrm{T}} = \hat{\boldsymbol{\alpha}}\mathbf{V}^{\mathrm{T}} \tag{10}$$

for changing the units of products and

$$\tilde{\mathbf{W}} = \hat{\boldsymbol{\beta}} W \tag{11}$$

for changing the units of the satellite rows. The axiom of unit invariance can now be stated as the requirement that

$$\Delta\tilde{\mathbf{w}} = \hat{\boldsymbol{\beta}}\Delta\mathbf{w} \tag{12}$$

So, we need to elaborate expressions for $\Delta\tilde{\mathbf{w}}^{(\mathrm{A})}$ until $\Delta\tilde{\mathbf{w}}^{(\mathrm{D})}$. Appendix B provides the proofs of the results that are presented in Table 5.

Thus, it is demonstrated that models A and C are unit invariant, while models B and D are not. In particular, models B and D are invariant for changes in the accounting units of the extensions ($\mathbf{W}$) but not for changes in the accounting units of the products ($\mathbf{V}^{\mathrm{T}}$ and $\mathbf{U}$). In other words, two different choices of the accounting unit of products in the SUT will lead to two different satellite vectors when model B or D is applied, but to identical results when model A or C is applied. This is not only true for a dimension-affecting change in units (e.g., from dollar to kg), but also for a dimension-preserving more trivial change in units, e.g., from kilogram to gram or pounds or from dollar to millions of dollar or yen.

In hindsight, there is some logic in this. The problem is in the vector $\mathbf{g}$ in Fig. 2, which is constructed by adding rows, so by adding products. But if the rows denote products measured in different units, this sum is not defined, mathematically. One cannot add kg and MJ, and neither can one add kg and tonne. And if one would neglect the units and just add the numbers, there is no way of converting the result of kg and MJ into that of kg and kWh. In fact, applying the equation of model B (Table 2) to a mixed-unit SUT leads to a representation that actually violates the industry technology assumption, i.e., the assumption that all co-products of an industry obey the same production

Table 5 Results of a change in accounting units in the expressions for the satellite vector for models A–D

Model	Unit-changed formula for impact analysis	Invariant
A/C (PTA-p*p/ISA-i*i)	$\Delta\tilde{\mathbf{w}}^{(\mathrm{A/C})} = \hat{\boldsymbol{\beta}}\Delta\mathbf{w}^{(\mathrm{A/C})}$	Yes
B/D (ITA-p*p/PSA-i*i)	$\Delta\tilde{\mathbf{w}}^{(\mathrm{B/D})} = \hat{\boldsymbol{\beta}}\mathbf{W}\left(\hat{\mathbf{q}}\mathbf{V}^{-1}\hat{\mathbf{g}} - \mathbf{U}\right)\Delta\mathbf{d} \neq \hat{\boldsymbol{\beta}}\Delta\mathbf{w}^{(\mathrm{B/D})}$	No

function. Model A does not contain a term with $\mathbf{g}$, while models B, C and D do. In model C, the terms with $\hat{\mathbf{g}}$ and $\hat{\mathbf{g}}^{-1}$ cancel in the formula for $\Delta\mathbf{w}$ in Appendix A. As a consequence, model C is, despite the presence of the ill-defined $\mathbf{g}$, dimensionally consistent, at least, at the level of the observable result $\Delta\mathbf{w}$. The issue of mixed-unit systems has been addressed before (see, e.g., Pauliuk et al. (2015), Majeau-Bettez et al. (2016)), but the real issue is not so much storing and balancing data, but using incommensurable data in a mathematical framework. Formulas like $\hat{\mathbf{q}}\mathbf{V}^{-1}\hat{\mathbf{g}} - \mathbf{U}$ do not make sense when $\mathbf{g}$ is invalid. While we recognize this theoretical limitation, we nevertheless proceed as if we are ignorant about it. This can be justified on the basis of mainstream documents, such Eurostat (2008), which do not explicitly warn us that the accounting units of the products must be equal. If SUTs are primarily used as accounting tables, mixed units should be used, materials in kg, electricity in kWh, services in dollar. That they may also be used for "analytical" purposes, IOT and IOA, is then creating problems, at least with models B and D.

Within the IO-literature, some authors (Kop Jansen and Ten Raa 1990; Rueda-Cantuche and Ten Raa 2009) have introduced changes in scale in a comparable axiomatic framework to sort out "correct" and "incorrect" models. Here, we offer a reinterpretation in terms of a change in the accounting unit of the products. In Appendix C, we show that the column-wise scale invariance axiom gives in the end the same result as the row-wise unit variance axiom: Models A and C satisfy both axioms, while models B and D violate it. In a different context, Dietzenbacher and Stage (2006) observe consistency problems in carrying out a structural decomposition analysis in IOTs with mixed units.

6 Impact analysis without IOT and without coefficient matrices

As noted above, there are different models for moving from a SUT to an IOT, and to construct a coefficient matrix from transaction matrices. Several authors have argued that it is well possible to do impact analysis without constructing an IOT and without calculating coefficient matrices (Rosenbluth 1968; Heijungs 2001; Suh et al. 2010; Lenzen and Rueda-Cantuche 2012). For instance, Rosenbluth (1968, p. 255) argued that "there is nothing [IOA] can do that cannot be done equally well by [SUT] analysis, and a good many things that the latter can do better," and Suh et al. (2010, p. 341) state that the IO-literature "has overlooked the fact that coefficient matrices… are rarely, if ever, used alone… [they] fulfill an intermediate function." On the other hand, Rueda-Cantuche (2011b, p. 36) observed that "there has been very little research on the application of supply and use tables to impact analysis." In a follow-up paper, however, Lenzen and Rueda-Cantuche (2012, p. 151) showed that "the use of supply–use tables in a common framework concerning product- and industry-related assumptions may overcome the undesirable limitations of symmetric input–output tables." In this section, we will discuss in more detail the connection with the LCA literature, where working without a coefficient form was already discussed much earlier.

