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Four mappings and generalized contractions

Lech Pasicki*

*Correspondence:
pasicki@agh.edu.pl
Faculty of Applied Mathematics, AGH University of Science and Technology, Al. Mickiewicza 30, Kraków, 30-059, Poland

Abstract

In the paper general theorems on common fixed point for four mappings are presented. The results are compact and they extend and unify the respective part of the fixed point theory.

MSC: 47H10; 54H25

Keywords: generalized contraction; dislocated metric; weakly compatible mappings; fixed point

1 Introduction

The present paper was inspired by the advanced and sophisticated article of Liu *et al.* [1]. Our assumptions are weaker, as the comparison function is much more general, and we do not assume the spaces under consideration to be metric. In addition the general contraction condition is compact and abstract. Also the proofs are relatively simple.

2 Theorems

Let us recall (see [2]) that $\boldsymbol{\Phi}$ is the family of all mappings $\varphi : [0,\infty) \to [0,\infty)$ such that $\varphi(\alpha) < \alpha$, $\alpha > 0$, and $\boldsymbol{\Phi_0}$ consists of mappings $\varphi \in \Phi$ such that $\varphi(0) = 0$. In turn, $\boldsymbol{\Phi}_P$ is the family of all mappings $\varphi : [0,\infty) \to [0,\infty)$ for which every sequence $(a_n)_{n\in\mathbb{N}}$ such that $a_{n+1} \le \varphi(a_n)$, $n \in \mathbb{N}$, converges to zero. It is well known ([2], Proposition 16) that $\Phi_P \subset \Phi_0$.

In turn, $\boldsymbol{\Psi}_P$ (see [3]) consists of all mappings of Φ for which every sequence $(a_n)_{n\in\mathbb{N}}$ such that $0 < a_{n+1} \le \varphi(a_n)$, $n \in \mathbb{N}$ converges to zero. It is known ([4], Corollary 2.4) that Ψ_P consists of all mappings $\varphi \in \Phi$ satisfying

$$\text{for each } \alpha > 0, \varphi(\cdot) \le \alpha \text{ on some interval } (\alpha, \alpha + \epsilon). \tag{1}$$

Clearly, $\Phi_P \subset \Psi_P$ holds and consequently, all members of Φ_P satisfy (1).

Lemma 2.1 *If a* $\varphi \in \Phi_0$ *satisfies* (1), *then* $\varphi \in \Phi_P$.

Proof Let $(a_n)_{n\in\mathbb{N}}$ be a sequence such that $a_{n+1} \le \varphi(a_n)$, $n \in \mathbb{N}$ for a $\varphi \in \Phi_0$. Then we have

$$a_{n+1} \le \varphi(a_n) \le a_n, \quad n \in \mathbb{N}.$$

Therefore, $(a_n)_{n\in\mathbb{N}}$ is nonincreasing and it converges, say to α. Suppose $\alpha > 0$. Then from (1) it follows that there exists an interval $(\alpha, \alpha + \epsilon)$ on which $\varphi(\cdot) \le \alpha$. For large n all a_n belong to this interval, as

$$0 < \alpha \le \varphi(a_{n+1}) \le a_{n+1} \le a_n,$$

and $\varphi \in \Phi$ yield $\alpha \le \varphi(a_{n+1}) < a_{n+1}$. Now, we have $\alpha < a_{n+1} \le \varphi(a_n) \le \alpha$, a contradiction. Consequently, $\alpha = 0$, *i.e.* $\varphi \in \Phi_P$. □

Corollary 2.2 *Φ_P consists of all mappings $\varphi \in \Phi_0$ satisfying* (1).

The notion of dislocated metric space presented below is due to Hitzler and Seda [5].
Let X be a nonempty set, and $p : X \times X \to [0, \infty)$ a mapping satisfying

$$p(x,y) = 0 \text{ yields } x = y, \quad x, y \in X, \tag{2a}$$

$$p(x,y) = p(y,x), \quad x, y \in X, \tag{2b}$$

$$p(x,z) \le p(x,y) + p(y,z), \quad x, y, z \in X. \tag{2c}$$

Then p is called a *dislocated metric* (briefly a d-metric), and (X,p) is called a *dislocated metric space* (briefly a d-metric space). The topology of (X,p) is generated by balls.

