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Quasiconsensus of fractional-order heterogeneous multiagent systems under event-triggered impulsive control method

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Abstract

This paper investigates the quasiconsensus problem of fractional-order heterogeneous multiagent systems, the distributed impulsive control protocol is designed for the multiagent system. In contrast to some existing results, the impulsive moments are determined by preset events, i.e., the event-triggered mechanism is used. Based on the fractional-order Lyapunov stability theory and fractional-order differential inequality, the quasiconsensus criteria are derived; furthermore, the prescribed error bound is given. Then, Zeno behavior for the considered event-triggered control method is excluded. Finally, numerical examples are given to shown the effectiveness of the proposed method.

Keywords: Quasiconsensus; Event-triggered; Impulsive control; Fractional-order multiagent systems; Heterogeneous

1 Introduction

Multiagent systems have been a hot topic in the past decades, due to their wide applications in many different fields, such as unmanned aerial vehicles, multirobot formations, distributed optimization, etc. There are some results reflecting that modeling by fractional-order differential equations would produce more accurate descriptions, for example, underwater robots that work on the ocean floor where microbes and sticky matter abound. Li has studied the consensus behavior of fractional-order multiagent systems in [1] and [2]. Since then, many results have focused on consensus of fractional-order multiagent systems (FOMASs), see for example [3–7] and references therein.

In a networked environment, communications among agents often block the channel under a continuous-transmission mechanism. Thus, discontinuous transmission mechanisms of information of agents have attracted much attention, in which, both time-triggered and event-triggered methods have produced many significant results [8–10]. There are several kinds of time-triggered methods, such as impulsive control, intermittent control, sampled-data control, etc. All of them have been widely applied in the fractional-order multiagent systems. See, for example [11–13] and references therein. The event-triggered control method has been proposed by Tabuada in [14], and was first used for fractional-order multiagent systems in [15]. Many outstanding results have been published recently [16–18].

Impulsive control makes the controlled systems convert their orbits just in some discrete instants and has extremely low cost. Noting that most of the existing results about impulsive control are time triggered, i.e., agents will change their states at some determined moments (periodic or aperiodic). A natural question is can we design the impulsive controllers based on an event-triggered mechanism? In other words, agents will change their states at some moments when the preset events occur. The so-called "Event-Triggered Impulsive Control (ETIC)" has aroused more and more attention in the last few years [19–21]. However, there is little research about ETIC for fractional-order systems [22, 23].

In networked systems, the mismatched parameters for the subsystems are difficult to avoid. This phenomenon caused the heterogeneous multiagent systems to be widely investigated by researchers. For the fractional-order multiagent systems, there are also a number of papers about heterogeneous models [24–26]. According to the discussion above, this paper will consider the quasiconsensus problem of fractional-order heterogeneous multiagent systems via the ETIC method. The main contributions of this manuscript can be summarized as follows:

(1) This manuscript studies the consensus problem of fractional-order heterogeneous multiagent systems using event-triggered impulsive control, while most existing works about cooperative control for fractional-order multiagent systems did not consider that the multiagent systems are heterogeneous.

(2) For the controllers given in this paper, the impulsive controllers based on an event-triggered mechanism are provided, which can avoid the situation that impulsive instants for all agents should be always identical. Furthermore, Zeno behavior is successfully excluded.

(3) Distributed impulsive controllers are used, which can reduce channel blocking, under which, the bounded consensus criteria are given by some lower-dimensional matrix inequalities and scalar inequalities and a prescribed error bound is given.

The remainder of this paper is organized as follows. The preliminaries of fractional-order calculus and problem formulation are introduced in Sect. 2. The quasiconsensus criteria for the considered fractional-order multiagent systems are derived in Sect. 3. In Sect. 4, the effectiveness and feasibility of the developed methods are shown by two numerical examples. A concise discussion is given in Sect. 5.

Notations Throughout this paper, I_n denotes an n-dimensional identity matrix. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ is the set of $m \times n$ real matrices. $*$ stands for the symmetrical part in a matrix. diag$\{\ldots\}$ stands for a diagonal matrix. $|x|$ denotes the absolute value of x. $\|\cdot\|$ denotes the Euclidean norm of the vector. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ stand for the largest eigenvalue and smallest eigenvalue of matrix P, respectively. $\sigma_{\max}(P)$ stands for the maximum singular value of matrix P.

2 Preliminaries and problem formulation

2.1 The Caputo fractional operator and Mittag–Leffler function

Definition 1 ([27]) The $\alpha > 0$ order integral is defined as:

$$
{}_{\varrho}\mathcal{D}_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{\varrho}^{t} \frac{f(s)}{(t-s)^{1-\alpha}}\, ds.
$$

Definition 2 ([27]) Caputo's $\alpha > 0$ order derivative is defined as:

$$ {}_{\varrho}^{C}\mathcal{D}_{t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{\varrho}^{t} \frac{f^{(n)}(s)}{(t-s)^{1+\alpha-n}}\, ds, $$

where $n - 1 < \alpha \leq n$, $n \in \mathbb{N}$.

In the following, we will consider Caputo's operation, by simply denoting:

$$ {}_{\varrho}\mathcal{D}^{\alpha} f(t) = {}_{\varrho}^{C}\mathcal{D}_{t}^{\alpha} f(t). $$

We just consider the case that $0 < \alpha < 1$, then, one has:

$$ {}_{\varrho}\mathcal{D}^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{\varrho}^{t} \frac{f'(s)}{(t-s)^{\alpha}}\, ds. $$

Noting that, for any constant C, one has ${}_{\varrho}\mathcal{D}^{\alpha} C = 0$. The Mittag–Leffler function is the basis function of fractional calculus, as the exponential function is to the integer-order calculus, which is defined as follows.

