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# Fractional calculus and integral transforms of the product of a general class of polynomial and incomplete Fox–Wright functions

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**Abstract**

Motivated by a recent study on certain families of the incomplete *H*-functions (Srivastava et al. in Russ. J. Math. Phys. 25(1):116–138, 2018), we aim to investigate and develop several interesting properties related to product of a more general polynomial class together with incomplete Fox–Wright hypergeometric functions $_p\Psi_q^{(\gamma)}(\mathfrak{t})$ and $_p\Psi_q^{(\Gamma)}(\mathfrak{t})$ including Marichev–Saigo–Maeda (M–S–M) fractional integral and differential operators, which contain Saigo hypergeometric, Riemann–Liouville, and Erdélyi–Kober fractional operators as particular cases regarding different parameter selection. Furthermore, we derive several integral transforms such as Jacobi, Gegenbauer (or ultraspherical), Legendre, Laplace, Mellin, Hankel, and Euler's beta transforms.

**MSC:** Primary 26A33; 33C20; secondary 33C05

**Keywords:** Incomplete Fox–Wright functions; Fractional calculus operators; Integral transforms

## 1 Introduction

In a variety of diverse fields, together with engineering science, material science, mathematical physics, chemistry, and biology, the concept of differential equations (including fractional order) and their application have played an important role, see [5, 14, 18, 41–48]. In addition, the special functions of one or more variables are also important because they occur as solutions to these simulated differential equations. Therefore, with the development of new problems in the area of technologies in engineering and applied sciences, the subject of special functions is very diverse and is continuously growing. As a result, a number of articles on these concepts and their future implementations have been made available in the literature, see [1–4, 39, 40]. Incomplete special functions have additionally been utilized to a wide range of problems, and numerous scientific studies on incomplete special functions, along with related higher transcendental special functions, have currently been published by various authors [6–12, 15, 20–23, 31, 35–38]. In particular, the incomplete Fox–Wright functions $_p\Psi_q^{(\gamma)}(\mathfrak{t})$ and $_p\Psi_q^{(\Gamma)}(\mathfrak{t})$ with $p$ numerator and $q$ denomi-

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nator parameters are stated as follows [23, 35]:

$$
{}_{p}\Psi_{q}^{(\gamma)}\left[\begin{array}{ll} (a_1, A_1, x), (a_j, A_j)_{2,p}; & \\ \quad (b_j, B_j)_{1,q}; & \mathfrak{t} \end{array}\right] = \sum_{\ell=0}^{\infty} \frac{\gamma(a_1 + A_1\ell, x)\prod_{j=2}^{p}\Gamma(a_j + A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j + B_j\ell)}\frac{\mathfrak{t}^{\ell}}{\ell!} \tag{1.1}
$$

and

$$
{}_{p}\Psi_{q}^{(\Gamma)}\left[\begin{array}{ll} (a_1, A_1, x), (a_j, A_j)_{2,p}; & \\ \quad (b_j, B_j)_{1,q}; & \mathfrak{t} \end{array}\right] = \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1 + A_1\ell, x)\prod_{j=2}^{p}\Gamma(a_j + A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j + B_j\ell)}\frac{\mathfrak{t}^{\ell}}{\ell!} \tag{1.2}
$$

$$
\left(x \geqq 0, A_j \in \mathbb{R}^+ \ (j = 1, \ldots, r); B_j \in \mathbb{R}^+ \ (j = 1, \ldots, s); 1 + \sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j \geqq 0\right),
$$

with

$$
|\mathfrak{t}| < \nabla := \left(\prod_{j=1}^{p} A_j^{-A_j}\right) \cdot \left(\prod_{j=1}^{q} B_j^{B_j}\right).
$$

The incomplete Fox–Wright functions ${}_{p}\Psi_{q}^{(\gamma)}(\mathfrak{t})$ and ${}_{p}\Psi_{q}^{(\Gamma)}(\mathfrak{t})$ in (1.1) and (1.2) satisfy the decomposition formula

$$
{}_{p}\Psi_{q}^{(\gamma)}[\mathfrak{t}] + {}_{p}\Psi_{q}^{(\Gamma)}[\mathfrak{t}] = {}_{p}\Psi_{q}[\mathfrak{t}], \tag{1.3}
$$

where ${}_{p}\Psi_{q}[\mathfrak{t}]$ is the Fox–Wright function [16].

Also, the normalized incomplete Fox–Wright functions ${}_{p}\Psi_{q}^{*(\gamma)}(\mathfrak{t})$ and ${}_{p}\Psi_{q}^{*(\Gamma)}(\mathfrak{t})$ are given by [23, 35]

$$
{}_{p}\Psi_{q}^{*(\gamma)}\left[\begin{array}{ll} (a_1, A_1, x), (a_j, A_j)_{2,p}; & \\ \quad (b_j, B_j)_{1,q}; & \mathfrak{t} \end{array}\right] = \frac{\prod_{j=1}^{q}\Gamma(b_j)}{\prod_{j=1}^{p}\Gamma(a_j)}{}_{p}\Psi_{q}^{(\gamma)}\left[\begin{array}{ll} (a_1, A_1, x), (a_j, A_j)_{2,p}; & \\ \quad (b_j, B_j)_{1,q}; & \mathfrak{t} \end{array}\right] \tag{1.4}
$$

and

$$
{}_{p}\Psi_{q}^{*(\Gamma)}\left[\begin{array}{ll} (a_1, A_1, x), (a_j, A_j)_{2,p}; & \\ \quad (b_j, B_j)_{1,q}; & \mathfrak{t} \end{array}\right] = \frac{\prod_{j=1}^{q}\Gamma(b_j)}{\prod_{j=1}^{p}\Gamma(a_j)}{}_{p}\Psi_{q}^{(\Gamma)}\left[\begin{array}{ll} (a_1, A_1, x), (a_j, A_j)_{2,p}; & \\ \quad (b_j, B_j)_{1,q}; & \mathfrak{t} \end{array}\right]. \tag{1.5}
$$

Let $S_n^m[\mathfrak{u}]$ denote the general family of polynomials made known through Srivastava [30]:

$$
S_n^m[\mathfrak{u}] = \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\mathfrak{u}^s \quad (n = 0, 1, 2, \ldots). \tag{1.6}
$$

Here, $m$ is a positive integer (arbitrary), the coefficients $A_{n,s} \in \mathbb{R}$ (or $\mathbb{C}$) are constants (arbitrary), and $(\lambda)_\nu = \frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)}$ $(\lambda, \nu \in \mathbb{C})$ denotes the shifted factorial (or the Pochhammer symbol). Also, the above polynomials provide a large number spectrum of well-known polynomials as one of its particular cases on appropriately specializing the coefficient $A_{n,s}$. Particularly, by setting $m = 1$, $A_{n,s} = \frac{s!}{(-n)_{ms}}$ for $s = k$ and $A_{n,s} = 0$ for $s \neq k$, the general class of polynomials leads to a power function, *i.e.*,

$$
S_n^m[\mathfrak{u}] = \mathfrak{u}^k \quad \left(k \in \mathbb{Z}^+ \text{ with } k \leq n\right). \tag{1.7}
$$

Taking into account formula (1.3), it is also appropriate to study the characteristics and properties of the incomplete Fox–Wright function $_p\Psi_q^{(\Gamma)}(\mathfrak{t})$.