Many authors applied sophisticated contraction conditions. To present a general idea let us consider a mapping $h : X^4 \to [0, \infty)$ satisfying the following requirements for each $\alpha > 0$:

$$(a = d \text{ or } b = c) \text{ yields } h(a,b,c,d) \le \max\{p(a,b), p(c,a), p(d,b)\}, \tag{3a}$$

$$\big(p(a,b) \to \alpha \text{ and } p(c,a), p(d,b) \to 0\big) \text{ yields } \limsup h(a,b,c,d) \le \alpha, \tag{3b}$$

$$\text{if } p(d,b) \ge \alpha \text{ and } p(a,b), p(c,a) \text{ are small, then } h(a,b,c,d) \le p(d,b), \tag{3c}$$

$$\text{if } p(c,a) \ge \alpha \text{ and } p(a,b), p(d,b) \text{ are small, then } h(a,b,c,d) \le p(c,a), \tag{3d}$$

$$h(a,b,a,b) \le p(a,b). \tag{3e}$$

In order to present an example let us recall the notion of partial metric due to Matthews ([6], Definition 3.1).

A *partial metric* is a mapping $p : X \times X \to [0, \infty)$ such that

$$x = y \text{ iff } p(x,x) = p(x,y) = p(y,y), \quad x, y \in X, \tag{4a}$$

$$p(x,x) \le p(x,y), \quad x, y \in X, \tag{4b}$$

$$p(x,y) = p(y,x), \quad x, y \in X, \tag{4c}$$

$$p(x,z) \le p(x,y) + p(y,z) - p(y,y), \quad x, y, z \in X. \tag{4d}$$

Conditions (4b), (4a) yield (2a) and therefore each partial metric is a d-metric. As regards the respective spaces, the situation is more complicated because their topologies are different.

Example 2.3 Let p be a partial metric on X and let

$$h(a,b,c,d) = \bigl[p(a,d) + p(b,c)\bigr]/2. \tag{5}$$

Then for $a = d$ we have

$$\begin{aligned}\bigl[p(a,d) + p(b,c)\bigr]/2 &\le \bigl[p(a,d) + p(b,d) + p(d,c) - p(d,d)\bigr]/2\\ &= \bigl[p(b,d) + p(a,c)\bigr]/2 \le \max\bigl\{p(c,a), p(d,b)\bigr\}.\end{aligned}$$

Similarly, for $b = c$ we obtain

$$\begin{aligned}\bigl[p(a,d) + p(b,c)\bigr]/2 &\le \bigl[p(a,b) + p(b,d) - p(b,b) + p(b,c)\bigr]/2\\ &= \bigl[p(a,b) + p(b,d)\bigr]/2 \le \max\bigl\{p(a,b), p(d,b)\bigr\}.\end{aligned}$$

Consequently, (3a) holds. From

$$\begin{aligned}&\bigl[p(a,d) + p(b,c)\bigr]/2\\ &\quad\le \bigl[p(a,b) + p(b,d) - p(b,b) + p(b,a) + p(a,c) - p(a,a)\bigr]/2\\ &\quad= p(a,b) + \bigl[p(d,b) - p(b,b) + p(c,a) - p(a,a)\bigr]/2\end{aligned}$$

we obtain (3b), (3c), (3d), and (3e).