Definition 3 ([27]) The two-parameter Mittag–Leffler function is defined as:

$$ E_{\alpha,\beta}(z) = \sum_{i=0}^{\infty} \frac{z^{i}}{\Gamma(\alpha i + \beta)}, $$

where $\alpha > 0$, $\beta > 0$, $\Gamma(.)$ is the Gamma function.

Definition 4 ([27]) The one-parameter Mittag–Leffler function is defined as:

$$ E_{\alpha}(z) = E_{\alpha,1}(z) = \sum_{i=0}^{\infty} \frac{z^{i}}{\Gamma(\alpha i + 1)}. $$

In the particular case when $\alpha = 1$, one has $E_1(z) = \exp(z)$.

Lemma 1 ([28]) *Let $x(t) \in R^n$ be a vector of differentiable functions. Then, for any time instant $t \geq \varrho$, the following relationship holds*

$$ {}_{\varrho}\mathcal{D}^{\alpha}\left(x^{T}(t)Px(t)\right) \leq 2x^{T}(t)P\,{}_{\varrho}\mathcal{D}^{\alpha}x(t), \quad \forall \alpha \in (0,1), \forall t \geq \varrho, $$

where $P \in \mathbb{R}^{n\times n}$ is a constant, square, symmetric, and positive-definite matrix.

Lemma 2 ([29]) *Suppose that $V(t)$ is a continuous function satisfying ${}_{t_k}\mathcal{D}_{t}^{\alpha} V(t) \leq \theta V(t)$ for $t > t_k$, then,*

$$ V(t) \leq V(t_k) E_{\alpha}\left(\theta (t - t_k)^{\alpha}\right), \quad t \geq t_k, $$

where $0 < \alpha < 1$ and θ is a constant.

2.2 Model formulation

Consider the nonlinear FOMASs consisting of N followers (labeled by $1, 2, \dots, N$), which are described by

$$ {}_{t_k}\mathcal{D}^{\alpha} x_i(t) = A_i x_i(t) + B_i g\big(x_i(t)\big) + u_i(t), \quad i = 1, 2, \dots, N, \tag{1} $$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \dots, x_{in}(t)]^T \in \mathbb{R}^n$ denotes the state of the ith follower, A_i and B_i are constant matrices, $g(x_i(t)) = [g_1(x_i(t)), g_2(x_i(t)), \dots, g_n(x_i(t))]^T$ is a vector value function with $g_k(\cdot) : \mathbb{R}^n \to \mathbb{R}$, and $u_i(t)$ is the communication protocol, which will be designed later. The dynamics of the leader (labeled by 0) is described by

$$ {}_{t_k}\mathcal{D}^{\alpha} x_0(t) = A_0 x_0(t) + B_0 f\big(x_0(t)\big), \tag{2} $$

where $x_0(t) = [x_{01}(t), x_{02}(t), \dots, x_{0n}(t)]^T \in \mathbb{R}^n$ denotes the state of the leader, $x_0(t)$ may be an equilibrium point, a periodic orbit or event a chaotic orbit.

The distributed impulsive control protocol is designed as

$$ u_i(t) = \sum_{k=1}^{\infty} \Bigg[-c\gamma_k \sum_{j=1}^{N} l_{ij} x_j(t) - c d_i \gamma_k \big(x_i(t) - x_0(t)\big) \Bigg] \delta(t - t_k), \tag{3} $$

where c is the coupling strength, $d_i \geq 0$ are the gain between leader and the ith follower, $i = 1, 2, \dots, N$, when $d_i = 0$, there is no directed path from the leader to the ith follower. Consequently, it can be seen as a pinning control method. $\delta(\cdot)$ is the Dirac delta function, and $\delta(t) = \lim_{r \to 0} \chi(t)$ with $\chi(t) = \frac{1}{r}$ when $0 \leq t < r$, and $\chi(t) = 0$ otherwise. γ_k is the impulsive gain in the kth impulsive moment; more information about the impulsive sequence $\{t_k\}$ and impulsive gain γ_k will be given later.

Let $e_i(t) = x_i(t) - x_0(t)$, $e(t) = [e_1(t), e_2(t), \dots, e_N(t)]^T$, then, the error dynamics can be described by

$$ {}_{t_k}\mathcal{D}^{\alpha} e_i(t) = A_i e_i(t) + B_i f\big(e_i(t), x_0(t)\big) + \varphi_i\big(x_0(t)\big) + u_i(t), \tag{4} $$

where $f(e_i(t), x_0(t)) = g(e_i(t) + x_0(t)) - g(x_0(t))$ and $\varphi_i(x_0(t)) = (A_i - A)x_0(t) + (B_i - B)g(x_0(t))$. Meanwhile, the control protocol can be rewritten as

$$ u_i(t) = \sum_{k=1}^{\infty} \Bigg[-c\gamma_k \sum_{j=1}^{N} l_{ij} e_j(t) - c d_i \gamma(k) e_i(t) \Bigg] \delta(t - t_k). $$

According to [30], let $\Delta e_i(t_k) = e_i(t_k^+) - e_i(t_k^-)$, and $e_i(t_k) = e_i(t_k^-) = \lim_{h \to 0^+} e_i(t_k - h)$, one can obtain the following error system:

$$ \begin{cases} {}_{t_k}\mathcal{D}^{\alpha} e_i(t) = A_i e_i(t) + B_i f(e_i(t), x_0(t)) + \varphi_i(x_0(t)), & t \in (t_{k-1}, t_k], \\ \Delta e_i(t_k) = -\frac{\gamma(k)}{\Gamma(1+\alpha)} [c \sum_{j=1}^{N} l_{ij} e_j(t_k) + c d_i e_i(t_k)]. \end{cases} \tag{5} $$

Throughout this paper, the nonlinear FOMASs are assumed to satisfy the following assumptions.