## 2 Fractional integral and differential operators

We recall a general pair of fractional integral and differential operators popularly known as Marichev–Saigo–Maeda (M-S-M), which involve, in their kernel, third Appell's two-variable hypergeometric function $F_3(.)$ and are defined by [32]

$$F_3\big(\sigma,\sigma',\rho,\rho';\eta;\mathfrak{X},\mathfrak{Y}\big) = \sum_{m,n=0}^{\infty} \frac{(\sigma)_m(\sigma')_n(\rho)_m(\rho')_n}{(\eta)_{m+n}} \frac{\mathfrak{X}}{m!}\frac{\mathfrak{Y}}{n!} \quad \big(\max\{|\mathfrak{X}|,|\mathfrak{Y}|<1\}\big). \quad (2.1)$$

Here, we mention and study left-hand-sided fractional integral and differential operators(see, for details, [19, 26, 28]).

**Definition 1** For $\sigma,\sigma',\rho,\rho',\eta \in \mathbb{C}$ and $x > 0$ with $\Re(\eta) > 0$, the left-hand-sided Marichev–Saigo–Maeda (M–S–M) fractional integral and differential operators are respectively defined as

$$\begin{aligned}
\big(\mathcal{I}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}f\big)(\mathfrak{X}) &= \frac{\mathfrak{X}^{-\sigma}}{\Gamma(\eta)}\int_0^{\mathfrak{X}}(\mathfrak{X}-\mathfrak{t})^{\eta-1}\mathfrak{t}^{-\sigma'} \\
&\quad \times F_3\bigg(\sigma,\sigma',\rho,\rho';\eta;1-\frac{\mathfrak{t}}{\mathfrak{X}},1-\frac{\mathfrak{X}}{\mathfrak{t}}\bigg)f(\mathfrak{t})\,d\mathfrak{t}
\end{aligned} \quad (2.2)$$

and

$$\begin{aligned}
\big(\mathcal{D}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}f\big)(\mathfrak{X}) &= \big(I_{0+}^{-\sigma',-\sigma,-\rho',-\rho,-\eta}f\big)(\mathfrak{X}) \\
&= \bigg(\frac{d}{d\mathfrak{X}}\bigg)^{\kappa}\big(\mathcal{I}_{0+}^{-\sigma',-\sigma,-\rho'+[\Re(\eta)]+1,-\rho,-\eta+\kappa+1}f\big)(\mathfrak{X}) \\
&= \frac{1}{\Gamma(\kappa-\eta)}\bigg(\frac{d}{d\mathfrak{X}}\bigg)^{\kappa}\mathfrak{X}^{\sigma'_1}\int_0^{\mathfrak{X}}(\mathfrak{X}-\mathfrak{t})^{\kappa-\eta-1}\mathfrak{t}^{\sigma} \\
&\quad \times F_3\bigg(-\sigma',-\sigma,\kappa-\rho',-\rho;\kappa-\eta;1-\frac{\mathfrak{t}}{\mathfrak{X}},1-\frac{\mathfrak{X}}{\mathfrak{t}}\bigg)f(\mathfrak{t})\,d\mathfrak{t},
\end{aligned} \quad (2.3)$$

where $\kappa = [\Re(\eta)] + 1$ and $[\Re(\eta)]$ denotes the integer part of $\Re(\eta)$.

The preceding results are well known and can be used as a proof of subsequent theorems.

**Lemma** *Let $\sigma,\sigma',\rho,\rho',\eta,\lambda \in \mathbb{C}$ such that $\Re(\eta) > 0$.*
(a) *If $\Re(\lambda) > \max\{0,\Re(\sigma'-\rho'),\Re(\sigma+\sigma'+\rho-\eta)\}$, then*

$$\begin{aligned}
&\big(\mathcal{I}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\mathfrak{t}^{\lambda-1}\big)(\mathfrak{X}) \\
&\quad = \frac{\Gamma(\lambda)\Gamma(-\sigma'+\rho'+\lambda)\Gamma(-\sigma-\sigma'-\rho+\eta+\lambda)}{\Gamma(\rho'+\lambda)\Gamma(-\sigma-\sigma'+\eta+\lambda)\Gamma(-\sigma'-\rho+\eta+\lambda)}\mathfrak{X}^{-\sigma-\sigma'+\eta+\lambda-1}.
\end{aligned} \quad (2.4)$$

(b)  *If* $\Re(\lambda) > \max\{0, \Re(-\sigma + \rho), \Re(-\sigma - \sigma' - \rho' + \eta)\}$, *then*

$$
\begin{aligned}
&\left(\mathcal{D}_{0+}^{\sigma,\sigma',\rho,\rho',\eta} t^{\lambda-1}\right)(\mathfrak{X}) \\
&= \frac{\Gamma(\lambda)\Gamma(\sigma - \rho + \lambda)\Gamma(\sigma + \sigma' + \rho' - \eta + \lambda)}{\Gamma(-\rho + \lambda)\Gamma(\sigma + \sigma' - \eta + \lambda)\Gamma(\sigma + \rho' - \eta + \lambda)} \mathfrak{X}^{\sigma+\sigma'-\eta+\lambda-1}.
\end{aligned}
\tag{2.5}
$$

Firstly, we shall investigate the left-hand-sided Marichev–Saigo–Maeda (M–S–M) fractional integral and differential operators of the product of a general family of polynomial along with incomplete Fox–Wright functions $_p\Psi_q^{(\Gamma)}(t)$.

**Theorem 1** *Let* $\sigma, \sigma', \rho, \rho', \eta, \lambda \in \mathbb{C}$ *be such that* $\Re(\eta), \mu, \nu > 0$ *and* $\Re(\lambda) > \max\{\Re(-s\mu), \Re(\sigma' - \rho' - s\mu), \Re(\sigma + \sigma' + \rho - \eta - s\mu)\}$. *Thereupon, for* $\mathfrak{X} > 0$,

$$
\begin{aligned}
&\left(\mathcal{I}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\left(t^{\lambda-1}S_n^m[t^\mu]_p\Psi_q^{(\Gamma)}[t^\nu]\right)\right)(\mathfrak{X}) \\
&= \mathfrak{X}^{\lambda-\sigma-\sigma'+\eta-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\mathfrak{X}^{s\mu} \\
&\quad \times {}_{p+3}\Psi_{q+3}^{(\Gamma)}\left[
\begin{array}{l}
(a_1,A_1,x),(\lambda + s\mu, \nu),(-\sigma' + \rho' + \lambda + s\mu, \nu), \\
(\rho' + \lambda + s\mu, \nu),(-\sigma - \sigma' + \eta + \lambda + s\mu, \nu),
\end{array}
\right. \\
&\qquad \left.
\begin{array}{l}
(-\sigma - \sigma' - \rho + \eta + \lambda + s\mu, \nu),(a_j,A_j)_{2,p}; \\
(-\sigma' - \rho + \eta + \lambda + s\mu, \nu),(b_j,B_j)_{1,q};
\end{array}
\ \mathfrak{X}^\nu
\right].
\end{aligned}
\tag{2.6}
$$

*Proof* For the sake of simplicity, let us consider

$$
\mathfrak{L} = \left(\mathcal{I}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\left(t^{\lambda-1}S_n^m[t^\mu]_p\Psi_q^{(\Gamma)}[t^\nu]\right)\right)(\mathfrak{X}).
\tag{2.7}
$$