The notion of a 0-*complete* d-metric space (or a set) was presented in [3], Definition 2.3 (condition (2.5)). Let us note that if $\lim_{n\to\infty} p(y,x_n) = \lim_{n\to\infty} p(x,x_n) = 0$ holds, then from $p(x,y) \le p(x,x_n) + p(y,x_n)$ it follows that $x = y$. In addition, $p(x,x) \le 2p(x,x_n)$ means that $p(x,x) = 0$. Therefore, condition (2.5) of [3] is equivalent to

$$\begin{aligned}&\text{for each sequence } (x_n)_{n\in\mathbb{N}} \text{ in } X \text{ with } \lim_{m,n\to\infty} p(x_n,x_m) = 0\\ &\text{there exists a unique } x \in X \text{ for which } \lim_{n\to\infty} p(x,x_n) = p(x,x) = 0.\end{aligned} \tag{6}$$

The idea of 0-completeness for partial metric spaces is due to Romaguera ([7], Definition 2.1). A partial metric space (X,p) is called 0-complete if any 0-Cauchy sequence $(x_n)_{n\in\mathbb{N}}$ in X (*i.e.* such that $\lim_{m,n\to\infty} p(x_n,x_m) = 0$), converges (in the topology of (X,p)) to a point $x \in X$ for which $p(x,x) = 0$.

It is well known (*e.g.* [6]) that $x \in \lim_{n\to\infty} x_n$ in a partial metric space (X,p) iff $\lim_{n\to\infty} p(x,x_n) = p(x,x)$. In addition, as it was noticed before, each partial metric is a d-metric. Hence we obtain the following.

Corollary 2.4 *Any partial metric space* (X,p) *is* 0*-complete iff* (X,p) *treated as a d-metric space is* 0*-complete, and iff* (6) *is satisfied.*

Proposition 2.5 *Let* (X,p) *be a d-metric space and let* $(x_n)_{n\in\mathbb{N}}$ *be a sequence of points of* X *such that*

$$p(x_{n+2},x_{n+1}) \le \varphi\bigl(p(x_{n+1},x_n)\bigr), \quad n \in \mathbb{N},$$

holds for a $\varphi \in \Phi_P$. *Then we have*

$$\lim_{n\to\infty} p(x_{n+1},x_n) = \lim_{n\to\infty} p(x_n,x_n) = 0. \tag{7}$$

Proof Let us adopt $a_n = p(x_{n+1},x_n)$. Clearly, $a_{n+1} \leq \varphi(a_n)$, $n \in \mathbb{N}$ holds, and $\lim_{n\to\infty} a_n = 0$, as $\varphi \in \Phi_P$. Now, $p(x_n,x_n) \leq 2p(x_{n+1},x_n) = 2a_n$ completes (7). □

In the following $f(X)$ is replaced by fX, $f(x)$ is replaced by fx, etc. The precise order in $\max\{p(a,b),p(c,a),p(d,b)\}$ informs on the variables of the mapping h, and they are not shown in the proofs.

Lemma 2.6 *Let* (X,p) *be a d-metric space and let* f,g,i,j *be self mappings in* X *satisfying the following conditions*:

$$fX \subset jX,\ gX \subset iX \text{ and at least one of these sets is } 0\text{-complete}, \tag{8}$$

$$p(fx,gy) \leq \varphi\bigl(\max\bigl\{p(ix,jy),p(fx,ix),p(gy,jy),h(ix,jy,fx,gy)\bigr\}\bigr), \tag{9}$$

for a $h : X^4 \to [0,\infty)$ *such that* (3a), (3b) *hold, and a* $\varphi \in \Phi_P$ (*see Corollary* 2.2). *Then there exist sequences* $(x_n)_{n\in\mathbb{N}}$, $(y_n)_{n\in\mathbb{N}}$ *in* X *such that*

$$\begin{aligned} &x_{2k+1} = gy_{2k} \quad \text{for } x_{2k} = jy_{2k} \quad \text{and} \\ &x_{2k} = fy_{2k-1} \quad \text{for } x_{2k-1} = iy_{2k-1}, \quad k \in \mathbb{N}, \end{aligned} \tag{10}$$

and $(x_n)_{n\in\mathbb{N}}$ *converges to a point* x *such that* $p(x,x) = 0$, *and*

$$\begin{aligned} x &= \lim_{n\to\infty} x_n = \lim_{k\to\infty} fy_{2k-1} = \lim_{k\to\infty} iy_{2k-1} \\ &= \lim_{k\to\infty} gy_{2k} = \lim_{k\to\infty} jy_{2k}. \end{aligned} \tag{11}$$