Assumption 1 There are nonnegative constants q_{ij} $(i,j = 1,2,\ldots,n)$ such that, for any $x = [x_1,x_2,\ldots,x_n] \in \mathbb{R}^n$ and $y = [y_1,y_2,\ldots,y_n] \in \mathbb{R}^n$, $|g_i(x) - g_i(y)| \leq \sum_{j=1}^{n} q_{ij}|x_j - y_j|$.

Assumption 2 $x_0(t)$ is bounded, that is, for any initial value $x_0(0)$, there is $\hat{T}(x_0(0))$ such that for any $t \geq \hat{T}(x_0(0))$, $\|x_0(t)\| \leq \varrho$, where ϱ is a positive constant.

Assumption 3 There is a directed spanning tree with the leader as the root in the communication topology of the FOMAS, that is, the leader has a path to every follower.

Remark 1 Let $Q = (q_{ij})_{n\times n}$. Then, for any diagonal matrices $\Lambda_g > 0$, Assumption 1 implies that $(x-y)^T Q^T \Lambda_g Q(x-y) \geq (g(x) - g(y))^T \Lambda_g (g(x) - g(y))$. Also, note that there are many systems that can be satisfied, such as Chua's circuit, and some chaotic neural networks. In addition, according to Assumption 1 and Assumption 2, $\varphi_i(x_0(t))$ is also bounded, that is, $\max_{t\geq\hat{T}} \|\varphi_i(x_0(t))\| = \varpi_i$, where $\varpi_i \geq 0$, $i = 1,2,\ldots,N$, are constants.

2.3 The design of the event-triggered impulsive controller (EIFC)

In this subsection, we will design the event-triggered impulsive controller. In the impulsive control method, the states of the system will be jumped at some determined moment t_k, however, when the states are converging at some impulsive moment, the states are unnecessary to jump. Therefore, the event-triggered mechanism will be adopted in this paper, which is related with the states of the system.

Let $T > 0$ be the check period and $0 = t_0$, $V(t) = \sum_{i=1}^{N} e_i^T(t) P e_i(t)$ and $P \in \mathbb{R}^{n\times n}$ is a positive-definite matrix, $\theta_1 > 1$ and $\theta_2 < 1$. Then, the kth jumped moment and impulsive gain $\gamma(k)$ are determined by the following algorithm ($k = 1,2,\ldots$):

Under the above EIFC, let $D = \mathrm{diag}\{d_1,d_2,\ldots,d_N\}$ be the pinning control matrix, one can rewrite the error dynamics in a matrix form when $t = t_k$:

$$e(t_k^+) = \left(I_N - \frac{c\mu_\nu}{\Gamma(1+\alpha)}(L+D)\otimes I_n\right)e(t_k), \quad \nu = 1,2,3. \tag{6}$$

3 Main results

In this section, we will prove that there is no Zeno behavior for the considered FOMAS with the EIFC. Then, some impulsive quasiconsensus criteria are established for FOMAS (1).

Theorem 1 *Consider the FOMAS* (1) *with the checked period* $T > 0$, *impulsive instants* t_k *for* $k = 1,2,\ldots$ *determined by the Algorithm* 1. *If Assumptions* 1–3 *hold, and there are*

Algorithm 1 Algorithm to determine t_k, $k = 1,2,\ldots$

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if ∃t ∈ (t_{k-1}, t_{k-1} + T] such that V(t) ≥ θ_1 V(t^+_{k-1}) then
    t_k = inf{t ∈ (t_{k-1}, t_{k-1} + T] | V(t) ≥ θ_1 V(t^+_{k-1})} and γ(k) = μ_1
else if ∃t ∈ (t_{k-1}, t_{k-1} + T] such that V(t) ≥ θ_2 V(t^+_{k-1}) then
    t_k = t_{k-1} + T and γ(k) = μ_2
else
    t_k = t_{k-1} + T and γ(k) = μ_3 = 0
end if
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positive matrices P, Ψ_{1i}, Ψ_{2i}, Ξ_{1i}, Ξ_{2i} and constants a_i, positive constants ξ_i, $i = 1, 2, \ldots, N$, such that

$$\Psi_{1i} \leq \Psi_{2i}, \tag{7}$$

$$\Xi_{1i} - \Xi_{2i} \leq \xi_i I_n. \tag{8}$$

Then, there is no Zeno behavior for the concerned FOMAS, that is, there is a constant $\tau > 0$ such that $\inf\{t_k - t_{k-1}\} \geq \tau > 0$, where

$$\Psi_{1i} = \begin{pmatrix} PA_i + A_i^T P + Q^T \Lambda_g Q & PB_i & P \\ * & -\Lambda_g & 0 \\ * & * & -\Xi_{1i} \end{pmatrix},$$

$$\Psi_{2i} = \begin{pmatrix} a_i P & 0 & 0 \\ * & 0 & 0 \\ * & * & -\Xi_{2i} \end{pmatrix},$$

and $a = \max_{1 \leq i \leq N}\{a_i\}$, Q and Λ_g are defined in Remark 1.

