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Revising the Hardy–Rogers–Suzuki-type Z-contractions

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Abstract

The aim of this study is to introduce a new interpolative contractive mapping combining the Hardy–Rogers contractive mapping of Suzuki type and $\mathcal{Z}$-contraction. We investigate the existence of a fixed point of this type of mappings and prove some corollaries. The new results of the paper generalize a number of existing results which were published in the last two decades.

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1 Introduction and preliminaries

A century ago, the notion of fixed point theory appeared in the papers that were written to solve certain differential equations. The first independent fixed point result was given by Banach [1] in the setting of a complete normed space. The analog of this result in the framework of the complete metric space was reported by Caccioppoli [2] in 1930. After then, metric fixed point theory has advanced in many directions in the setting of several abstract spaces. Regarding the appearance of the notion, fixed point theory is one of the useful and crucial tools in several disciplines. Most of the daily life problems can be restated in the context of fixed point theory, see, e.g., the book of Rus [3] for interesting examples.

In the last fourth decades, an enormous number of publications were reported on the advances of metric fixed point theory regarding very distinct aspects in various settings, see, e.g., [3–41] and related reference therein. As a natural consequence of this fact, some authors proposed new notions to combine and unify this tremendous number of publications in the literature. Here, we mention and use three interesting notions that were proposed for this purpose, namely simulation function (see, e.g., [19–29]), admissible mapping (see, e.g., [9–18]), and Suzuki-type contraction (see [4, 5]).

In 2014, Popescu [21] suggested an interesting notion, the so-called ω-orbital admissible mappings, which is a smart expansion of the notion of α-admissible mappings, see Samet et al. [19]. In this work, Popescu [21] showed that each admissible mapping is an ω-orbital admissible mapping, but the converse is not true.

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Definition 1 ([21]) Let $\omega : Y \times Y \to [0, \infty)$ be a function where Y is a any nonempty set. A self mapping H on Y is called ω-orbital admissible if for all u in Y, we have

$$\omega(u, Hu) \geq 1 \quad \implies \quad \omega\big(Hu, H^2 u\big) \geq 1.$$

One of the interesting uses of ω-admissible mapping is that it is ω-regular in the setting of metric spaces. This was a condition that helps refine the continuity condition on the self-mapping accompanied with some additional conditions; see, e.g., [19].

Definition 2 A metric space (Y, d) is called ω-regular if for every sequence $\{u_n\}$ in Y, which converges to some $z \in Y$ and satisfies $\omega(u_n, u_{n+1}) \geq 1$ for each $n \in \mathbb{N}$, we have $\omega(u_n, z) \geq 1$.

Later in 2015, the concept of a simulation function had been introduced by Khojasteh et al. [9]. These functions cover many types of the existing contractions. We give now the definition of simulation function as it was redefined by Argoubi [11].

Definition 3 A simulation function is a mapping $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ satisfying the following conditions:
(ζ_1) $\zeta(t, s) < s - t$ for all $t, s > 0$;
(ζ_2) if $\{t_n\}, \{s_n\}$ are sequences in $(0, \infty)$ such that $\lim_{n \to \infty} t_n = \lim_{n \to \infty} s_n > 0$, then

$$\limsup_{n \to \infty} \zeta(t_n, s_n) < 0. \tag{1.1}$$

We demonstrate here some examples of simulation functions from [10–18].