Proof From (8), (9), and (3a) (for $b = c$) it follows that

for each $x \in X$ there exists a $y \in X$ such that $fx = jy$ and

$$p(gy,fx) = p(fx,gy) \leq \varphi\bigl(\max\bigl\{p(ix,fx),p(gy,fx)\bigr\}\bigr),$$

and (for $a = d$)

for each $x \in X$ there exists a $y \in X$ such that $gx = iy$ and

$$p(fy,gx) \leq \varphi\bigl(\max\bigl\{p(gx,jx),p(fy,gx)\bigr\}\bigr).$$

If *e.g.* $p(fx,ix) \leq p(gy,fx)$ holds, then $p(gy,fx) \leq \varphi(p(gy,fx))$ means that $p(gy,fx) = 0$, as $\varphi \in \Phi_0$. Consequently, (8) and (9) yield

$$\begin{aligned} &\text{for each } x \in X \text{ there exists a } y \in X \text{ such that} \\ &fx = jy \text{ and } p(gy,fx) \leq \varphi\bigl(p(fx,ix)\bigr) \end{aligned} \tag{12}$$

and

$$\begin{aligned}&\text{for each } x \in X \text{ there exists a } y \in X \text{ such that}\\ &gx = iy \text{ and } p(fy, gx) \le \varphi\bigl(p(gx, jx)\bigr).\end{aligned} \tag{13}$$

For an $x_0 \in X$ let us adopt $x_1 = gx_0 = iy_1$, $x_2 = fy_1$, where $p(x_2, x_1) \le \varphi(p(x_1, jx_0))$ (see (13)). Now, we define $x_3 = gy_2$ for y_2 such that $x_2 = fy_1 = jy_2$ and $p(x_3, x_2) \le \varphi(p(x_2, x_1))$ (see (12)). In turn $x_4 = fy_3$ for y_3 such that $x_3 = gy_2 = iy_3$ and $p(x_4, x_3) \le \varphi(p(x_3, x_2))$ (see (13)). By induction we define two sequences $(x_n)_{n\in\mathbb{N}}$, $(y_n)_{n\in\mathbb{N}}$ satisfying (10) and

$$p(x_{n+2}, x_{n+1}) \le \varphi\bigl(p(x_{n+1}, x_n)\bigr), \quad n \in \mathbb{N}.$$

In view of Proposition 2.5 we have $\lim_{n\to\infty} p(x_{n+1}, x_n) = 0$.

Suppose that there exists an infinite set $\mathbb{K} \subset \mathbb{N}$ such that for each $k \in \mathbb{K}$ there exists an $n \in \mathbb{N}$ for which $0 < \alpha < p(x_{2k}, x_{2n+1+2k})$ holds. Let $n = n(k) > 0$ be the smallest numbers satisfying this inequality for $k \in \mathbb{K}$. We have (see (2c))

$$\begin{aligned}&\alpha - p(x_{2k}, x_{2k-1}) - p(x_{2n+2k}, x_{2n+1+2k})\\ &\quad< p(x_{2k}, x_{2n+1+2k}) - p(x_{2k}, x_{2k-1}) - p(x_{2n+2k}, x_{2n+1+2k})\\ &\quad\le p(x_{2k-1}, x_{2n+2k})\\ &\quad\le p(x_{2k-1}, x_{2k}) + p(x_{2k}, x_{2n-1+2k}) + p(x_{2n-1+2k}, x_{2n+2k})\\ &\quad\le p(x_{2k-1}, x_{2k}) + \alpha + p(x_{2n-1+2k}, x_{2n+2k})\end{aligned}$$