Proof Choose a Lyapunov function as $V(t) = \sum_{i=1}^N e_i^T(t) P e_i(t)$, according to Lemma 1 and Remark 1 for any $t \in (t_{k-1}, t_k]$, $k = 1, 2, \ldots$, one has

$$\begin{aligned} {}_{t_k}\mathcal{D}^\alpha V(t)|_{(5)} &\leq 2 \sum_{i=1}^N e_i^T(t) P\big[A_i e_i(t) + B_i f\big(e_i(t)\big) + \varphi_i\big(x_0(t)\big)\big] \\ &\times \sum_{i=1}^N \big[\eta_i(t)^T \Psi_{1i} \eta_i(t) + \varphi_i^T\big(x_0(t)\big) \Xi_{1i} \varphi_i\big(x_0(t)\big)\big], \end{aligned} \tag{9}$$

where $\eta_i(t) = [e_i^T(t), f^T(e_i(t)), \varphi_i^T(x_0(t))]^T$, according to (7) and (8), one has

$${}_{t_k}\mathcal{D}^\alpha V(t) \leq aV(t) + \sum_{i=1}^N \xi_i \varpi_i^2. \tag{10}$$

Noting that, ${}_{t_k}\mathcal{D}^\alpha C = 0$ for any constant C, then, we have

$${}_{t_k}\mathcal{D}^\alpha \left(V(t) + \frac{\sum_{i=1}^N \xi_i \varpi_i^2}{a}\right) \leq a\left(V(t) + \frac{\sum_{i=1}^N \xi_i \varpi_i^2}{a}\right).$$

According to Lemma 2, one has

$$V(t) \leq -\frac{\sum_{i=1}^N \xi_i \varpi_i^2}{a} + \left(V\big(t_{k-1}^+\big) + \frac{\sum_{i=1}^N \xi_i \varpi_i^2}{a}\right) E_\alpha\big(a(t - t_{k-1})^\alpha\big), \quad t \in (t_{k-1}, t_k]. \tag{11}$$

Taking any $(t_{k-1}, t_k]$, let us consider the event at $t = t_k$. Based on Algorithm 1, if the first condition "$\exists t \in (t_{k-1}, t_{k-1} + T]$ such that $V(t) \geq \theta_1 V(t_{k-1}^+)$" is not met, then $t_k - t_{k-1} = T > 0$, it is obvious that there is no Zeno behavior. Consequently, we should investigate the case that "$\exists t \in (t_{k-1}, t_{k-1} + T]$ such that $V(t) \geq \theta_1 V(t_{k-1}^+)$", if this event occurs at t_k, we have $V(t_k) = \theta_1 V(t_{k-1}^+)$, combined with $\theta_1 > 1$ and (11), we have

$$V\big(t_{k-1}^+\big) + \frac{\sum_{i=1}^N \xi_i \varpi_i^2}{a} < \theta_1 V\big(t_{k-1}^+\big) + \frac{\sum_{i=1}^N \xi_i \varpi_i^2}{a} \leq \left(V\big(t_{k-1}^+\big) + \frac{\sum_{i=1}^N \xi_i \varpi_i^2}{a}\right) E_\alpha\big(a(t_k - t_{k-1})^\alpha\big).$$

Thus, we have $E_\alpha(a(t_k - t_{k-1})^\alpha) > 1$, then, we obtain $t_k - t_{k-1} > 0$. That is, Zeno behavior is excluded for the system. The proof is completed. □

Theorem 2 *Consider the FOMAS (1) with the checked period $T > 0$, impulsive instants t_k for $k = 1, 2, \ldots$ determined by Algorithm 1. If Assumptions 1–3, (7), (8) hold, and parameters of the FOMAS are satisfied by*

$$\sigma_{\max}^2 \left(I_N - \frac{c\mu_\nu}{\Gamma(\alpha+1)}(L+D)^T\right) \leq \rho, \quad \nu = 1, 2, \tag{12}$$

$$\rho\theta_1 \leq \theta_2, \tag{13}$$

then, the trajectory of the error system (5) can exponentially converge into a ball $\mathbb{M}$ with a convergence rate $\frac{\ln(\theta_2)}{2T}$, where $\mathbb{M} = \{e(t) | \|e(t)\| \leq \sqrt{\frac{(\eta-1)\sum_{i=1}^N \xi_i \varpi_i^2}{a\lambda_{\min}(P)}}\}$, in which,

$$\eta = \begin{cases} E_\alpha(a\tau^\alpha) & a \leq 0, \\ E_\alpha(aT^\alpha) & a > 0. \end{cases}$$

Proof Choose a Lyapunov function as $V(t) = \sum_{i=1}^N e_i^T(t) P e_i(t)$. If "$\exists t \in (t_{k-1}, t_{k-1} + T]$ such that $V(t) \geq \theta_1 V(t_{k-1}^+)$", according to (12) and definition of t_k, we have

$$\begin{aligned}
V\big(t_k^+\big) &= e^T\big(t_k^+\big)(I_N \otimes P) e\big(t_k^+\big) \\
&= e^T(t_k)\left(\left(I_N - \frac{c\mu_1}{\Gamma(\alpha+1)}(L+D)\right) \otimes I_n\right)^T (I_N \otimes P) \\
&\quad \times \left(\left(I_N - \frac{c\mu_1}{\Gamma(\alpha+1)}(L+D)\right) \otimes I_n\right) e(t_k) \\
&= \rho e^T(t_k)\left(\left(\left(I_N - \frac{c\mu_1}{\Gamma(\alpha+1)}(L+D)^T\right)\right.\right. \\
&\quad \times \left.\left.\left(I_N - \frac{c\mu_1}{\Gamma(\alpha+1)}(L+D)\right)\right) \otimes P\right) e(t_k) \\
&\leq \sigma_{\max}^2 \left(I_N - \frac{c\mu_1}{\Gamma(\alpha+1)}(L+D)^T\right) e^T(t_k)(I_N \otimes P) e(t_k) \\
&\leq \rho V(t_k) \leq \rho\theta_1 V\big(t_{k-1}^+\big) \leq \theta_2 V\big(t_{k-1}^+\big).
\end{aligned}$$