Indeed, in the context of LCA, the mainstream approach is a linear modeling on the basis of a product-by-industry format without the construction of a symmetric IO-table, and sometimes without the construction of coefficient forms. Heijungs and Suh (2002) present a modeling framework which in the present notation amounts to

$$\Delta\mathbf{w}^{(\mathrm{LCA})} = \mathbf{W}\mathbf{N}^{-1}\Delta\mathbf{d} \quad (13)$$

where $\mathbf{U}$ and $\mathbf{V}^{\mathrm{T}}$ have been consolidated into one matrix $\mathbf{N}$:

$$\mathbf{N} = \mathbf{V}^{\mathrm{T}} - \mathbf{U} \quad (14)$$

This matrix $\mathbf{N}$ can be interpreted as a "net" supply matrix: gross supply by industries ($\mathbf{V}^{\mathrm{T}}$) minus what is used up by industries in the process of manufacturing ($\mathbf{U}$). In general, a column of the net supply matrix $\mathbf{N}$ will have positive elements for the products it produces and negative elements for the products it uses.

A crucial observation is that the LCA framework naturally allows for systems where the matrix $\mathbf{N}$ contains more than one positive element in the same column. Such a column corresponds to a co-producing industry, i.e., an industry that produces more than one product. In the usual SUT-to-IOT conversions, two consecutive steps are made:

- from product-by-industry ($\mathbf{U}$ and $\mathbf{V}$) to product-by-product ($\mathbf{S}$) or industry-to-industry ($\mathbf{B}$) format;
- from transaction ($\mathbf{S}$ or $\mathbf{B}$) to coefficient ($\mathbf{A}$) format.

Both steps are associated with a difficult choice in case of co-production:

- assignment to the correct column: is a cow breeding company that produces dairy and meat a dairy producer or a meat producer?
- division by the correct total: should we specify its inputs per unit of dairy, per unit of meat or per unit of undifferentiated output?

As noted by Eurostat (2008, p. 327): "the issue has been debated a lot in literature, but a truly satisfactory solution has not yet been found." The interesting aspect of adding the LCA model by this paper [and by Suh et al. (2010)] is that it works without making the step to a symmetric table and without making the step to a coefficient table. In doing so, it avoids to face the choice of model.

The basic idea underlying the use of $\mathbf{N} = \mathbf{V}^{\mathrm{T}} - \mathbf{U}$ partly coincides with the by-product technology assumption, also referred to as Stone's method (Eurostat 2008), the by-product technology model (Suh et al. 2010) or the by-product technology construct (Majeau-Bettez et al. 2014), which assumes that co-products within one industry are produced in fixed ratios. The LCA model, like the by-product technology assumption, considers such extra outputs as negative inputs. As a consequence, its use in impact analysis may yield negatives, which is natural because this model coincides with models A and C, which were already known to potentially yield negatives. So, one might wonder, is the LCA model not the same as models A and/or C? The answer is negative: While model A uses $\mathbf{A} = \mathbf{U}\mathbf{V}^{-\mathrm{T}}$ and $\mathbf{R} = \mathbf{W}\mathbf{V}^{-\mathrm{T}}$ and model C uses $\mathbf{A} = \hat{\mathbf{g}}\mathbf{V}^{-\mathrm{T}}\mathbf{U}\hat{\mathbf{q}}^{-1}$ and $\mathbf{R} = \mathbf{W}\hat{\mathbf{g}}^{-1}$, the LCA model refrains from constructing $\mathbf{A}$ and $\mathbf{R}$ altogether and is only interested in their implicit combination through $\mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}$. This avoidance of making $\mathbf{A}$ and $\mathbf{R}$ is precisely which makes the difference with the by-product technology model, which still produces $\mathbf{A} = \left(\mathbf{U} - \mathbf{V}_{\mathrm{od}}^{\mathrm{T}}\right)\mathbf{V}_{\mathrm{d}}^{-1}$ and $\mathbf{R} = \mathbf{W}\mathbf{V}_{\mathrm{d}}^{-1}$, where the subscripts d and od code for diagonal and off-diagonal entries [Suh et al. (2010, p.

340)]. Indeed, in the by-product technology model one still needs to decide on what is the main product and what are the co-products of an industry: The model "assumes that production of co-products is fully dependent on the production of the primary product of a process" (Suh et al. 2010, p. 339). Without that choice, we cannot figure out which numbers are on the diagonal and which are off-diagonal.

In that respect, two remarkable and confusing things should be mentioned from the LCA literature:

- It is standard practice in LCA to remodel co-producing industries into industries with one output. For instance, the ISO-standard on LCA (ISO 2006, p. 14) prescribes that "the inputs and outputs shall be allocated to the different products."
- Most of the LCA literature does not recognize the advantage of not needing a coefficient form and still insists on making this conversion step. For instance, ISO (2006 p. 13) states that "an appropriate flow shall be determined for each unit process. The quantitative input and output data of the unit process shall be calculated in relation to this flow."