Example 4 For $i = 1, 2$, we define the mappings $\zeta_i : [0, \infty) \times [0, \infty) \to \mathbb{R}$, as follows:
(i) $\zeta_1(t, s) = \phi_1(s) - \phi_2(t)$ for all $t, s \in [0, \infty)$, where $\phi_1, \phi_2 : [0, \infty) \to [0, \infty)$ are two continuous functions such that $\phi_1(t) = \phi_2(t) = 0$ if and only if $t = 0$ and $\phi_1(t) < t \leq \phi_2(t)$ for all $t > 0$.
If we take $\phi_2(t) = t$, $\phi_1(t) = \lambda t$ where $\lambda \in [0, 1)$, we get the special case $\zeta_B = \lambda s - t$ for all $s, t \in [0, \infty)$.
(ii) $\zeta_2(t, s) = \eta(s) - t$ for all $s, t \in [0, \infty)$, where $\eta : [0, \infty) \to [0, \infty)$ is an upper semicontinuous mapping such that $\eta(t) < t$ for all $t > 0$ and $\eta(0) = 0$.
It is clear that each function ζ_i ($i = 1, 2$) forms a simulation function.

The next definition presents the Suzuki-type contraction mappings.

Definition 5 ([5]) A self-mapping H on a metric space (Y, d) is called a Suzuki-type contraction if for all $x, y \in Y$ with $x \neq y$, we have

$$\frac{1}{2} d(x, Hx) \leq d(x, y) \quad \implies \quad d(Hx, Hy) \leq d(x, y).$$

One of the interesting results in metric fixed point theory was given by Karapınar [39], which involves interpolation. After these initial results, interpolative contraction has been investigated by several authors, e.g., [15, 30–41]. Recently, interpolative Hardy–Rogers-type contractions have been investigated by many authors (see [6–8]). In particular, in

[38], Karapınar used simulation functions to introduce the notion of interpolative Hardy–Rogers-type $\mathcal{Z}$-contraction mappings and prove some related fixed point results. The aim of our work is to combine the latter contractions with those of the Suzuki-type and investigate the existence of fixed points of this new type of mappings under some conditions.

Karapınar's definition that introduced the notion of interpolative Hardy–Rogers-type $\mathcal{Z}$-contraction mappings is given as follows.

Definition 6 ([38]) Let H be a self-mapping defined on a metric space (Y, d). If there exist $\alpha, \beta, \gamma \in (0, 1)$ with $\alpha + \beta + \gamma < 1$, and $\zeta \in \mathcal{Z}$ such that

$$\zeta\big(d(Hx, Hy), C(x, y)\big) \geq 0,$$

for all $x, y \in Y \setminus \mathrm{Fix}(H)$, where $\mathrm{Fix}(H)$ is the set of all fixed point of H, and

$$C(x, y) := \big[d(x, y)\big]^{\beta} \cdot \big[d(x, Hx)\big]^{\alpha} \cdot \big[d(y, Hy)\big]^{\gamma} \cdot \left[\frac{1}{2}\big(d(x, Hy) + d(y, Hx)\big)\right]^{1-\alpha-\beta-\gamma}, \quad (1.2)$$

then we say that H is an interpolative Hardy–Rogers-type $\mathcal{Z}$-*contraction* with respect to ζ.

2 Main results

We introduce now our new contraction type mapping in the following definition.

Definition 7 Let H be a self-mapping on a metric space (Y, d). We say that H is an interpolative Hardy-Rogers–Suzuki-type $\mathcal{Z}$-contraction with respect to some $\zeta \in \mathcal{Z}$ if there exists $\alpha, \beta, \gamma \in (0, 1)$ with $\alpha + \beta + \gamma < 1$, $\zeta \in \mathcal{Z}$ and a function $\omega : Y \times Y \to [0, \infty)$ such that

$$\frac{1}{2}d(x, Hx) \leq d(x, y)$$
$$\implies \quad \zeta\big(\omega(x, y)d(Hx, Hy), C(x, y)\big) \geq 0 \qquad (2.1)$$

for all $x, y \notin \mathrm{Fix}(H)$ where $C(x, y)$ is given by (1.2).