If "$\exists t \in (t_{k-1}, t_{k-1} + T]$ such that $V(t) \geq \theta_1 V(t_{k-1}^+)$" is not met, but "$\exists t \in (t_{k-1}, t_{k-1} + T]$ such that $V(t) \geq \theta_2 V(t_{k-1}^+)$", similarly, we have

$$V\big(t_k^+\big) \leq \theta_2 V\big(t_{k-1}^+\big).$$

If "$\exists t \in (t_{k-1}, t_{k-1}+T]$ such that $V(t) \geq \theta_1 V(t_{k-1}^+)$" is not met, and "$\exists t \in (t_{k-1}, t_{k-1}+T]$ such that $V(t) \geq \theta_2 V(t_{k-1}^+)$" is also not met, one can conclude that

$$V\bigl(t_k^+\bigr) = V(t_k) \leq \theta_2 V\bigl(t_{k-1}^+\bigr).$$

According to (11), one has

$$\begin{cases} V(t) \leq \eta V(t_{k-1}^+) + \zeta, & t \in (t_{k-1}, t_k], \\ V(t_k^+) \leq \theta_2 V(t_{k-1}^+), \end{cases} \tag{14}$$

where $\zeta = \varepsilon(\eta - 1)$, $\varepsilon = \frac{\sum_{i=1}^{N} \xi_i \varpi_i^2}{a}$. By mathematical induction, we can derive that

$$V(t) \leq \eta \theta_2^k V(0) + \zeta, \quad t \in (t_{k-1}, t_k].$$

Noting that $\tau \leq t_k - t_{k-1} \leq T$ and for any t, there must be k such that $t \in (t_{k-1}, t_k]$, one has $\frac{t}{T} \leq k \leq \frac{t}{\tau}$, which implies that

$$V(t) \leq \eta V(0) e^{\frac{\ln \theta_2}{T} t} + \zeta.$$

Therefore, one can conclude that

$$\bigl\|e(t)\bigr\| \leq \sqrt{\frac{\eta V(0)}{\lambda_{\min}(P)}} e^{\frac{\ln(\theta_2)}{2T} t} + \sqrt{\frac{\zeta}{\lambda_{\min}(P)}}.$$

Then, as $t \to +\infty$, the error $e(t)$ converges exponentially into the ball $\mathbb{M} = \{e(t) | \|e(t)\| \leq \sqrt{\frac{(\eta-1)\sum_{i=1}^{N} \xi_i \varpi_i^2}{a \lambda_{\min}(P)}}\}$ at a convergence rate $\frac{\ln(\theta_2)}{2T}$. The proof is completed. □

Remark 2 Note that conditions in Theorem 1 are independent of the order α; however, α effects the value of $E_\alpha(a(t_k - t_{k-1})^\alpha)$, which implies that α will impact the time interval of two successive triggers. In addition, $E_\alpha(a\tau^\alpha)$ is also related with α, which is significant in Theorem 2. Consequently, the consensus results in this paper are closely related to the order α.

Remark 3 In the above, the topology structure of the network is considered as a directed graph. When the topology is undirected, one has a symmetric Laplacian matrix L, then, the condition (12) can be replaced as $\lambda_{\max}^2(I_N - \frac{c}{\Gamma(\alpha+1)}(L+D)^T) \leq \rho$. In addition, if the FOMAS is homogeneous, which means that all nodes are identical, then it is easy to obtain $\varpi_i = 0$, $i = 1, 2, \ldots, N$, according to the above, one can obtain the complete exponential consensus.

Remark 4 More detailed results about error estimation, optimization for quasiconsensus of heterogeneous dynamic networks via distributed impulsive control have been discussed in [31], in which, the pinning strategy also has been investigated. Some similar results also can be derived in this paper, therefore, we omit them here.

Remark 5 Compared with some existing results about impulsive control or the distributed impulsive control method, this paper has considered the event-triggered mechanism.

Conditions in this manuscript are unrelated to the checked period T, which is important, the checked period T just effects the converge rate. Furthermore, due to the event-triggered mechanism, some unnecessary impulsive jumping can be avoided, which would be verified in the simulation part.

Remark 6 There are some results about impulsive control with an event-triggered mechanism. In [32–35], the event-based impulsive control method has been investigated, in which, the impulsive instants are determined by a certain event. However, the feedback controllers are also used in the systems, which is different from this paper. Distributed impulsive control for heterogeneous multiagent systems based on an event-triggered scheme has been studied in [36], compared with which, events and impulsive controllers are simpler. Furthermore, this paper has discussed a FOMAS with fractional-order dynamics. Of course, letting $\alpha = 1$, the corresponding results about consensus of integer-order multiagent system can be obtained.

Remark 7 The consensus problem has been analyzed in this paper, results about synchronization of a coupled dynamical network or master–slave system can be derived easily. For example, if there is only one follower, then the consensus problem converts to the synchronization problem of a master–slave system directly, an example will be given in the simulation part.

4 Numerical simulations

In this section, three examples will be given to show the effectiveness of the above theoretical results. A master–slave system with mismatched parameters and a heterogeneous FOMAS will be studied in two examples. The predictor–corrector algorithm has been used to simulate the fractional-order dynamical networks in this paper [37] with step 0.001.

Example 1 Consider $N = 1$, then, the consensus problem of a leader-following FOMAS (1) becomes a synchronization problem between $x_1(t)$ and $x_0(t)$. Let $n = 3$, for any $z \in \mathbb{R}^3$,

$$g_i(z) = \frac{|z_i + 1| - |z_i - 1|}{2},$$

$$A_0 = -I_3, \qquad A_1 = I_3,$$

$$B_0 = \begin{pmatrix} 1.25 & -3.2 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 1 & -3 & -3 \\ -3 & 1 & 4 \\ -3 & 4 & 0 \end{pmatrix}.$$

Without any control, the chaotic behavior of the leader $x_0(t)$ and the error response are shown in Fig. 1 and Fig. 2, respectively. In which, the initial values are selected as $x_0(0) = [0.1, 0.2, 0.3]^T$ and $x_1(0) = [-1, 3, -4]^T$.