Clearly, both practices refer to unnecessary actions. The expression $\mathbf{WN}^{-1}$ just works whenever $\mathbf{N}$ is square and invertible, even when some of the off-diagonal elements are positive. And as long as $\mathbf{N} = \mathbf{V}^{\mathrm{T}} - \mathbf{U}$ and $\mathbf{W}$ are standardized by the same (nonzero) vector (say, $\mathbf{c}$), we have $(\mathbf{W}\hat{\mathbf{c}}^{-1})\left((\mathbf{V}^{\mathrm{T}} - \mathbf{U})\hat{\mathbf{c}}^{-1}\right)^{-1} = \mathbf{W}\left(\mathbf{V}^{-\mathbf{T}} - \mathbf{U}\right)$, so the choice of $\mathbf{c}$ does not matter. Tricks are only needed when $\mathbf{N}$ or $\left(\mathbf{V}^{\mathrm{T}}, \mathbf{U}\right)$ is not square. But that is no different for models A and C, as these models work with $\mathbf{V}^{-\mathrm{T}}$ and therefore are restricted to square SUTs as well. Finally, we mention the fact that ISO's (2006) co-product allocation has spawned a large literature which bears a lot of similarities with that of IOA. This literature features terms such as "partitioning," "substitution," "avoided impacts" and "system expansion." Suh et al. (2010) and Majeau-Bettez et al. (2014, 2018) contain an extensive treatment, which we will not repeat here.

Observe that we have added a superscript (LCA) which will allow us to make easy comparisons with the earlier frameworks based on models A–D. Also observe that matrix $\mathbf{N}$ is often referred to by the symbol $\mathbf{A}$ (Heijungs and Suh 2002); here we choose for another letter to avoid confusion with the $\mathbf{A}$ in the Leontief inverse $(\mathbf{I} - \mathbf{A})^{-1}$.

Given the results in the previous sections, we can conclude that

$$\Delta\mathbf{w}^{(\mathrm{LCA})} = \Delta\mathbf{w}^{(\mathrm{A})} = \Delta\mathbf{w}^{(\mathrm{C})} \tag{15}$$

which implies that the results obtained from satellite multipliers produced by models A and C agree with each other and moreover agree with those produced without the construction of coefficient matrices, directly applying supply and use tables (so using the "LCA model").

Suh et al. (2010) demonstrate that the form in (13) is equivalent to model A. They also add a section on the historic origins of this observation (p. 348). They in fact show moreover that it is also equivalent to the by-product model, another variant besides Eurostat's A–D. They, however, do not discuss the equivalence with the product-by-product version C.

Of course, we could argue that models B and D can be done without coefficient tables as well:

$$\Delta \mathbf{w}^{(\mathrm{B/D})} = \mathbf{W}\mathbf{M}^{-1}\Delta \mathbf{d} \tag{16}$$

where now $\mathbf{U}$ and $\mathbf{V}^{\mathrm{T}}$ have been consolidated into one matrix $\mathbf{M}$:

$$\mathbf{M} = \hat{\mathbf{q}}\mathbf{V}^{\mathrm{T}}\hat{\mathbf{g}} - \mathbf{U} \tag{17}$$

The point is, however, that the LCA-format with $\mathbf{N} = \mathbf{V}^{\mathrm{T}} - \mathbf{U}$ appears naturally when we look at the net production of a sector: When sector j produces an amount v_{ji} of product i and to do so uses an amount u_{ij} of the same product i, its net production is simply $n_{ij} = v_{ji} - u_{ij}$. There is no such a natural interpretation of $m_{ij} = q_j v_{ji} g_i - u_{ij}$. Indeed, we are not aware of fields of science where a quantity like $\mathbf{M}$ has been proposed to measure the net effect of production. It has only been constructed with the aim of constructing coefficient matrices, which are—as argued here—an intermediate step at most.

7 Illustrative case study

Above, we theoretically analyzed the points of agreement and disagreement between the various models. But in case of a disagreement, we could only conclude a theoretical inequality, without indicating how large the disagreement was. Below, we apply the formulas to the supply–use tables published by Eurostat for the year 2010 for the Netherlands (Eurostat 2017b). The tables were converted into input–output tables following models A, B, C and D. These input–output tables distinguish 65 products (models A/B) or 65 industry sectors (models C/D). Next, the total domestic output related to 1 million Euro of each of the 65 products was calculated, using the four models. The results are compared by creating scatter plots as shown in Fig. 4.

As predicted by the theoretical analysis, the total output calculated for models A and C and that for models B and D are the same, while for models A and B and for models C and D there are sometimes substantial differences, up to 50%. An analysis of the size of these differences is beyond the scope of the present paper. For an example based on supply–use tables of Andalusia, see Rueda-Cantuche and Ten Raa (2013). Similar calculations have been done for 16 other European countries, all giving a similar outcome. These additional results are available in SI.

Next, we study the impact analysis at the level of the satellite part. Figure 5 illustrates this for the CO_2 emission in Netherlands using the same Eurostat data source. For each of the four models A–D, the change in CO_2 emissions related to a domestic final demand change of 1 million Euro of each of the 65 outputs was calculated. The results for models A–D are compared by creating scatterplots.

The graphs confirm the theoretical prediction that the satellite multipliers are equal for models A and C and that they are equal for models B and D. They also show that the differences between models A/C and B/D can be substantial in a concrete case.

Although the choice between A/C and between B/D has no effect on the satellite multipliers, it does have an effect on the way such multipliers are attributed to individual products of industries. To illustrate such differences, the change in final demand for 1 million Euro wholesale services in the Netherlands is examined in more detail.