Our main result is the following theorem:

Theorem 8 *Let (Y, d) be a complete metric space and let H be a self-mapping on Y. Assume that*
   (i) *H is an interpolative Hardy–Rogers–Suzuki-type $\mathcal{Z}$-contraction with respect to some $\zeta \in \mathcal{Z}$;*
  (ii) *H is ω-orbital admissible;*
 (iii) *there exists $u_0 \in Y$ such that $\omega(u_0, Hu_0) \geq 1$;*
 (iv) *Y is ω-regular.*
 Then H has a fixed point.

Proof Define the sequence u_n by $u_n = H^n u_0$. If there exists $k \in \mathbb{N}$ such that $u_k = u_{k+1}$, then u_k is a fixed point of H. Assume that $u_n \neq u_{n+1}$ for all $n \in \mathbb{N}$. Now as $\omega(u_0, Hu_0) \geq 1$ and H is ω-orbital admissible, $\omega(u_n, u_{n+1}) \geq 1$ for all $n \in \mathbb{N}$. And as H is an interpolative Hardy–Rogers–Suzuki-type $\mathcal{Z}$-contraction with respect to some $\zeta \in \mathcal{Z}$ with

$\frac{1}{2}d(u_n, Hu_n) = \frac{1}{2}d(u_n, u_{n+1}) \leq d(u_n, u_{n+1})$, we have

$$\zeta\left(\omega(u_n, u_{n+1})d(u_{n+1}, u_{n+2}), C(u_n, u_{n+1})\right) \geq 0$$

which turns into

$$\begin{aligned}
0 &\leq \zeta\left(\omega(u_n, u_{n+1})d(u_{n+1}, u_{n+2}), C(u_n, u_{n+1})\right) \\
&< C(u_n, u_{n+1}) - \omega(u_n, u_{n+1})d(u_{n+1}, u_{n+2}) \\
&\implies \quad \omega(u_n, u_{n+1})d(u_{n+1}, u_{n+2}) < C(u_n, u_{n+1}).
\end{aligned}$$

As $\omega(u_n, u_{n+1}) \geq 1$ for all $n \in \mathbb{N}$, we have

$$d(u_{n+1}, u_{n+2}) \leq \omega(u_n, u_{n+1})d(u_{n+1}, u_{n+2}) < C(u_n, u_{n+1}), \tag{2.2}$$

which implies that

$$\begin{aligned}
d(u_{n+1}, u_{n+2}) &< d(u_n, u_{n+1})^\alpha d(u_n, u_{n+1})^\beta d(u_{n+1}, u_{n+2})^\gamma \\
&\quad \times \left[\frac{1}{2}\left(d(u_n, u_{n+2}) + d(u_{n+1}, u_{n+1})\right)\right]^{1-\alpha-\beta-\gamma},
\end{aligned} \tag{2.3}$$

and, using the triangular inequality with the fact that the function $f(x) = x^{1-\alpha-\beta-\gamma}$ is increasing for $x > 0$, we obtain

$$\left[\frac{1}{2}d(u_n, u_{n+2})\right]^{1-\alpha-\beta-\gamma} \leq \left[\frac{1}{2}\left(d(u_n, u_{n+1}) + d(u_{n+1}, u_{n+2})\right)\right]^{1-\alpha-\beta-\gamma}.$$

So from (2.3) we have

$$\begin{aligned}
d(u_{n+1}, u_{n+2}) &< d(u_n, u_{n+1})^{\alpha+\beta} d(u_{n+1}, u_{n+2})^\gamma \\
&\quad \times \left[\frac{1}{2}\left(d(u_n, u_{n+1}) + d(u_{n+1}, u_{n+2})\right)\right]^{1-\alpha-\beta-\gamma}.
\end{aligned} \tag{2.4}$$