Consider the event-triggered impulsive controllers that have been designed in this paper, one can let $T = 1$, $\theta_1 = 25$, $\theta_2 = 0.9$, $P = I_3$, $\mu_1 = 0.8$, $\mu_2 = 0.5$, then, the consensus states are shown in Fig. 3. Furthermore, the errors are shown in Fig. 4, and the event-triggered

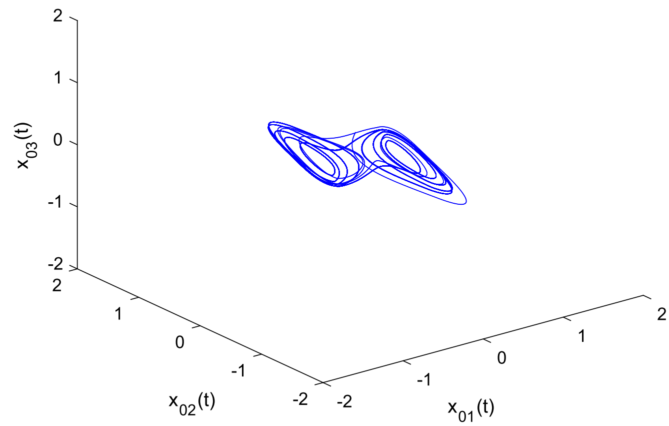

Figure 1 Chaotic behavior of the leader $x_0(t)$

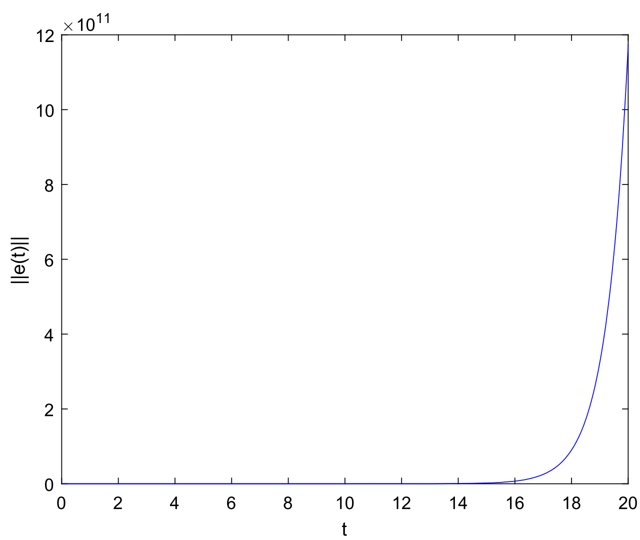

Figure 2 Consensus error $\|e(t)\|$ without any control

instants and the interval between this triggered moment and the next triggered moment is shown in Fig. 5.

Example 2 Let us consider $N = 4$, $n = 3$ in this example, for any

$$z \in \mathbb{R}^3, \quad g_i(z) = \left[\frac{|z_i + 1| - |z_i - 1|}{2}, 0, 0\right]^T.$$

Also,

$$A_0 = \begin{pmatrix} -2.5 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -18 & 0 \end{pmatrix}, \qquad B_0 = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

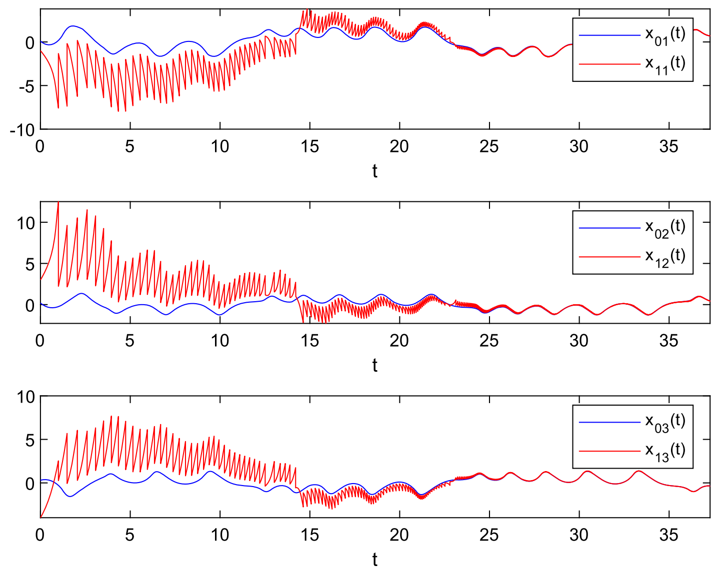

Figure 3 Consensus states with control

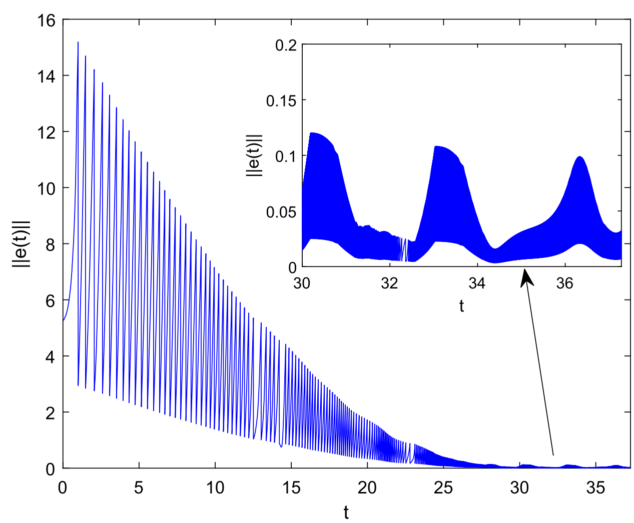

Figure 4 Consensus error $\|e(t)\|$ with control

$$A_1 = \begin{pmatrix} -2.5 & 10 & 1 \\ 1 & -1 & 1 \\ 0 & -18 & 0.1 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -2.5 & 5 & 0 \\ 1 & -0.5 & 1 \\ 0 & -17 & 0 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} -2.5 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -18 & 0 \end{pmatrix}, \qquad B_3 = \begin{pmatrix} 5.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