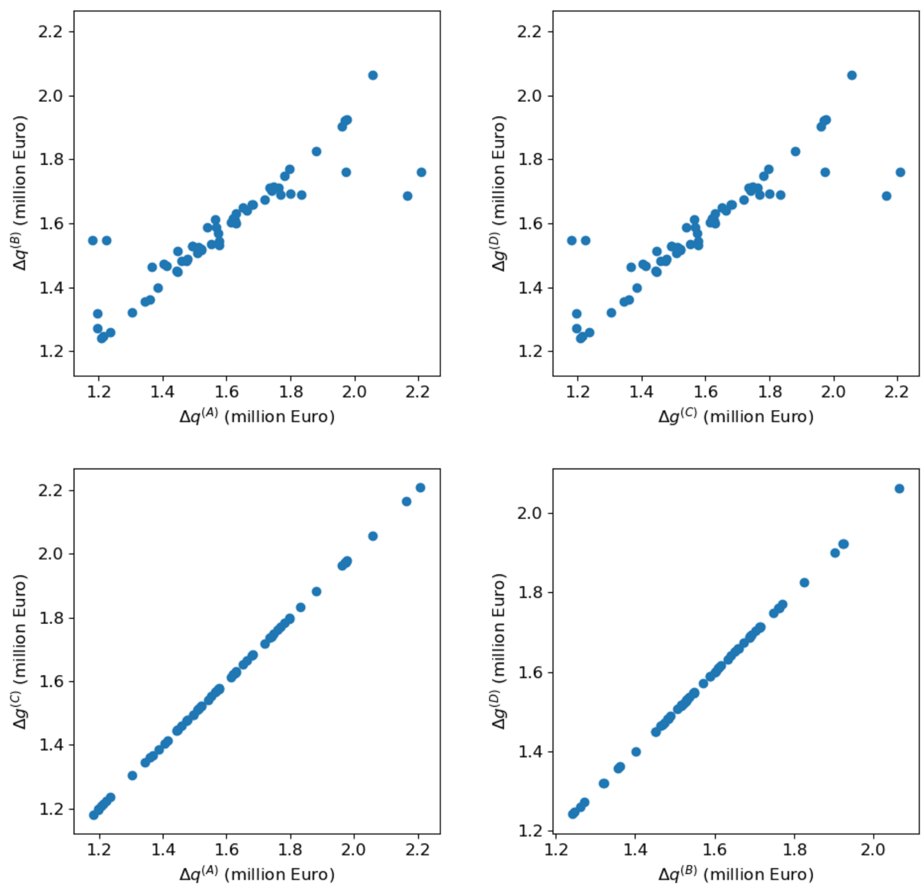

Fig. 4 Scatterplots of the change in output levels ($\Delta \mathbf{q}$ for the product-by-product tables and $\Delta \mathbf{g}$ for the industry-by-industry tables) using the four models A–D. Each dot represents the change in output level when demand for that output is increased by 1 million Euro

Wholesale services are chosen because about one-fifth (20.4%) of the supply to that sector is in the form of by-products, so this provides an interesting test case. The change in CO_2 emissions attributed to each product or industry related to a change in final demand of wholesale services is shown in Fig. 6. We can observe that in models B and D no negative emission contributions are calculated as expected and that negative emission contributions are calculated for models A and C. Furthermore, although the models B and D give an equal total emission change $\Delta \mathbf{w}$, they will calculate different contributions by each product or industry to this total. The same can be observed for models A and C. Figure 5 is about $\Delta \mathbf{w}$ for a number of different choices of $\Delta \mathbf{d}$: Models A and C fully agree, as was also proved mathematically. By contrast, Fig. 6 shows for one choice of $\Delta \mathbf{d}$ (namely 1 million Euro wholesale services) the attribution to the different contributing sectors. Here, models A and C give different results. Notice that we do not imply to say that model A or C is in any way superior to the other one. Both agree in their prediction of observable results (Fig. 5). But there is a difference in the non-observable attribution of such a prediction to individual sectors (Fig. 6).

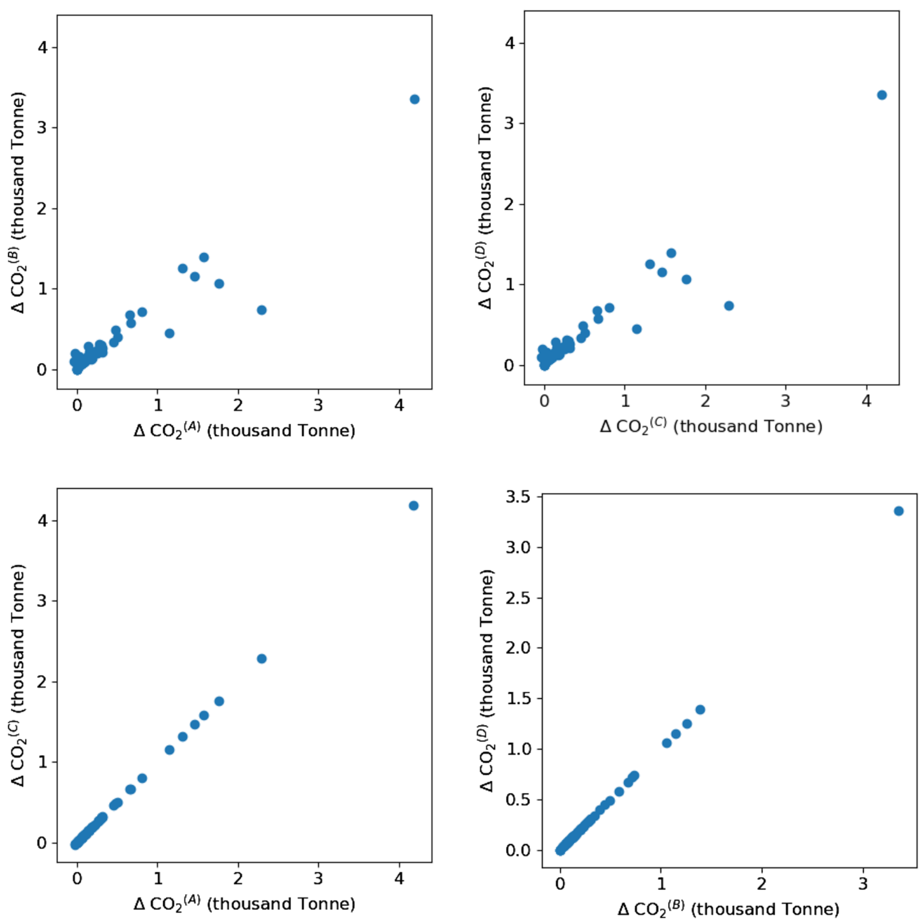

Fig. 5 Scatterplots of the change in satellite results ($\Delta\mathbf{w}$) using the four models (A)–(D). Each dot represents the change in CO_2 emissions when demand for an output is increased by 1 million Euro

8 Discussion

This paper started by summarizing the main four models for transforming a SUT to an IOT prior to using the data for impact analysis. We performed a mathematical analysis ("Appendices 1 and 2") and carried out an empirical study (Sect. 7). We were able to prove:

- that models A and C give identical results, and so do models B and D,
- that models A and C are unit invariant while models B and D are not,
- that it is possible to do impact analysis without an IOT and without coefficient matrices (referred to as the "LCA model") and that the result of the latter analysis is identical to the results of models A and C.