If we suppose that $d(u_n, u_{n+1}) < d(u_{n+1}, u_{n+2})$ for all $n \in \mathbb{N}$, then (2.4) yields

$$d(u_{n+1}, u_{n+2})^{1-\gamma} < d(u_n, u_{n+1})^{\alpha+\beta} + d(u_{n+1}, u_{n+2})^{1-\alpha-\beta-\gamma},$$

which implies that

$$d(u_{n+1}, u_{n+2})^{\alpha+\beta} < d(u_n, u_{n+1})^{\alpha+\beta},$$

a contradiction. Hence $d(u_{n+1}, u_{n+2}) \leq d(u_n, u_{n+1})$ for all $n \in \mathbb{N}$. So, we deduce that the sequence $\{d(u_n, u_{n+1})\}$ is nonincreasing, and as $d(u_n, u_{n+1}) \geq 0$ for all $n \in \mathbb{N}$, $\{d(u_n, u_{n+1})\}$ is a bounded monotone sequence of real numbers, which implies that there exists $t \geq 0$ such that $\lim_{n\to\infty} d(u_n, u_{n+1}) = t$. We have to prove that $t = 0$. It is easy to see that

$\lim_{n \to \infty} C(u_n, u_{n+1}) = t$. So from (2.2) we have $\lim_{n \to \infty} \omega(u_n, u_{n+1}) d(u_{n+1}, u_{n+2}) = t$ by the squeeze theorem. Accordingly, if we suppose that $t > 0$, we can apply ζ_2 to get

$$0 \le \zeta \big(\omega(u_n, u_{n+1}) d(u_{n+1}, u_{n+2}), C(u_n, u_{n+1}) \big) < 0,$$

which is a contradiction. Hence $t = 0$, which implies that $\{u_n\}$ is a Cauchy sequence. By completeness of Y, there exists $v \in Y$ such that $\lim_{n \to \infty} u_n = v$. We will prove that v is a fixed point of H. Note that as Y is ω-regular and $\omega(u_n, u_{n+1}) \ge 1$ for all $n \in \mathbb{N}$, so $\omega(u_n, v) \ge 1$ for all $n \in \mathbb{N}$. Now either

$$\frac{1}{2} d(u_n, H u_n) \le d(u_n, v) \tag{2.5}$$

or

$$\frac{1}{2} d\big(H u_n, H^2 u_n \big) \le d(H u_n, v), \tag{2.6}$$

for if we suppose that $\frac{1}{2} d(u_n, H u_n) > d(u_n, v)$ and $\frac{1}{2} d(H u_n, H^2 u_n) > d(H u_n, v)$ then, using the triangular inequality together with the fact that $\{d(u_n, u_{n+1})\}$ is a nonincreasing sequence, we will get

$$d(u_n, u_{n+1}) = d(u_n, H u_n) \le d(u_n, v) + d(v, H u_n)$$
$$< \frac{1}{2} d(u_n, H u_n) + \frac{1}{2} d\big(H u_n, H^2 u_n \big)$$
$$= \frac{1}{2} d(u_n, u_{n+1}) + \frac{1}{2} d(u_{n+1}, u_{n+2})$$
$$\le \frac{1}{2} d(u_n, u_{n+1}) + \frac{1}{2} d(u_n, u_{n+1})$$
$$= d(u_n, u_{n+1}),$$

which is a contradiction. So either (2.5) or (2.6) holds. If we assume that (2.5) holds and v is not a fixed point of H, then by ω-regularity of Y we have

$$0 \le \zeta \big(\omega(u_n, v) d(H u_n, H v), C(u_n, v) \big).$$

Using ζ_2, we have

$$0 \le C(u_n, v) - \omega(u_n, v) d(u_n, H v)$$
$$\implies \quad d(u_n, H v) \le \omega(u_n, v) d(u_n, H v)$$
$$\le C(u_n, v)$$
$$= \big[d(u_n, u_{n+1}) \big]^\alpha \big[d(u_n, v) \big]^\beta \big[d(v, H v) \big]^\gamma$$
$$\times \left[\frac{1}{2} \big(d(u_n, H v) + d(v, u_{n+1}) \big) \right]^{1 - \alpha - \beta - \gamma}.$$

As the limit of the right-hand side of the previous inequality as $n \to \infty$ is zero, by the squeeze theorem, $\lim_{n \to \infty} d(u_n, H v) = 0$. Hence, by the uniqueness of the limit, we have $v = H v$. Similarly, if (2.6) holds, we can prove that v is a fixed point of H, as wanted. $\qquad \square$

2.1 Consequences

We get the following corollaries by using different examples of the function ζ.