Now, using (1.2) and (1.6) in (2.7) and then taking advantage of relationship (2.4), we find for $\mathfrak{X} > 0$

$$
\begin{aligned}
\mathfrak{L} &= \left(\mathcal{I}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\left(t^{\lambda-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}t^{s\mu}\sum_{\ell=0}^{\infty}\frac{\Gamma(a_1 + A_1\ell, x)\prod_{j=2}^{p}\Gamma(a_j + A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j + B_j\ell)}\frac{t^{\ell\nu}}{\ell!}\right)\right)(\mathfrak{X}) \\
&= \sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\sum_{\ell=0}^{\infty}\frac{\Gamma(a_1 + A_1\ell, x)\prod_{j=2}^{p}\Gamma(a_j + A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j + B_j\ell)\ell!} \times \left(\mathcal{I}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\left\{t^{\lambda+s\mu+\ell\nu-1}\right\}\right)(\mathfrak{X}) \\
&= \sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\sum_{\ell=0}^{\infty}\frac{\Gamma(a_1 + A_1\ell, x)\prod_{j=2}^{p}\Gamma(a_j + A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j + B_j\ell)\ell!} \times \mathfrak{X}^{-\sigma-\sigma'+\eta+\lambda+s\mu+\ell\nu-1} \\
&\quad \times \frac{\Gamma(\lambda + s\mu + \ell\nu)\Gamma(-\sigma' + \rho' + \lambda + s\mu + \ell\nu)\Gamma(-\sigma - \sigma' - \rho + \eta + \lambda + s\mu + \ell\nu)}{\Gamma(\rho' + \lambda + s\mu + \ell\nu)\Gamma(-\sigma - \sigma' + \eta + \lambda + s\mu + \ell\nu)\Gamma(-\sigma' - \rho + \eta + \lambda + s\mu + \ell\nu)}.
\end{aligned}
$$

Finally, in opinion of (1.2) interpretation, we get (2.6) as a desired outcome.                   $\square$

**Theorem 2** *Let $\sigma, \sigma', \rho, \rho', \eta, \lambda, c \in \mathbb{C}$ be such that $\mu, \nu > 0$ and $\Re(\lambda) > \max\{\Re(-s\mu), \Re(-\sigma + \rho - s\mu), \Re(-\sigma - \sigma' - \rho' + \eta - s\mu)\}$. Then, for $\mathfrak{X} > 0$,*

$$
\left(\mathcal{D}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\left(\mathfrak{t}^{\lambda-1} S_n^m[\mathfrak{t}^\mu]_p \Psi_q^{(\Gamma)}[\mathfrak{t}^\nu]\right)\right)(\mathfrak{X})
$$

$$
= \mathfrak{X}^{\lambda+\sigma+\sigma'-\eta-1} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \mathfrak{X}^{s\mu}
$$

$$
\times \; {}_{p+3}\Psi_{q+3}^{(\Gamma)} \left[ \begin{array}{l} (a_1, A_1, x), (\lambda + s\mu, \nu), (\sigma - \rho + \lambda + s\mu, \nu), \\ (-\rho + \lambda + s\mu, \nu), (\sigma + \sigma' - \eta + \lambda + s\mu, \nu), \end{array} \right.
$$

$$
\left. \begin{array}{l} (\sigma + \sigma' + \rho' - \eta + \lambda + s\mu, \nu), (a_j, A_j)_{2,p}; \\ (\sigma + \rho' - \eta + \lambda + s\mu, \nu), (b_j, B_j)_{1,q}; \end{array} \quad \mathfrak{X}^\nu \right]. \tag{2.8}
$$

*Proof* For the sake of simplicity, let us consider

$$
\mathfrak{T} = \left(\mathcal{D}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\left(\mathfrak{t}^{\lambda-1} S_n^m[\mathfrak{t}^\mu]_p \Psi_q^{(\Gamma)}[\mathfrak{t}^\nu]\right)\right)(\mathfrak{X}). \tag{2.9}
$$

Now using (1.2) and (1.6) in (2.9) and then taking advantage of relationship (2.5), for $\mathfrak{X} > 0$, we acquire

$$
\mathfrak{T} = \left(\mathcal{D}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\left(\mathfrak{t}^{\lambda-1} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \mathfrak{t}^{s\mu} \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1 + A_1\ell, x) \prod_{j=2}^{p} \Gamma(a_j + A_j\ell)}{\prod_{j=1}^{q} \Gamma(b_j + B_j\ell)} \frac{\mathfrak{t}^{\ell\nu}}{\ell!}\right)\right)(\mathfrak{X})
$$

$$
= \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1 + A_1\ell, x) \prod_{j=2}^{p} \Gamma(a_j + A_j\ell)}{\prod_{j=1}^{q} \Gamma(b_j + B_j\ell)\ell!} \times \left(\mathcal{D}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\{\mathfrak{t}^{\lambda+s\mu+\ell\nu-1}\}\right)(\mathfrak{X})
$$

$$
= \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1 + A_1\ell, x) \prod_{j=2}^{p} \Gamma(a_j + A_j\ell)}{\prod_{j=1}^{q} \Gamma(b_j + B_j\ell)\ell!} \times \mathfrak{X}^{\sigma+\sigma'-\eta+\lambda+s\mu+\ell\nu-1}
$$

$$
\times \frac{\Gamma(\lambda + s\mu + \ell\nu)\Gamma(\sigma + \sigma' + \rho' - \eta + \lambda + s\mu + \ell\nu)\Gamma(\sigma - \rho + \lambda + s\mu + \ell\nu)}{\Gamma(-\rho + \lambda + s\mu + \ell\nu)\Gamma(\sigma + \rho' - \eta + \lambda + s\mu + \ell\nu)\Gamma(\sigma + \sigma' - \eta + \lambda + s\mu + \ell\nu)}.
$$

Finally, in opinion of the (1.2) interpretation, we get (2.8) as a desired outcome.    $\square$

The $F_3$ Appell function in (2.1) tends to reduce to the hypergeometric function ${}_2F_1$ of Gauss as follows:

$$
{}_2F_1(\sigma, \rho; \eta; \mathfrak{X}) = F_3(\sigma, \sigma', \rho, \rho'; \eta; \mathfrak{X}, 0)
$$

$$
= F_3(\sigma, 0, \rho, \rho'; \eta; \mathfrak{X}, \mathfrak{Y})
$$

$$
= F_3(\sigma, \sigma', \rho, 0; \eta; \mathfrak{X}, \mathfrak{Y}). \tag{2.10}
$$

In the light of the reduction formula (2.10), the Marichev–Saigo–Maeda operators (2.2) including (2.3) reduce to *Saigo's* hypergeometric fractional operators. If we take $\sigma = \sigma + \rho$, $\sigma' = \rho' = 0$, $\rho = -\eta$, $\eta = \sigma$, we immediately obtain *Saigo's* fractional integral along with differential operators in conjunction with the hypergeometric function ${}_2F_1$ [26, 27]:

$$
\left(\mathcal{I}_{0,+}^{\sigma,\rho,\eta}f\right)(\mathfrak{X}) = \frac{\mathfrak{X}^{-\sigma-\rho}}{\Gamma(\sigma)} \int_0^{\mathfrak{X}} (\mathfrak{X} - \mathfrak{t})^{\sigma-1} {}_2F_1\left(\sigma + \rho, -\eta; \sigma; 1 - \frac{\mathfrak{t}}{\mathfrak{X}}\right) f(\mathfrak{t}) \, d\mathfrak{t}, \tag{2.11}
$$

and

$$
\begin{aligned}
\left(\mathcal{D}_{0,+}^{\sigma,\rho,\eta}f\right)(\mathfrak{X}) &= \left(\mathcal{I}_{0,+}^{-\sigma,-\rho,\sigma+\eta}f\right)(\mathfrak{X}) \\
&= \left(\frac{d}{d\mathfrak{X}}\right)^{\kappa}\left(\mathcal{I}_{0,+}^{-\sigma+\kappa,-\rho-\kappa,\sigma+\eta-\kappa}f\right)(\mathfrak{X}) \quad \left(\kappa = [\Re(\sigma)] + 1\right).
\end{aligned}
\tag{2.12}
$$