From an industrial ecologist's or ecological economist's perspective where a proper physical representation is important, models A, C and LCA are preferred over models B and D because they are unit invariant. Of course, being a model, they are not correct in absolute terms. Any model is as weak as the validity of its assumptions, and the models presuppose, among others, a square SUT, besides the usual IO-assumptions of linear

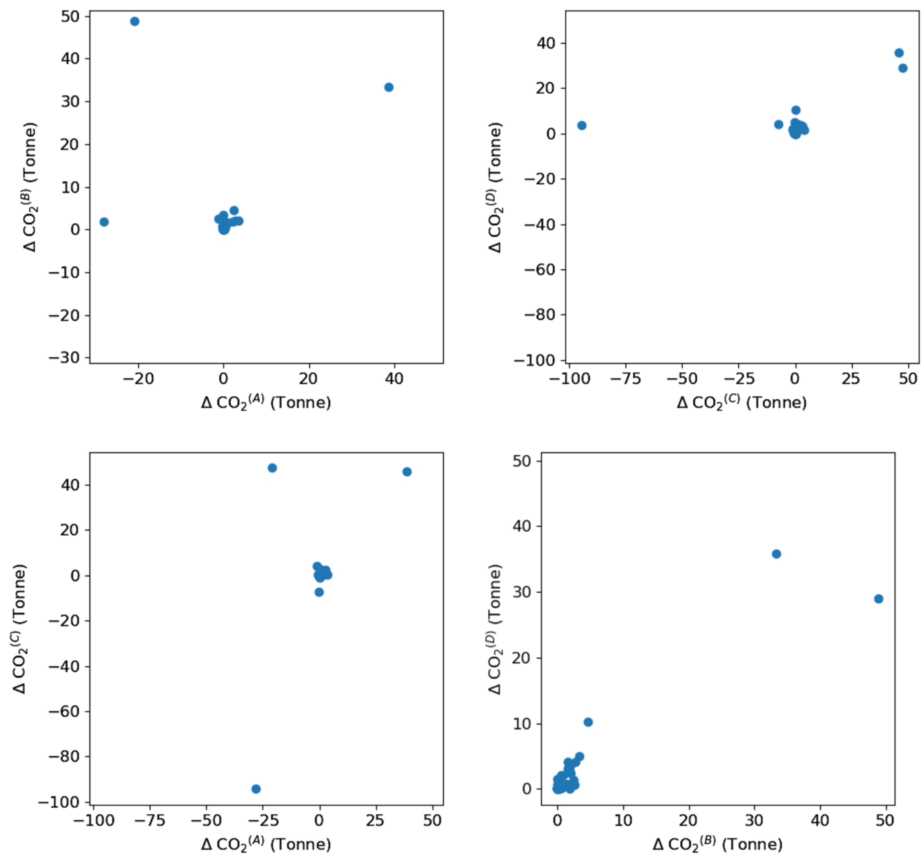

Fig. 6 Scatterplots of the change in satellite results per product or industry as a result of a change in wholesale services of 1 million Euro, using the four models (A)–(D). Each dot represents the contribution made by a particular product of sector to the total change $\Delta \mathbf{w}$

technology and full market clearing. But models A, C and LCA survive at least a number of critical tests.

However, models A and C are not identical, so how can both be correct? As a matter of fact, at the level of a breakdown of a result into the contributing products or sectors, model A deviates from model C and LCA, so there are real differences. A closer consideration of the problem reveals that a product-by-product structure (model A) and an industry-by-industry structure (model C) are in fact both difficult to interpret. The issue is that households and governments exert a demand for products, while the value added (and the emissions) is coming from the industries. A product-by-product structure reallocates the value added (and the emissions) to products, while an industry-by-industry structure reallocates the final demand to industries. Both are artificial (Rueda-Cantuche 2011a). The LCA model does not do this: The demand is for products and the value added (and emissions) belongs to industries. The LCA model is thus closer to economic accounting structure. Recall that this was precisely the reason for switching from IOT to SUT by Stone: "the SUTs framework provides the natural statistical framework" (Eurostat 2008, p. 51). Our analysis shows that we can maintain this natural framework not only in accounting, but also in analysis, at least for the case of marginal changes, so for satellite multipliers.