Corollary 9 *Let (Y, d) be a complete metric space and let H be a self-mapping on Y. Assume that*

(i) *there exists $\alpha, \beta, \gamma \in (0, 1)$, $\lambda \in [0, 1)$ with $\alpha + \beta + \gamma < 1$, and a function $\omega : Y \times Y \to [0, \infty)$ such that*

$$\frac{1}{2} d(x, Hx) \leq d(x, y)$$
$$\implies \quad \omega(x, y) d(Hx, Hy) \leq \lambda C(x, y); \tag{2.7}$$

(ii) *H is ω-orbital admissible;*
(iii) *there exists $u_0 \in Y$ such that $\omega(u_0 H u_0) \geq 1$;*
(iv) *Y is ω-regular.*
Then H has a fixed point.

Sketch of the proof It is sufficient to replace $\zeta = \lambda s - t$ in Theorem 8 where $\lambda \in [0, 1)$ for all $s, t \in [0, \infty)$.

Corollary 10 *Let (Y, d) be a complete metric space and let H be a self-mapping on Y. Assume that*

(i) *there exists $\alpha, \beta, \gamma \in (0, 1)$, with $\alpha + \beta + \gamma < 1$, a function $\omega : Y \times Y \to [0, \infty)$ and an upper semi-continuous mapping $\eta : [0, \infty) \to [0, \infty)$ with $\eta(t) < t$ for all $t > 0$ and $\eta(0) = 0$ such that*

$$\frac{1}{2} d(x, Hx) \leq d(x, y)$$
$$\implies \quad \omega(x, y) d(Hx, Hy) \leq \eta\big(C(x, y)\big); \tag{2.8}$$

(ii) *H is ω-orbital admissible;*
(iii) *there exists $u_0 \in Y$ such that $\omega(u_0 H u_0) \geq 1$;*
(iv) *Y is ω-regular.*
Then H has a fixed point.

Sketch of the proof It is sufficient to replace $\zeta(t, s) = \eta(s) - t$ for all $s, t \in [0, \infty)$ in Theorem 8.

We can obtain more results by reducing the terms in Theorem 8 as follows:

Theorem 11 *Let (Y, d) be a complete metric space and let H be a self-mapping on Y. Assume that there exists $\alpha, \beta, \in (0, 1)$ with $\alpha + \beta < 1$, $\zeta \in \mathcal{Z}$ and a function $\omega : Y \times Y \to [0, \infty)$ such that*

(i) $\quad \dfrac{1}{2} d(x, Hx) \leq d(x, y)$
$$\implies \quad \zeta\big(\omega(x, y) d(Hx, Hy), D(x, y)\big) \geq 0 \tag{2.9}$$

for all x, y ∉ Fix(*H*) *where*

$$D(x,y) := \left[d(x,y)\right]^{\beta}\left[d(x,Hx)\right]^{\alpha}\left[d(y,Hy)\right]^{1-\alpha-\beta};\tag{2.10}$$

  (ii) *H is ω-orbital admissible*;
 (iii) *there exists $u_0 \in Y$ such that $\omega(u_0, Hu_0) \geq 1$*;
 (iv) *Y is ω-regular.*
Then H has a fixed point.

Proof By analogue of the proof of Theorem 8.   □

3 Conclusion

In conclusion, we can use the results of the paper to generate more results by using different examples of the simulation function. Moreover, we can follow the same argument of the proof of the main result to prove more results with less terms; this will enrich the fixed point theory.

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