for $n = n(k)$, and therefore (see (7)),

$$\lim_{k\in\mathbb{K}} p(x_{2k-1}, x_{2n+2k}) = \alpha.$$

Now, (9), (10), (7), and the above equality yield

$$\begin{aligned}p(x_{2k}, x_{2n+1+2k}) &= p(fy_{2k-1}, gy_{2n+2k})\\ &\le \varphi\bigl(\max\bigl\{p(iy_{2k-1}, jy_{2n+2k}), p(fy_{2k-1}, iy_{2k-1}), p(gy_{2n+2k}, jy_{2n+2k}), h(\cdot)\bigr\}\bigr)\\ &= \varphi\bigl(\max\bigl\{p(x_{2k-1}, x_{2n+2k}), p(x_{2k}, x_{2k-1}), p(x_{2n+1+2k}, x_{2n+2k}), h(\cdot)\bigr\}\bigr)\\ &= \varphi\bigl(\max\bigl\{p(x_{2k-1}, x_{2n+2k}), h(\cdot)\bigr\}\bigr)\end{aligned}$$

for large k. In turn, (3b) yields

$$\limsup_{k\in\mathbb{K}} h(\cdot) \le \lim_{k\in\mathbb{K}} p(x_{2k-1}, x_{2n+2k}) = \alpha,$$

and hence we obtain $\max\{p(x_{2k-1}, x_{2n+2k}), h(\cdot)\} < \alpha + \epsilon$ for large k. From $\varphi(\beta) < \alpha$ for $\beta \le \alpha$, and (1) we get

$$\alpha < p(x_{2k}, x_{2n+1+2k}) \le \varphi\bigl(\max\bigl\{p(x_{2k-1}, x_{2n+2k}), h(\cdot)\bigr\}\bigr) \le \alpha,$$

for large k, a contradiction. Therefore

$$\lim_{k,n\to\infty} p(x_{2k}, x_{2n+1+2k}) = 0$$

holds, and (7) with the triangle inequality (2c) yield $\lim_{m,n\to\infty} p(x_n, x_m) = 0$. Consequently (see (6)), there exists an $x \in X$ such that $\lim_{n\to\infty} p(x, x_n) = p(x, x) = 0$, and (11) holds. □

Let us recall (see [8]) that a pair (f, i) of mappings $f, i : X \to X$ is called *weakly compatible* if

$$\text{for each } x \in X, fx = ix \text{ yields } fix = ifx.$$

Now, we are ready to prove our theorem.

Theorem 2.7 *Let* (X, p) *be a d-metric space such that* (4b) *holds. Assume that* f, g, i, j *are self mappings in* X *satisfying*

$$(f, i), (g, j) \quad \text{are weakly compatible,} \tag{14}$$

(8), *and* (9) *for a* $\varphi \in \Phi_P$ *(see Corollary* 2.2) *and a mapping* $h : X^4 \to [0, \infty)$ *for which the system of conditions* (3a)-(3e) *holds. Then* f, g, i, j *have a unique common fixed point, say* x, *and* $p(x, x) = 0$.

Proof Let $(x_n)_{n\in\mathbb{N}}$, $(y_n)_{n\in\mathbb{N}}$, and x be as in Lemma 2.6. Assume *e.g.* that $x \in jX$ (for 0-complete jX or fX). Then there exists a v such that $x = jv$. Let us prove that $gv = x$. We have

$$p(gv, x) = p(x, gv) \le p(x, x_{2k}) + p(x_{2k}, gv).$$

Suppose $p(gv, x) > 0$. Then we obtain

$$\begin{aligned} p(x_{2k}, gv) &= p(fy_{2k-1}, gv) \\ &\le \varphi\bigl(\max\bigl\{p(iy_{2k-1}, jv), p(fy_{2k-1}, iy_{2k-1}), p(gv, jv), h(\cdot)\bigr\}\bigr) \\ &= \varphi\bigl(\max\bigl\{p(x_{2k-1}, x), p(x_{2k}, x_{2k-1}), p(gv, x), h(\cdot)\bigr\}\bigr) = \varphi\bigl(p(gv, x)\bigr), \end{aligned}$$

for large k (see (9), (3c). Hence,

$$p(gv, x) \le \lim_{k\to\infty} p(x, x_{2k}) + \varphi\bigl(p(gv, x)\bigr)$$

yields $p(gv, x) = 0$ ($\varphi \in \Phi_0$), and $gv = x$. Thus we have $x = gv = jv$.