Furthermore, by specializing the parameters in (2.11) and (2.12), we obtain *Riemann–Liouville* and *Erdélyi–Kober* fractional operators. Setting $\rho = -\sigma$ in (2.11) and (2.12) yields the familiar *Riemann–Liouville* integrals and derivatives of fractional order $\sigma \in \mathbb{C}$ beside $\Re(\sigma) > 0$ as well as $\mathfrak{X} \in \mathbb{R}^+$ (see, e.g., [17]):

$$
\left(\mathcal{I}_{0,+}^{\sigma,-\sigma,\eta}f\right)(\mathfrak{X}) = \left(\mathcal{I}_{0,+}^{\sigma}f\right)(\mathfrak{X}) \equiv \frac{1}{\Gamma(\sigma)}\int_0^{\mathfrak{X}}(\mathfrak{X}-\mathfrak{t})^{\sigma-1}f(\mathfrak{t})\,d\mathfrak{t}
\tag{2.13}
$$

and

$$
\begin{aligned}
\left(\mathcal{D}_{0,+}^{\sigma,-\sigma,\eta}f\right)(\mathfrak{X}) &= \left(\mathcal{D}_{0,+}^{\sigma}f\right)(\mathfrak{X}) \\
&= \left(\frac{d}{d\mathfrak{X}}\right)^{\kappa}\left(\mathcal{I}_{0,+}^{\kappa-\sigma}f\right)(\mathfrak{X}) \quad \left(\kappa = [\Re(\sigma)] + 1\right).
\end{aligned}
\tag{2.14}
$$

Again setting $\rho = 0$ in (2.11) and (2.12) provides the so-called *Erdélyi–Kober* integrals and derivatives of fractional order $\sigma \in \mathbb{C}$ along with $\Re(\sigma) > 0$ and $\mathfrak{X} \in \mathbb{R}^+$ (see, e.g., [34]):

$$
\left(\mathcal{I}_{0,+}^{\sigma,0,\eta}f\right)(\mathfrak{X}) = \left(\mathcal{I}_{\eta,\sigma}^{+}f\right)(\mathfrak{X}) = \frac{\mathfrak{X}^{-\sigma-\eta}}{\Gamma(\sigma)}\int_0^{\mathfrak{X}}(\mathfrak{X}-\mathfrak{t})^{\sigma-1}\mathfrak{t}^{\eta}f(\mathfrak{t})\,d\mathfrak{t}
\tag{2.15}
$$

and

$$
\begin{aligned}
\left(\mathcal{D}_{0,+}^{\sigma,0,\eta}f\right)(\mathfrak{X}) &= \left(\mathcal{D}_{\eta,\sigma}^{+}f\right)(\mathfrak{X}) \\
&= \left(\frac{d}{d\mathfrak{X}}\right)^{n}\left(\mathcal{I}_{0,+}^{\sigma+\kappa,-\sigma,\sigma+\eta-\kappa}f\right)(\mathfrak{X}) \quad \left(\kappa = [\Re(\sigma)] + 1\right).
\end{aligned}
\tag{2.16}
$$

Here, we mention these results in form of Corollaries 2.1 to 2.6.

**Corollary 2.1** *Let* $\sigma, \rho, \eta, \lambda \in \mathbb{C}$ *be such that* $\Re(\sigma), \mu, \nu > 0$ *and* $\Re(\lambda) > \max\{\Re(-s\mu), \Re(\rho - \eta - s\mu)\}$. *Thereupon, for* $\mathfrak{X} > 0$,

$$
\begin{aligned}
&\left(\mathcal{I}_{0+}^{\sigma,\rho,\eta}\left(\mathfrak{t}^{\lambda-1}S_n^m[\mathfrak{t}^{\mu}]_p\Psi_q^{(\Gamma)}[\mathfrak{t}^{\nu}]\right)\right)(\mathfrak{X}) \\
&= \mathfrak{X}^{-\rho+\lambda-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\mathfrak{X}^{s\mu} \\
&\quad \times {}_{p+2}\Psi_{q+2}^{(\Gamma)}\left[\begin{array}{c}(a_1,A_1,x),(\lambda+s\mu,\nu),(-\rho+\eta+\lambda+s\mu,\nu),(a_j,A_j)_{2,p}; \\ (\sigma+\eta+\lambda+s\mu,\nu),(-\rho+\lambda+s\mu,\nu),(b_j,B_j)_{1,q};\end{array}\ \mathfrak{X}^{\nu}\right].
\end{aligned}
\tag{2.17}
$$

**Corollary 2.2** *Suppose* $\sigma, \eta, \lambda \in \mathbb{C}$ *to be such that* $\Re(\sigma), \mu, \nu > 0$ *and* $\Re(\lambda) > \max\{\Re(-s\mu),$ $\Re(-\sigma - \eta - s\mu)\}$. *Thereupon, for* $\mathfrak{X} > 0$,

$$
\left(\mathcal{I}_{0+}^{\sigma,-\sigma,\eta}\left(\mathfrak{t}^{\lambda-1}S_n^m[\mathfrak{t}^\mu]_p\Psi_q^{(\Gamma)}[\mathfrak{t}^\nu]\right)\right)(\mathfrak{X})
$$

$$
= \mathfrak{X}^{\sigma+\lambda-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\mathfrak{X}^{s\mu}
$$

$$
\times {}_{p+1}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c}(a_1,A_1,x),(\lambda+s\mu,\nu),(a_j,A_j)_{2,p}; \\ (\sigma+\lambda+s\mu,\nu),(b_j,B_j)_{1,q};\end{array}\ \mathfrak{X}^\nu\right]. \tag{2.18}
$$

**Corollary 2.3** *Let* $\sigma, \eta, \lambda \in \mathbb{C}$ *be such that* $\Re(\sigma), \mu, \nu > 0$ *and* $\Re(\lambda) > \max\{\Re(-s\mu), \Re(-\eta - s\mu)\}$. *Thereupon, for* $\mathfrak{X} > 0$,

$$
\left(\mathcal{I}_{\eta,\sigma}^{+}\left(\mathfrak{t}^{\lambda-1}S_n^m[\mathfrak{t}^\mu]_p\Psi_q^{(\Gamma)}[\mathfrak{t}^\nu]\right)\right)(\mathfrak{X})
$$

$$
= \mathfrak{X}^{\lambda-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\mathfrak{X}^{s\mu}
$$

$$
\times {}_{p+1}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c}(a_1,A_1,x),(\eta+\lambda+s\mu,\nu),(a_j,A_j)_{2,p}; \\ (\sigma+\eta+\lambda+s\mu,\nu),(b_j,B_j)_{1,q};\end{array}\ \mathfrak{X}^\nu\right]. \tag{2.19}
$$

**Corollary 2.4** *Let* $\sigma, \rho, \eta, \lambda \in \mathbb{C}$ *be such that* $\mu, \nu > 0$ *and* $\Re(\lambda) > \max\{\Re(-s\mu), \Re(-\sigma - \rho - \eta - s\mu)\}$. *Then, for* $\mathfrak{X} > 0$,

$$
\left(\mathcal{D}_{0+}^{\sigma,\rho,\eta}\left(\mathfrak{t}^{\lambda-1}S_n^m[\mathfrak{t}^\mu]_p\Psi_q^{(\Gamma)}[\mathfrak{t}^\nu]\right)\right)(\mathfrak{X})
$$

$$
= \mathfrak{X}^{\rho+\lambda-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\mathfrak{X}^{s\mu}
$$

$$
\times {}_{p+2}\Psi_{q+2}^{(\Gamma)}\left[\begin{array}{c}(a_1,A_1,x),(\lambda+s\mu,\nu),(\sigma+\rho+\eta+\lambda+s\mu,\nu),(a_j,A_j)_{2,p}; \\ (\eta+\lambda+s\mu,\nu),(\rho+\lambda+s\mu,\nu),(b_j,B_j)_{1,q};\end{array}\ \mathfrak{X}^\nu\right]. \tag{2.20}
$$