Given this acknowledged superiority of the product-by-industry format over the industry-by-industry and the product-by-product format for accounting purposes, it seems natural to also for analytical purposes prefer a mathematical model $\boldsymbol{\phi}(\cdot)$ like

$$\Delta\mathbf{w}^{(\mathrm{LCA})} = \boldsymbol{\phi}^{(\mathrm{LCA})}(\mathbf{V},\mathbf{U},\mathbf{W})\Delta\mathbf{d} \tag{18}$$

to the conventional model

$$\Delta\mathbf{w}^{(\mathrm{A/C})} = \boldsymbol{\phi}^{(\mathrm{A/C})}(\mathbf{A},\mathbf{R})\Delta\mathbf{d} \tag{19}$$

where an intermediate step involving $\mathbf{A}$ and $\mathbf{R}$ is made. Without this intermediate step, we cannot calculate the IO-tables in coefficient form ($\mathbf{A}$) and the Leontief inverse ($(\mathbf{I}-\mathbf{A})^{-1}$). But we can calculate a matrix

$$\boldsymbol{\Lambda}^{(\mathrm{LCA})} = \mathbf{W}\left(\mathbf{V}^{\mathrm{T}}-\mathbf{U}\right)^{-1} = \mathbf{W}\mathbf{N}^{-1} \tag{20}$$

that has been referred to as the intensity matrix by Heijungs and Suh (2002). It represents the intensity of the satellite by industry per unit of final demand of product. In that sense, it provides a bridge between product demand and industry satellite, without the need of choosing between the product-by-product or industry-by-industry format, and without choosing between PTA and ITA or between ISA and PSA. Ten Raa and Rueda-Cantuche (2007) use this form (in their notation it is $\boldsymbol{\lambda}^{\mathrm{T}} = \mathbf{l}^{\mathrm{T}}\left(\mathbf{V}^{\mathrm{T}}-\mathbf{U}\right)^{-1}$) to derive a row vector of labor multipliers. In more general impact analysis studies, the intensity matrix $\boldsymbol{\Lambda}^{(\mathrm{LCA})}$ can easily be used for calculating the change in value added or emissions

$$\Delta\mathbf{w}^{(\mathrm{LCA})} = \boldsymbol{\Lambda}^{(\mathrm{LCA})}\Delta\mathbf{d} \tag{21}$$

The debate on models A/B and C/D has in part been fueled by the fact that models A and C may yield negatives. That is, it may happen that the input coefficient matrix ($\mathbf{A}^{(\mathrm{A/C})}$) contains negative entries. As a consequence, the Leontief inverse ($\mathbf{L}$) the product output ($\mathbf{q}$), the industry output ($\mathbf{g}$), or the value added (or emissions) ($\mathbf{w}$) may contain negative elements. Such elements point to a negative amount of product, negative industrial activity, negative value added or negative emissions. Obviously, these negatives do not make sense; hence, the quest for transformations without negatives seems to be well grounded (Konijn 1994; Ten Raa and Rueda-Cantuche 2003). However, when we restrict the scope to small changes as in the case of multipliers, the argument gets much weaker (Suh et al. 2010). A marginal increase in final demand for one product may well decrease the marginal activity of some industries and thereby decrease the value added or the emissions. In a context of multipliers, indicating the effect of marginal changes, the issue of negatives is therefore not a valid argument in the choice of model.

The argument against negatives is much more valid in the context of larger changes or scenario analysis. Use of $\mathbf{w} = \mathbf{R}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{d}$ for predicting the values added (or emissions) of an economy-wide scenario $\mathbf{d}$ is an entirely different case. It is anyhow doubtful to what extent the underlying assumption of a linear technology with fixed coefficients is valid for such types of study. Whenever the result of a scenario study returns negative production, negative value added or negative emissions, we can be sure that the model was not appropriate. A choice for model B/D will only mask the intrinsic defects of the model by "saving the phenomena," an expression introduced by

Duhem (1969) to characterize the complex additions to save Ptolemy's astronomical system, while it should have in fact been replaced by an entirely different system.

In the context of SUT, Konijn (1994, p. 7) has observed that "we cannot perform input–output analysis in the way we used to, without (re-)constructing an input–output table from the system of [supply] and use matrices." As we have demonstrated, this is true, but we should reflect on the question if we need to perform input–output analysis at all, since the LCA model is simpler, devoid of fundamental questions and equally effective for (environmental) impact analysis, at least when we restrict the discussion to square matrices. While the calculation of an IOT, a coefficient matrix, a Leontief inverse or an industry output is useful for certain types of analysis (such as structural path analysis), it is not necessary for the calculation of satellite multipliers, including environmental impact analysis, at least in the case of marginal changes. This paper argues that the intensity matrix $\mathbf{\Lambda}^{(\mathrm{LCA})} = \mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}$ is crucial, for such analyses. It is uniquely available, obviating the choice between models A, B, C and D, or any of the other models [including those mentioned by Kop Jansen and Ten Raa (1990) and Eurostat (2008)]. So even when we have shown that models A and C are from a satellite point of view identical, we can still do without these models, and in fact we propose to abandon them entirely and use the less problematic LCA model, because they are different in their intermediate results and because they need assumptions that are problematic.

A final remaining issue with the intensity matrix $\mathbf{\Lambda}^{(\mathrm{LCA})}$ is that it involves an inverse, so that the SUT must be square. This can be a problem (Konijn 1994; Duchin and Levine 2011), but most alternative approaches also start by assuming a square SUT [see, e.g., Kop Jansen and Ten Raa (1990, p. 213), Rueda-Cantuche and Ten Raa (2009, p. 364), Rueda-Cantuche et al. (2009, p. 63), Suh et al. (2010, p. 339)]. Models B and D have two seemingly attractive sides: They do not yield negatives (discussed above) and they rely on formulas that do not require a square SUT [see also Lenzen and Rueda-Cantuche (2012)]. However, they have been shown from a dimensional perspective to suffer from theoretical shortcomings, so the mere fact they can be employed for rectangular cases by no means justifies their use. Probably, the topic of the construction of an IOT from a rectangular SUT can benefit from a renewed analysis similar to the one undertaken in this paper for the square case. Ingredients are again an emphasis on how the IOT is used to calculate empirically observable results, instead of which intermediate results are traditionally constructed, and on the formulation and application of consistency requirements, such as that of dimensional invariance.

Additional file

Additional file 1. ZIP file containing input data, Python script and output results used in the illustrative case study of Sect. 7.

Authors' contributions
RH conceived the argument and provided the mathematical proofs. AdK provided the data and code for the graphs. RH and AdK wrote the manuscript together.