From $gX \subset iX$ it follows that there exists a w such that $x = iw$. Let us show that $fw = x$. We have

$$p(fw, x) \le p(fw, gy_{2k}) + p(gy_{2k}, x) = p(fw, gy_{2k}) + p(x_{2k+1}, x).$$

Suppose $p(fw,x) > 0$. Then we have

$$\begin{aligned} p(fw,gy_{2k}) &\le \varphi\bigl(\max\bigl\{p(iw,jy_{2k}),p(fw,iw),p(gy_{2k},jy_{2k}),h(\cdot)\bigr\}\bigr) \\ &= \varphi\bigl(\max\bigl\{p(x,x_{2k}),p(fw,x),p(x_{2k+1},x_{2k})\bigr\}\bigr) = \varphi\bigl(p(fw,x)\bigr) \end{aligned}$$

for large k (see (9)), (3d)). Hence, we obtain

$$p(fw,x) \le \varphi\bigl(p(fw,x)\bigr) + \lim_{k\to\infty} p(x_{2k+1},x) = \varphi\bigl(p(fw,x)\bigr),$$

and $p(fw,x) = 0$, *i.e.* $x = fw = iw$.

Now, (14) yields

$$jx = jgv = gjv = gx \quad \text{and} \quad ix = ifw = fiw = fx.$$

Let us show that $fx = gx$. We have (see (9), (3e), (4b))

$$\begin{aligned} p(fx,gx) &\le \varphi\bigl(\max\bigl\{p(ix,jx),p(fx,ix),p(gx,jx),h(\cdot)\bigr\}\bigr) \\ &= \varphi\bigl(\max\bigl\{p(fx,gx),p(fx,fx),p(gx,gx)\bigr\}\bigr) = \varphi\bigl(p(fx,gx)\bigr), \end{aligned}$$

and therefore $p(fx,gx) = 0$, *i.e.* $fx = gx$ (let us note that $p(fx,fx) = 0$ not necessarily holds if p is a d-metric).

Now, it is clear that $fx = gx = ix = jx$ holds.

Let us prove that $gx = x$. We have (see (9), (3e), (4b))

$$\begin{aligned} p(x,gx) = p(fw,gx) &\le \varphi\bigl(\max\bigl\{p(iw,jx),p(fw,iw),p(gx,jx),h(\cdot)\bigr\}\bigr) \\ &= \varphi\bigl(\max\bigl\{p(x,gx),p(x,x),p(gx,gx)\bigr\}\bigr) = \varphi\bigl(p(x,gx)\bigr), \end{aligned}$$

and consequently, $p(x,gx) = 0$, *i.e.* $gx = x$.

We have proved that $x = fx = gx = ix = jx$.

If y is a common fixed point of our mappings, then (see (9), (3e), (4b))

$$p(x,y) = p(fx,gy) \le \varphi\bigl(\max\bigl\{p(ix,jy),p(fx,ix),p(gy,jy),h(\cdot)\bigr\}\bigr) = \varphi\bigl(p(x,y)\bigr)$$

yields $p(x,y) = 0$, *i.e.* $x = y$. □

The authors of [1] consider (in metric spaces) formulas M_1, M_2, and M_3 instead of 'max…' from our condition (9). It would be rather exhausting to cite M_1 or M_2, nevertheless all three formulas are applied in conditions that are particular cases of (9) for h satisfying (3a)-(3e).

The next theorem is a consequence of Corollary 2.4, and Theorem 2.7 (see also Example 2.3).

Theorem 2.8 *Let (X,p) be a partial metric space. Assume that f, g, i, j are self mappings in X satisfying* (14), (8), *and* (9) *for a $\varphi \in \Phi_P$ (see Corollary* 2.2) *and a mapping $h : X^4 \to [0,\infty)$ for which the system of conditions* (3a)-(3e) *holds. Then f, g, i, j have a unique common fixed point, say x, and $p(x,x) = 0$.*

The above theorem is a far extension of [9], Theorem 2.8 in its part concerning a unique common fixed point; our mapping φ need not be continuous (see also Example 2.3).

Competing interests
The author declares that he has no competing interests.

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