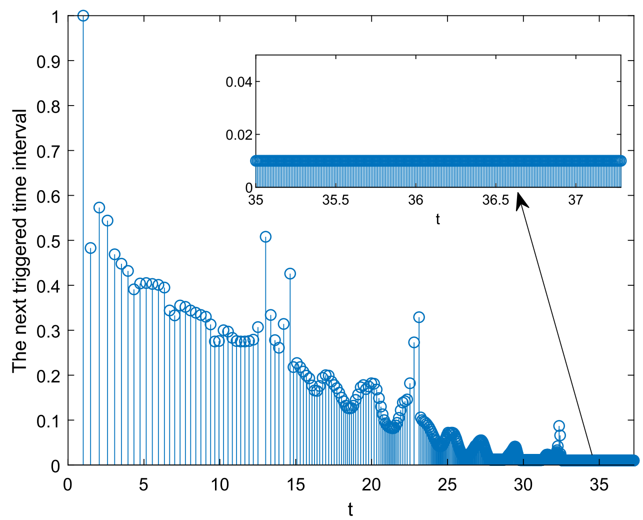

Figure 5 The release instants and release interval

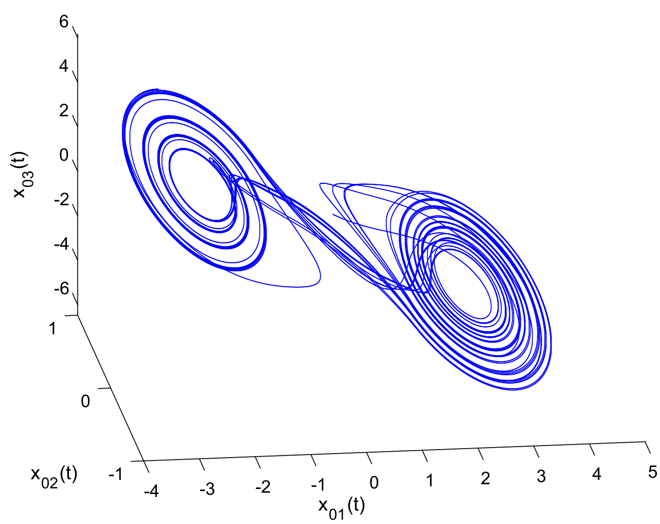

Figure 6 Chaotic behavior of the leader $x_0(t)$

$$A_4 = \begin{pmatrix} -2.5 & 10 & 1 \\ 1 & -1 & 1 \\ 1 & -18 & -1 \end{pmatrix}, \qquad B_4 = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Without any control, the chaotic behavior of the leader $x_0(t)$ is shown in Fig. 6, and the phase spaces of the followers can be seen in Fig. 7. One can see that the chaotic, stable, unstable or periodic behaviors have been shown for the followers. Obviously, without any control, the consensus can not be achieved, the error response is shown in Fig. 8.

The topology of the multiagent system is shown in Fig. 9, obviously, just the 1st and 2nd agents have been selected to be controlled. Let $d_1 = d_2 = 1$, $d_3 = d_4 = 0$, $T = 1$, $\theta_1 = 1.2$, $\theta_2 = 0.9$, $P = I_3$, $\mu_1 = 0.95$, $\mu_2 = 0.8$, then, the consensus states are shown in Fig. 10. Furthermore, the errors are shown in Fig. 11, and the event-triggered instants

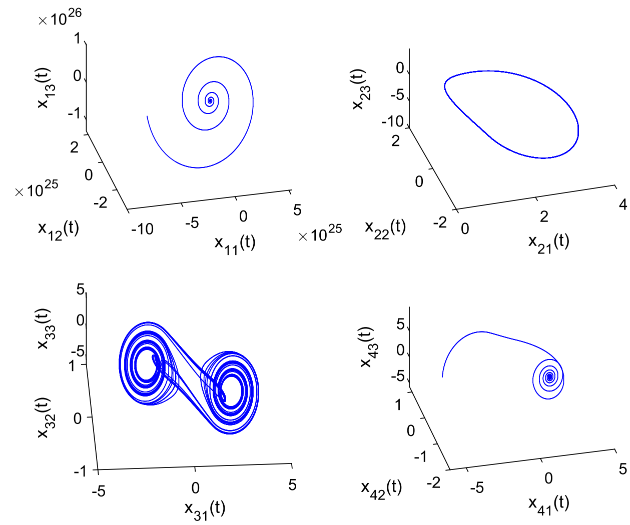

Figure 7 Phase spaces of followers without control

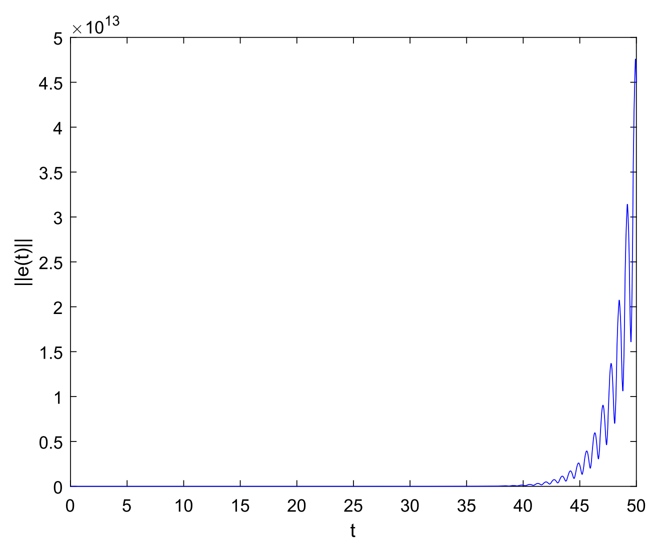

Figure 8 Consensus error $\|e(t)\|$ without any control

and the interval between this triggered moment and the next triggered moment is shown in Fig. 12.