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Competing interests
The authors declare that they have no competing of interests.

Availability of data and materials
The online version contains a downloadable zip file (Additional file 1) with the input data, routines and outputs used in the illustrative example.

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Publisher's Note

Appendix 1
Here, we provide the proofs of the formulas in Table 4. For clarity, we have suppressed the superscripts (A)–(D) in these proofs and worked out the four models separately.

Model A:

$$\begin{aligned}\Delta\mathbf{e} &= \mathbf{R}(\mathbf{I}-\mathbf{A})^{-1}\Delta\mathbf{d} = \mathbf{W}\mathbf{V}^{-\mathrm{T}}\left(\hat{\mathbf{q}}\hat{\mathbf{q}}^{-1} - \mathbf{U}\mathbf{V}^{-\mathrm{T}}\hat{\mathbf{q}}\hat{\mathbf{q}}^{-1}\right)^{-1}\Delta\mathbf{d}\\ &= \mathbf{W}\mathbf{V}^{-\mathrm{T}}\left(\mathbf{I} - \mathbf{U}\mathbf{V}^{-\mathrm{T}}\right)^{-1}\Delta\mathbf{d} = \mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}\end{aligned}$$

Model B:

$$\Delta\mathbf{e} = \mathbf{R}(\mathbf{I}-\mathbf{A})^{-1}\Delta\mathbf{d} = \mathbf{W}\hat{\mathbf{g}}^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1}\left(\mathbf{I} - \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1}\right)^{-1}\Delta\mathbf{d} = \mathbf{W}\left(\hat{\mathbf{q}}\mathbf{V}^{-1}\hat{\mathbf{g}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}$$

Model C:

$$\Delta\mathbf{w} = \mathbf{R}(\mathbf{I}-\mathbf{A})^{-1}\Delta\mathbf{h} = \mathbf{W}\hat{\mathbf{g}}^{-1}\left(\mathbf{I} - \hat{\mathbf{g}}\mathbf{V}^{-\mathrm{T}}\mathbf{U}\hat{\mathbf{g}}^{-1}\right)^{-1}\hat{\mathbf{g}}\mathbf{V}^{-\mathrm{T}}\Delta\mathbf{d} = \mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}$$

Model D:

$$\Delta\mathbf{w} = \mathbf{R}(\mathbf{I}-\mathbf{A})^{-1}\Delta\mathbf{h} = \mathbf{W}\hat{\mathbf{g}}^{-1}\left(\mathbf{I} - \mathbf{V}\hat{\mathbf{q}}^{-1}\mathbf{U}\hat{\mathbf{g}}^{-1}\right)^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1}\Delta\mathbf{d} = \mathbf{W}\left(\hat{\mathbf{q}}\mathbf{V}^{-1}\hat{\mathbf{g}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}$$

Appendix 2
Here, we provide the proofs of the formulas in Table 5. For clarity, we have suppressed the superscripts (A)–(D) in these proofs and worked out the four models separately. Please see Kop Jansen and Ten Raa (1990, pp. 216–217) for a similar derivation.

Results needed for several models on the basis of $\tilde{\mathbf{U}} = \hat{\boldsymbol{\alpha}}\mathbf{U}$, $\tilde{\mathbf{V}}^{\mathrm{T}} = \hat{\boldsymbol{\alpha}}\mathbf{V}^{\mathrm{T}}$, $\tilde{\mathbf{d}} = \hat{\boldsymbol{\alpha}}\mathbf{d}$ and $\tilde{\mathbf{W}} = \hat{\boldsymbol{\beta}}\mathbf{W}$ as well as $\mathbf{q} = \mathbf{V}^{\mathrm{T}}\mathbf{1}$ and $\mathbf{g}^{\mathrm{T}} = \mathbf{1}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$.

For $\tilde{\mathbf{V}}$ and $\tilde{\mathbf{V}}^{-1}$:

$$\tilde{\mathbf{V}} = \left(\tilde{\mathbf{V}}^{\mathrm{T}}\right)^{\mathrm{T}} = \left(\hat{\boldsymbol{\alpha}}\mathbf{V}^{\mathrm{T}}\right)^{\mathrm{T}} = \mathbf{V}\hat{\boldsymbol{\alpha}}^{\mathrm{T}} = \mathbf{V}\hat{\boldsymbol{\alpha}}; \quad \tilde{\mathbf{V}}^{-1} = \left(\mathbf{V}\hat{\boldsymbol{\alpha}}\right)^{-1} = \hat{\boldsymbol{\alpha}}^{-1}\mathbf{V}^{-1}$$

For $\tilde{\mathbf{q}}$ and $\tilde{\hat{\mathbf{q}}}$:

$$\tilde{\mathbf{q}} = \tilde{\mathbf{V}}^{\mathrm{T}}\mathbf{1} = \hat{\boldsymbol{\alpha}}\mathbf{V}^{\mathrm{T}}\mathbf{1} = \hat{\boldsymbol{\alpha}}\mathbf{q}; \quad \tilde{\hat{\mathbf{q}}} = \hat{\boldsymbol{\alpha}}\hat{\mathbf{q}}$$

For $\tilde{\mathbf{g}}$ and $\tilde{\hat{\mathbf{g}}}$:

$$\tilde{\mathbf{g}} = \left(\tilde{\mathbf{g}}^{\mathrm{T}}\right)^{\mathrm{T}} = \left(\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{V}}^{\mathrm{T}}\right)^{\mathrm{T}} = \tilde{\mathbf{V}}\mathbf{1} = \mathbf{V}\hat{\boldsymbol{\alpha}}\mathbf{1} = \mathbf{V}\boldsymbol{\alpha}; \quad \tilde{\hat{\mathbf{g}}} = \mathbf{V}\hat{\boldsymbol{\alpha}}$$