**Corollary 2.5** *Let* $\sigma, \eta, \lambda \in \mathbb{C}$ *be such that* $\mu, \nu > 0$ *and* $\Re(\lambda) > \max\{\Re(-s\mu), \Re(-\eta - s\mu)\}$. *Then, for* $\mathfrak{X} > 0$,

$$
\left(\mathcal{D}_{0+}^{\sigma,-\sigma,\eta}\left(\mathfrak{t}^{\lambda-1}S_n^m[\mathfrak{t}^\mu]_p\Psi_q^{(\Gamma)}[\mathfrak{t}^\nu]\right)\right)(\mathfrak{X})
$$

$$
= \mathfrak{X}^{-\sigma+\lambda-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\mathfrak{X}^{s\mu}
$$

$$
\times {}_{p+1}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c}(a_1,A_1,x),(\lambda+s\mu,\nu),(a_j,A_j)_{2,p}; \\ (-\sigma+\lambda+s\mu,\nu),(b_j,B_j)_{1,q};\end{array}\ \mathfrak{X}^\nu\right]. \tag{2.21}
$$

**Corollary 2.6** *Let* $\sigma, \eta, \lambda \in \mathbb{C}$ *be such that* $\mu, \nu > 0$ *and* $\Re(\lambda) > \max\{\Re(-s\mu), \Re(-\sigma - \eta - s\mu)\}$. *Thereupon, for* $\mathfrak{X} > 0$,

$$\left(\mathcal{D}_{\eta,\sigma}^{+}\left(\mathfrak{t}^{\lambda-1} S_n^m[\mathfrak{t}^\mu]_p\Psi_q^{(\Gamma)}[\mathfrak{t}^\nu]\right)\right)(\mathfrak{X})$$

$$= \mathfrak{X}^{\lambda-1} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\mathfrak{X}^{s\mu}$$

$$\times \,_{p+1}\Psi_{q+1}^{(\Gamma)}\left[\begin{matrix}(a_1,A_1,x),(\sigma+\eta+\lambda+s\mu,\nu),(a_j,A_j)_{2,p}; \\ (\eta+\lambda+s\mu,\nu),(b_j,B_j)_{1,q};\end{matrix}\ \ \mathfrak{X}^\nu\right]. \tag{2.22}$$

## 3 Integral transforms

In this part, several integral transforms such as Jacobi, Gegenbauer (or ultraspherical), Legendre, Laplace, Mellin, Hankel, and Euler's beta transforms of a product of a general polynomial class and incomplete Fox–Wright function $_p\Psi_q^{(\Gamma)}(\mathfrak{t})$ are presented.

### 3.1 Jacobi and related integral transforms

The classical orthogonal Jacobi polynomial $P_n^{(\mathfrak{h},\mathfrak{g})}(\mathfrak{t})$ is given by the following (see, for example, [33]):

$$P_n^{(\mathfrak{h},\mathfrak{g})}(\mathfrak{t}) = (-1)^n(-\mathfrak{t}) = \binom{\mathfrak{h}+n}{n} \,_2F_1\left[\begin{matrix}-n,\mathfrak{h}+\mathfrak{g}+n+1 \\ \mathfrak{h}+1\end{matrix}\ \middle|\ \frac{1-\mathfrak{t}}{2}\right], \tag{3.1}$$

where $_2F_1$ is the Gauss hypergeometric function [25].

**Definition 2** The Jacobi transformation of a $f(\mathfrak{t})$ function is set as follows (see, *e.g.*, [13, p. 501]):

$$\mathbb{J}^{(\mathfrak{h},\mathfrak{g})}\left[f(\mathfrak{t}); n\right] = \int_{-1}^{1}(1-\mathfrak{t})^{\mathfrak{h}}(1+\mathfrak{t})^{\mathfrak{g}}P_n^{(\mathfrak{h},\mathfrak{g})}(\mathfrak{t})f(\mathfrak{t})\,d\mathfrak{t} \tag{3.2}$$

$$\left(\min\{\Re(\mathfrak{h}),\Re(\mathfrak{g})\} > -1; n \in \mathbb{N}_0\right),$$

provided that the $f(\mathfrak{t})$ function seems to be so limited that only the integral exists in (3.2).

The Jacobi transform of the power function $\mathfrak{t}^{\rho-1}$ is given by (see, *e.g.*, [37, p. 128, Eq. (18)])

$$\mathbb{J}^{(\mathfrak{h},\mathfrak{g})}\left[\mathfrak{t}^{\rho-1}; n\right] = \int_{-1}^{1}(1-\mathfrak{t})^{\xi-1}(1+\mathfrak{t})^{\eta-1}P_n^{(\mathfrak{h},\mathfrak{g})}(\mathfrak{t})\mathfrak{t}^{\rho-1}\,d\mathfrak{t}$$

$$= 2^{\xi+\eta-1}\binom{\mathfrak{h}+n}{n}B(\xi,\eta)F_{1:1;0}^{1:2;1}\left[\begin{matrix}\xi:-n,\mathfrak{h}+\mathfrak{g}+n+1;1-\rho; \\ \xi+\eta:\quad\ \mathfrak{h}+1;\quad\ —;\end{matrix}\ \ 1,2\right] \tag{3.3}$$

$$\left(\min\{\Re(\xi),\Re(\eta)\} > 0; \rho \in \mathbb{C}; n \in \mathbb{N}_0\right),$$

where $F_{p:l;\mu}^{q:m;\nu}$ corresponds to Kampé de Fériet's function in two variables (see, *e.g.*, [32, p. 22, Eq. 1.3(2)] and [32, p. 37, Eq. 1.4(21)]). In fact, this last integral formula (3.3) will be

reduced instantly to the preceding form when specifying $\xi = \mathfrak{h} + 1$ and $\eta = \mathfrak{g} + 1$:

$$
\begin{aligned}
\mathbb{J}^{(\mathfrak{h},\mathfrak{g})}\big[t^{\rho-1}; n\big] &= \int_{-1}^{1} (1-t)^{\mathfrak{h}}(1+t)^{\mathfrak{g}} P_n^{(\mathfrak{h},\mathfrak{g})}(t) t^{\rho-1}\, dt \\
&= 2^{\mathfrak{h}+\mathfrak{g}+1}\binom{\mathfrak{h}+n}{n} B(\mathfrak{h}+1,\mathfrak{g}+1) \\
&\quad \times F_{1:1;0}^{1:2;1}\left[\begin{array}{c} \mathfrak{h}+1: -n, \mathfrak{h}+\mathfrak{g}+n+1; 1-\rho; \\ \mathfrak{h}+\mathfrak{g}+2: \quad \mathfrak{h}+1; \qquad \text{---}; \end{array}\ 1,2\right]
\end{aligned}
\tag{3.4}
$$

$$
\big(\min\{\mathfrak{R}(\mathfrak{h}),\mathfrak{R}(\mathfrak{g})\} > -1; \rho \in \mathbb{C}; n \in \mathbb{N}_0\big).
$$