5 Conclusion

The quasiconsensus problem of a fractional-order multiagent system has been studied in this paper, the heterogeneous case is considered for the multiagent system. By using the designed event-triggered impulsive control protocol, the quasiconsensus can be reached under some conditions that are formulated by a number of lower-dimensional matrix inequalities and scalar inequalities. The upper bound of the consensus error was estimated precisely. Furthermore, Zeno behavior was excluded successfully. Numerical simulation examples have been given to check the validity of the theoretical results. Noting that the

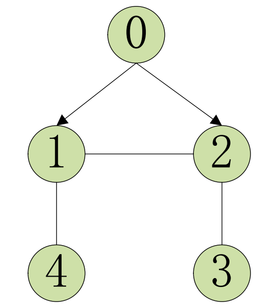

Figure 9 Topology of FOMAs in this example

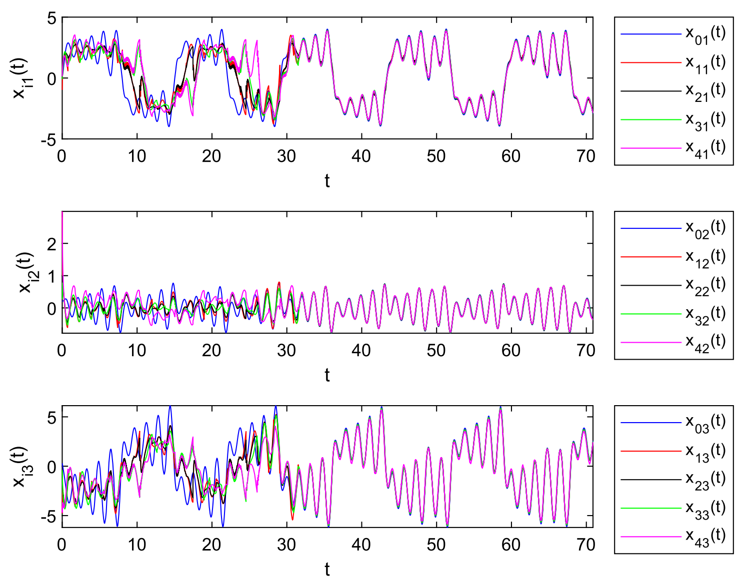

Figure 10 Consensus states with control

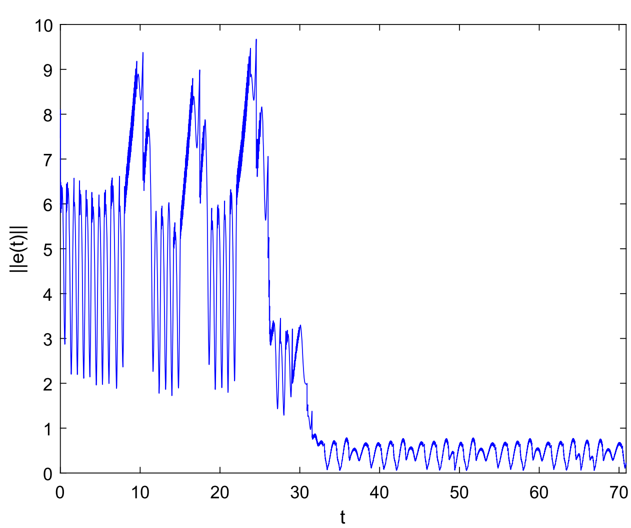

Figure 11 Consensus error $\|e(t)\|$ with control

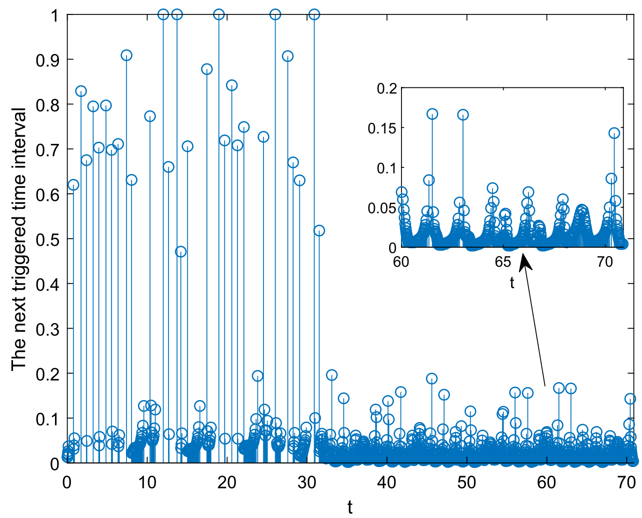

Figure 12 The release instants and release interval

centralized control method has been used in this paper, however, the distributed strategy will be more robust, thus, we will pay more attention to the distributed control methods in our future works. As is known, time delays are difficult to avoid in real-world networked systems, thus, the fractional-order multiagent system with time delays based on the control method in this manuscript will be researched in our future works.

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Abbreviations
ETIC, Event-Triggered Impulsive Control; FOMAs, fractional-order multiagent systems.

Availability of data and materials
All data generated or analyzed during this study are included in this article.

Declarations

Consent for publication
This article has not been published previously; it is not under consideration for publication elsewhere.

Competing interests
The authors declare that they have no competing interests.

Author contribution
All authors contributed equally to this article. They read and approved the final manuscript.

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