For $\tilde{\mathbf{d}}$ and $\Delta\tilde{\mathbf{d}}$:

$$\tilde{\mathbf{d}} = \tilde{\mathbf{Y}}\mathbf{1} = \hat{\boldsymbol{\alpha}}\mathbf{Y}\mathbf{1} = \hat{\boldsymbol{\alpha}}\mathbf{d}; \quad \Delta\tilde{\mathbf{d}} = \hat{\boldsymbol{\alpha}}\Delta\mathbf{d}$$

Unit transformation of models A/C:

$$\Delta\tilde{\mathbf{w}} = \tilde{\mathbf{W}}\left(\tilde{\mathbf{V}}^{\mathrm{T}} - \tilde{\mathbf{U}}\right)^{-1}\Delta\tilde{\mathbf{d}} = \hat{\boldsymbol{\beta}}\mathbf{W}\left(\hat{\boldsymbol{\alpha}}\mathbf{V}^{\mathrm{T}} - \hat{\boldsymbol{\alpha}}\mathbf{U}\right)^{-1}\hat{\boldsymbol{\alpha}}\Delta\mathbf{d} = \hat{\boldsymbol{\beta}}\mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d} = \hat{\boldsymbol{\beta}}\Delta\mathbf{w}$$

Unit transformation of models B/D:

$$\begin{aligned}\Delta\tilde{\mathbf{w}} &= \tilde{\mathbf{W}}\left(\tilde{\hat{\mathbf{q}}}\tilde{\mathbf{V}}^{-1}\tilde{\hat{\mathbf{g}}} - \tilde{\mathbf{U}}\right)^{-1}\Delta\tilde{\mathbf{d}} = \hat{\boldsymbol{\beta}}\mathbf{W}\left(\hat{\boldsymbol{\alpha}}\hat{\mathbf{q}}\hat{\boldsymbol{\alpha}}^{-1}\mathbf{V}^{-1}\mathbf{V}\hat{\boldsymbol{\alpha}} - \hat{\boldsymbol{\alpha}}\mathbf{U}\right)^{-1}\hat{\boldsymbol{\alpha}}\Delta\mathbf{d} \\ &= \hat{\boldsymbol{\beta}}\mathbf{W}\left(\hat{\mathbf{q}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d} = \hat{\boldsymbol{\beta}}\mathbf{W}\left(\hat{\mathbf{q}} - \mathbf{U}\right)^{-1}\left(\hat{\mathbf{q}}\mathbf{V}^{-1}\hat{\mathbf{g}} - \mathbf{U}\right)\mathbf{W}^{-1}\Delta\mathbf{w} \neq \hat{\boldsymbol{\beta}}\Delta\mathbf{w}\end{aligned}$$

Appendix 3

The scale invariance axiom (Kop Jansen and Ten Raa 1990) refers to an effective rescaling of the columns of the SUT. We might think of this as filling the SUT with data that is for every sector defined or scaled differently. For instance, sector 1 (column 1) of $\mathbf{U}$, $\mathbf{V}^{\mathrm{T}}$ and $\mathbf{W}$ is included on a per-year basis, sector 2 on a per-month basis, sector on a per-million euro basis and so on.

While our novelty lies in developing a unit invariance ("Appendix 2"), which applies to rows of the SUT, the original column-wise scale invariance still makes sense. Here, we show that it still holds for our framework, for models A and C.

Now we rescale $\mathbf{U}$ and $\mathbf{V}^{\mathrm{T}}$ through a post-multiplication: $\tilde{\mathbf{U}} = \mathbf{U}\hat{\boldsymbol{\alpha}}$, $\tilde{\mathbf{V}}^{\mathrm{T}} = \mathbf{V}^{\mathrm{T}}\hat{\alpha}$ and $\tilde{\mathbf{W}} = \mathbf{W}\hat{\boldsymbol{\alpha}}$. $\mathbf{d}$ obviously does not change by scaling industries. For $\mathbf{g}$, we have likewise $\tilde{\mathbf{g}} = \mathbf{g}\hat{\boldsymbol{\alpha}}$.

The expression $\Delta\mathbf{w} = \mathbf{W}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d}$ for models A and C then transforms into

$$\Delta\tilde{\mathbf{w}} = \tilde{\mathbf{W}}\left(\tilde{\mathbf{V}}^{\mathrm{T}} - \tilde{\mathbf{U}}\right)^{-1}\Delta\tilde{\mathbf{d}} = \mathbf{W}\hat{\alpha}\left(\mathbf{V}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} - \mathbf{U}\hat{\boldsymbol{\alpha}}\right)^{-1}\Delta\mathbf{d} = \mathbf{W}\hat{\alpha}\hat{\boldsymbol{\alpha}}^{-1}\left(\mathbf{V}^{\mathrm{T}} - \mathbf{U}\right)^{-1}\Delta\mathbf{d} = \Delta\mathbf{w}$$

For models B and D, the case is more complicated, because there is no clear formula for $\tilde{\mathbf{q}}\Delta\tilde{\mathbf{w}} = \tilde{\mathbf{W}}\left(\tilde{\hat{\mathbf{q}}}\tilde{\mathbf{V}}^{\mathrm{T}}\tilde{\hat{\mathbf{g}}} - \tilde{\mathbf{U}}\right)^{-1}\Delta\tilde{\mathbf{d}} = \mathbf{W}\hat{\alpha}\left(\tilde{\hat{\mathbf{q}}}\mathbf{V}^{\mathrm{T}}\hat{\alpha}\widehat{\mathbf{g}\hat{\alpha}} - \mathbf{U}\hat{\alpha}\right)^{-1}\Delta$. We haveSo, the scale invariance axiom strengthens our finding that models A and C satisfy the unit invariance axiom.

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