However, in its additional limited case where $\rho = m + 1$ ($m \in \mathbb{N}_0$), (3.4) brings the established consequence about the $t^m$ ($m \in \mathbb{N}_0$) Jacobi transform studied by [25, p. 261, Eq. (14) along with (15)]:

$$
\mathbb{J}^{(\mathfrak{h},\mathfrak{g})}\big[t^m; n\big]
$$

$$
= \int_{-1}^{1} (1-t)^{\mathfrak{h}}(1+t)^{\mathfrak{g}} P_n^{(\mathfrak{h},\mathfrak{g})}(t) t^m\, dt
$$

$$
= \begin{cases}
0 & (m = 0,1,2,\ldots,n-1), \\
2^{\mathfrak{h}+\mathfrak{g}+n+1} B(\mathfrak{h}+n+1,\mathfrak{g}+n+1) & (m = n), \\
2^{\mathfrak{h}+\mathfrak{g}+n+1}\binom{m}{n} B(\mathfrak{h}+n+1,\mathfrak{g}+n+1) \\
\quad \cdot\, {}_2F_1\left[\begin{smallmatrix} n-m,\mathfrak{h}+n+1 \\ \mathfrak{h}+\mathfrak{g}+2n+2 \end{smallmatrix}\,\big|\, 2\right], & (m = n+1, n+2, n+3, \ldots)
\end{cases}
\tag{3.5}
$$

$$
\big(\min\{\mathfrak{R}(\mathfrak{h}),\mathfrak{R}(\mathfrak{g})\} > -1; m, n \in \mathbb{N}_0\big).
$$

Specifying the parameters $\mathfrak{h}$ and $\mathfrak{g}$, the Jacobi polynomials $P_n^{(\mathfrak{h},\mathfrak{g})}(t)$ exhibit, like in their individual cases, other such recognized orthogonal polynomials being the Gegenbauer (or ultraspherical) polynomials $C_n^{\nu}(t)$, the Legendre (or spherical) polynomials $P_n(t)$, and the Tchebycheff polynomials $T_n(t)$ and $U_n(t)$ of the first kind and second kind (see, for details, [33]). In addition, we have the accompanying established connections with the Gegenbauer polynomials $C_n^{\nu}(t)$ as well as the Legendre polynomials $P_n(t)$:

$$
C_n^{\nu}(t) = \binom{\nu + n - \frac{1}{2}}{n}^{-1}\binom{2\nu + n - 1}{n} P_n^{(\nu - \frac{1}{2}, \nu - \frac{1}{2})}(t)
\tag{3.6}
$$

and

$$
P_n(t) = C_n^{\frac{1}{2}}(t) = P_n^{(0,0)}(t),
\tag{3.7}
$$

respectively, which, in conjunction with (3.2), brings the Gegenbauer transform $\mathbb{G}^{(\nu)}[f(t); n]$ as follows:

$$
\begin{aligned}
\mathbb{G}^{(\nu)}\big[f(t); n\big] &= \binom{\nu + n - \frac{1}{2}}{n}^{-1}\binom{2\nu + n - 1}{n} \mathbb{J}^{(\nu - \frac{1}{2}, \nu - \frac{1}{2})}\big[f(t); n\big] \\
&= \int_{-1}^{1} \big(1-t^2\big)^{\nu - \frac{1}{2}} C_n^{\nu}(t) f(t)\, dt \quad \left(\mathfrak{R}(\nu) > -\frac{1}{2}; n \in \mathbb{N}_0\right),
\end{aligned}
\tag{3.8}
$$

and the resulting Legendre transform $\mathbb{L}[f(\mathfrak{t}); n]$ which is described by

$$\mathbb{L}\big[f(\mathfrak{t}); n\big] = \mathbb{G}^{(\frac{1}{2})}\big[f(\mathfrak{t}); n\big] = \int_{-1}^{1} P_n(\mathfrak{t}) f(\mathfrak{t})\, d\mathfrak{t} \quad (n \in \mathbb{N}_0). \tag{3.9}$$

We are now generating three new results that provide the relations between Jacobi, Gegenbauer, and Legendre transforms with the following incomplete Fox–Wright function $_p\Psi_q^{(\Gamma)}$ (1.2).

**Theorem 3** *The preceding formula for Jacobi transform is valid under the condition stated in* (1.2):

$$\mathbb{J}^{(\mathfrak{h},\mathfrak{g})}\big\{\mathfrak{t}^{\rho-1} S_{n'}^{m'}[\omega\mathfrak{t}]\, _p\Psi_q^{(\Gamma)}[\omega\mathfrak{t}]; n\big\}$$

$$= 2^{\mathfrak{h}+\mathfrak{g}+1}\binom{\mathfrak{h}+n}{n} B(\mathfrak{h}+1, \mathfrak{g}+1)$$

$$\times \sum_{s=0}^{[n'/m']} \frac{(-n')_{m's}}{s!} A_{n',s}\omega^s \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1+A_1\ell, x)\prod_{j=2}^{p}\Gamma(a_j+A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j+B_j\ell)}$$

$$\times F_{1:1;0}^{1:2;1}\left[\begin{matrix} \mathfrak{h}+1: -n, \mathfrak{h}+\mathfrak{g}+n+1; 1-\rho-s-\ell; \\ \mathfrak{h}+\mathfrak{g}+2: \qquad \mathfrak{h}+1; \qquad \text{---}; \end{matrix} \; 1, 2 \right] \frac{\omega^\ell}{\ell!} \tag{3.10}$$

$$\big(x \geqq 0; n \in \mathbb{N}_0; \min\{\Re(\mathfrak{h}), \Re(\mathfrak{g})\} > -1; \rho \in \mathbb{C}; p, q \in \mathbb{N}_0\big),$$

*where the Jacobi transform into* (3.4) *is assumed to exist.*

*Proof* By employing the concept of (3.2) together with (1.2), we get

$$\mathbb{J}^{(\mathfrak{h},\mathfrak{g})}\big\{\mathfrak{t}^{\rho-1} S_{n'}^{m'}[\omega\mathfrak{t}]\, _p\Psi_q^{(\Gamma)}[\omega\mathfrak{t}]; n\big\}$$

$$= \int_{-1}^{1} \mathfrak{t}^{\rho-1}(1-\mathfrak{t})^{\mathfrak{h}}(1+\mathfrak{t})^{\mathfrak{g}} P_n^{(\mathfrak{h},\mathfrak{g})}(\mathfrak{t}) S_{n'}^{m'}[\omega\mathfrak{t}]\, _p\Psi_q^{(\Gamma)}[\omega\mathfrak{t}]\, d\mathfrak{t}$$

$$= \int_{-1}^{1} \mathfrak{t}^{\rho-1}(1-\mathfrak{t})^{\mathfrak{h}}(1+\mathfrak{t})^{\mathfrak{g}} P_n^{(\mathfrak{h},\mathfrak{g})}(\mathfrak{t}) \sum_{s=0}^{[n'/m']} \frac{(-n')_{m's}}{s!} A_{n',s}(\omega\mathfrak{t})^s$$

$$\times \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1+A_1\ell, x)\prod_{j=2}^{p}\Gamma(a_j+A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j+B_j\ell)} \frac{(\omega\mathfrak{t})^\ell}{\ell!}\, d\mathfrak{t}, \tag{3.11}$$

where, when adjusting the order of integration and summation (that might be easily explained by absolute convergence), we make use of the Jacobi transform formula (3.4) along with the parameter $\rho$ substituted by $\rho + k$ ($\rho \in \mathbb{C}; k \in \mathbb{N}_0$). □

By employing the Jacobi transform formula (3.5), we can simplify the assertion (3.10) of Theorem 3 in their limiting case when $\rho = m + 1$ ($m \in \mathbb{N}_0$). Furthermore, in light

of connection (3.6), Theorem 3 gives the subsequent corollary by considering $\mathfrak{h} = \mathfrak{g} = \nu - \frac{1}{2}$.

**Corollary 3.1** *The following Gegenbauer transform formula holds true under the condition stated in* (1.2):

$$
\begin{aligned}
&\mathbb{G}^{(\nu)}\big[t^{\rho-1} S_{n'}^{m'}[\omega t]_p \Psi_q^{(\Gamma)}[\omega t]; n\big]\\
&\quad = 2^{2\nu} \binom{2\nu + n - 1}{n} B\left(\nu + \frac{1}{2}, \nu + \frac{1}{2}\right)\\
&\qquad \times \sum_{s=0}^{[n'/m']} \frac{(-n')_{m's}}{s!} A_{n',s} \omega^s \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1 + A_1 \ell, x) \prod_{j=2}^{p} \Gamma(a_j + A_j \ell)}{\prod_{j=1}^{q} \Gamma(b_j + B_j \ell)}\\
&\qquad \times F_{1:1;0}^{1:2;1}\left[\begin{array}{c} \nu + \frac{1}{2} : -n, 2\nu + n; 1 - \rho - \ell; \\ 2\nu + 1 : \qquad \nu + \frac{1}{2}; \qquad \text{---}; \end{array} \; 1, 2\right] \frac{\omega^\ell}{\ell!}
\end{aligned}
\tag{3.12}
$$

$$(x \geqq 0; n \in \mathbb{N}_0; \rho \in \mathbb{C}; p, q \in \mathbb{N}_0),$$

*where it is assumed that the Gegenbauer transform in* (3.12) *exists.*

A special case of Theorem 3 when $\mathfrak{h} = \mathfrak{g} = 0$ (or, alternatively, Corollary 3.1 with $\nu = \frac{1}{2}$) gives the following result for the Legendre transform described by (3.9).

**Corollary 3.2** *The subsequent Legendre transform formula holds true under the condition stated in* (1.2):

$$
\begin{aligned}
&\mathbb{L}\big[t^{\rho-1} S_{n'}^{m'}[\omega t]_p \Psi_q^{(\Gamma)}[\omega t]; n\big]\\
&\quad = 2 \sum_{s=0}^{[n'/m']} \frac{(-n')_{m's}}{s!} A_{n',s} \omega^s \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1 + A_1 \ell, x) \prod_{j=2}^{p} \Gamma(a_j + A_j \ell)}{\prod_{j=1}^{q} \Gamma(b_j + B_j \ell)}\\
&\qquad \times F_{1:1;0}^{1:2;1}\left[\begin{array}{c} 1 : -n, n + 1; 1 - \rho - \ell; \\ 2 : \qquad 1; \qquad \text{---}; \end{array} \; 1, 2\right] \frac{\omega^\ell}{\ell!}
\end{aligned}
\tag{3.13}
$$

$$(x \geqq 0; n \in \mathbb{N}_0; \rho \in \mathbb{C}; p, q \in \mathbb{N}_0),$$

*where it is assumed that the Legendre transform in* (3.13) *exists.*

## 3.2 Laplace transform

The Laplace transform of a given function $f(t)$ is defined as follows [13, 29]:

$$
L\{f(t); \omega\} = \int_0^\infty e^{-\omega t} f(t)\, dt \quad (\Re(\omega) > 0),
\tag{3.14}
$$

if the improper integral exists.

**Theorem 4** *If $\Delta > 0$, $\alpha > 0$, $\beta > 0$, and $\Re(\omega) > 0$, then the Laplace transform of incomplete Fox–Wright function $_p\Psi_q^{(\Gamma)}$ is given as follows*:

$$
L\{t^{\lambda-1}S_n^m[t^\alpha]_p\Psi_q^{(\Gamma)}[t^\beta];\omega\}
$$

$$
= \frac{1}{\omega^\lambda}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\frac{1}{\omega^{s\alpha}}
$$

$$
\times\,_{p+1}\Psi_q^{(\Gamma)}\left[\begin{array}{cc}(a_1,A_1,x),(\lambda+s\alpha,\beta),(a_j,A_j)_{2,p}; & \dfrac{1}{\omega^\beta}\\ (b_j,B_j)_{1,q}; & \end{array}\right]. \tag{3.15}
$$

*Proof* Using the definition of (1.2) and (1.6) in the left-hand side of (3.15) and applying the definition of Laplace transform (3.14), we obtain

$$
L\{t^{\lambda-1}S_n^m[t^\alpha]_p\Psi_q^{(\Gamma)}[t^\beta];\omega\}
$$

$$
= L\left\{t^{\lambda-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}t^{s\alpha}\sum_{\ell=0}^{\infty}\frac{\Gamma(a_1+A_1\ell,x)\prod_{j=2}^{p}\Gamma(a_j+A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j+B_j\ell)}\frac{t^{\ell\beta}}{\ell!};\omega\right\}
$$

$$
= \sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\sum_{\ell=0}^{\infty}\frac{\Gamma(a_1+A_1\ell,x)\prod_{j=2}^{p}\Gamma(a_j+A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j+B_j\ell)\ell!}\int_0^\infty e^{-\omega t}t^{\lambda+s\alpha+\ell\beta-1}\,dt
$$

$$
= \sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\sum_{\ell=0}^{\infty}\frac{\Gamma(a_1+A_1\ell,x)\prod_{j=2}^{p}\Gamma(a_j+A_j\ell)}{\prod_{j=1}^{q}\Gamma(b_j+B_j\ell)\ell!}\frac{\Gamma(\lambda+s\alpha+\ell\beta)}{\omega^{\lambda+s\alpha+\ell\beta}}.
$$

Now, employing the *gamma function* formula [25]

$$
\Gamma(\eta)\xi^{-\eta} = \int_0^\infty e^{-\xi t}t^{\eta-1}\,dt, \quad \Re(\xi) > 0, \Re(\eta) > 0, \tag{3.16}
$$

and then using (1.2), we arrive at the desired result in Theorem 4.  □

### 3.3 Mellin transform

The Mellin transform of a given function $f(t)$ is represented as follows [13, 29]:

$$
M\{f(t);\omega\} = \int_0^\infty t^{\omega-1}f(t)\,dt \quad (\Re(\omega) > 0), \tag{3.17}
$$

given that the improper integral exists.

**Theorem 5** *If $\Delta > 0$, $\mathfrak{h} > 0$, $\mathfrak{g} > 0$, $\alpha > 0$, $\beta > 0$, and $\Re(\omega) > 0$, then the Mellin transform of incomplete Fox–Wright function $_p\Psi_q^{(\Gamma)}$ is given as follows*:

$$
M\left\{S_n^m\left[\frac{\mathfrak{h}}{(1+t)^\alpha}\right]_p\Psi_q^{(\Gamma)}\left[\frac{\mathfrak{g}}{(1+t)^\beta}\right];\omega\right\}
$$

$$
= \Gamma(\omega)\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\mathfrak{h}^s
$$

$$
\times\,_{p+1}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{cc}(a_1,A_1,x),(-\omega+s\alpha,\beta),(a_j,A_j)_{2,p}; & \mathfrak{g}\\ (s\alpha,\beta),(b_j,B_j)_{1,q}; & \end{array}\right]. \tag{3.18}
$$

*Proof* Using the definition of (1.2), (1.6) and applying Mellin transform (3.17) in the left-hand side of (3.18) and then employing the *beta function* formula [25]

$$
B(\alpha, \beta) = \begin{cases} \int_0^\infty \frac{t^{\alpha-1}}{(1-t)^{\alpha+\beta}} \, dt & (\Re(\alpha) > 0; \Re(\beta) > 0), \\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} & (\alpha, \beta \notin \mathbb{Z}_0^-), \end{cases}
\tag{3.19}
$$

after using definition (1.2), we obtain the desired formula in Theorem 5. □

### 3.4 Hankel transform

The Hankel transform of a given function $f(t)$ is characterized as follows [13, 29]:

$$
H_\nu\{f(t); \omega\} = \int_0^\infty t J_\nu(\omega t) f(t) \, dt \quad (\Re(\omega) > 0),
\tag{3.20}
$$

provided that the improper integral exists, $J_\nu(\omega t)$ is the Bessel function of order $\nu$.

**Theorem 6** *If $\Delta > 0$, $\alpha > 0$, $\beta > 0$, $\lambda > 0$, and $\Re(\omega) > 0$, then the Hankel transform of incomplete Fox–Wright function $_p\Psi_q^{(\Gamma)}$ is given as follows*:

$$
\begin{aligned}
&H_\nu\{t^{\lambda-1} S_n^m[t^\alpha]_p\Psi_q^{(\Gamma)}[t^\beta]; \omega\} \\
&= \frac{2}{\omega^\lambda} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \left(\frac{2}{\omega}\right)^{s\alpha} \\
&\quad \times {}_{p+1}\Psi_{q+1}^{(\Gamma)} \left[ \begin{matrix} (a_1, A_1, x), (\frac{\lambda+\nu+s\alpha}{2}, \frac{\beta}{2}), (a_j, A_j)_{2,p}; \\ (\frac{2+\nu-\lambda-s\alpha}{2}, -\frac{\beta}{2}), (b_j, B_j)_{1,q}; \end{matrix} \quad \left(\frac{2}{\omega}\right)^\beta \right].
\end{aligned}
\tag{3.21}
$$

*Proof* Using the definition of (1.2), (1.6) and applying Hankel transform (3.20) in the left-hand side of (3.21) and then employing the formula (Prudnikov, Brychkov, and Marichev [24, (2.44)])

$$
\int_0^\infty t^{\lambda-1} J_\nu(at) \, dt = 2^{\lambda-1} a^{-\lambda} \frac{\Gamma(\frac{\lambda+\nu}{2})}{\Gamma(1 + \frac{\nu-\lambda}{2})} \quad (a > 0, -\Re(\nu) < \Re(\lambda) < 3/2),
\tag{3.22}
$$

we are led easily to the right-hand side of the assertion (3.21) of Theorem 6. The details are omitted here. □

### 3.5 Euler's beta transform

The integral transform of Euler's beta type for a given function $f(t)$ is characterized as follows [13, 29]:

$$
B\{f(t) : \alpha, \beta\} = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} f(t) \, dt \quad (\Re(\alpha) > 0, \Re(\beta) > 0).
\tag{3.23}
$$

**Theorem 7** *If $\Delta > 0$, $\mathfrak{h} > 0$, $\mathfrak{g} > 0$, $\mu > 0$, $\nu > 0$, $\xi > 0$, $\eta > 0$, and $\Re(\alpha), \Re(\beta) > 0$, then Euler's beta type transform of incomplete Fox–Wright function $_p\Psi_q^{(\Gamma)}$ is given as follows:*

$$
B\big\{S_n^m\big[\mathfrak{h}\mathfrak{t}^\mu(1-\mathfrak{t})^\nu\big]_p\Psi_q^{(\Gamma)}\big[\mathfrak{g}\mathfrak{t}^\xi(1-\mathfrak{t})^\eta\big] : \alpha, \beta\big\}
$$

$$
= \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \mathfrak{h}^s
$$

$$
\times\ {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{matrix} (a_1, A_1, x), (\alpha + s\mu, \xi), (\beta + s\nu, \eta), (a_j, A_j)_{2,p}; \\ (\alpha + \beta + s(\mu + \nu), \xi + \eta), (b_j, B_j)_{1,q}; \end{matrix}\ \ \mathfrak{g}\right]. \tag{3.24}
$$

*Proof* Using the definition of (1.2), (1.6) and applying Euler's beta transform (3.23) in the left-hand side of (3.24) and then employing the *beta function* formula [25]

$$
B(\alpha, \beta) = \begin{cases} \int_0^1 \mathfrak{t}^{\alpha-1}(1-\mathfrak{t})^{\beta-1}\,d\mathfrak{t} & (\Re(\alpha) > 0; \Re(\beta) > 0), \\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} & (\alpha, \beta \notin \mathbb{Z}_0^-), \end{cases} \tag{3.25}
$$

after using definition (1.2), we arrive at the desired formula in Theorem 7. □

## 4 Concluding remarks and observations

In our present study, with the aid of the incomplete Fox–Wright functions $_p\Psi_q^{(\gamma)}(\mathfrak{t})$ and $_p\Psi_q^{(\Gamma)}(\mathfrak{t})$, we have studied several interesting properties, such as Marichev–Saigo–Maeda left-handed fractional integral and differential operators, which include Saigo hypergeometric fractional integral and differential operators, Riemann–Liouville, and Erdélyi–Kober fractional integral and differential operators as particular cases for various choices of parameter. In a similar pattern, one can derive results for right-hand-sided Marichev–Saigo–Maeda, Saigo hypergeometric fractional, Riemann–Liouville, and Erdélyi–Kober fractional integral and differential operators. Furthermore, we derive several integral transforms such as Jacobi, Gegenbauer (or ultraspherical), Legendre, Laplace, Mellin, Hankel, and Euler's beta transforms. Specific cases of derived findings can be developed by suitably specializing the coefficient $A_{n,s}$ to obtain a large number of spectrum of the known polynomials (see, *e.g.* [30]). Here we give three main results, and we left the remaining ones for interested readers. If we set $n = 0$ and $A_{0,0}$ (the polynomial family $S_0^m$ will reduce to unity) in Theorems 1, 2, and 3, we get the following corollaries.

**Corollary 7.1** *Let $\sigma, \sigma', \rho, \rho', \eta, \lambda \in \mathbb{C}$ be such that $\Re(\eta), \mu, \nu > 0$ and $\Re(\lambda) > \max\{0, \Re(\sigma' - \rho'), \Re(\sigma + \sigma' + \rho - \eta)\}$. Thereupon, for $\mathfrak{X} > 0$,*

$$
\big(\mathcal{I}_{0+}^{\sigma,\sigma',\rho,\rho',\eta}\big(\mathfrak{t}^{\lambda-1}{}_p\Psi_q^{(\Gamma)}\big[\mathfrak{t}^\nu\big]\big)\big)(\mathfrak{X})
$$

$$
= \mathfrak{X}^{\lambda-\sigma-\sigma'+\eta-1}{}_{p+3}\Psi_{q+3}^{(\Gamma)}\left[\begin{matrix} (a_1, A_1, x), (\lambda, \nu), (-\sigma' + \rho' + \lambda, \nu), \\ (\rho' + \lambda, \nu), (-\sigma - \sigma' + \eta + \lambda, \nu), \end{matrix}\right.
$$

$$
\left.\begin{matrix} (-\sigma - \sigma' - \rho + \eta + \lambda, \nu), (a_j, A_j)_{2,p}; \\ (-\sigma' - \rho + \eta + \lambda, \nu), (b_j, B_j)_{1,q}; \end{matrix}\ \ \mathfrak{X}^\nu\right]. \tag{4.1}
$$

**Corollary 7.2** *Let* $\sigma, \sigma', \rho, \rho', \eta, \lambda, c \in \mathbb{C}$ *be such that* $\mu, \nu > 0$ *and* $\Re(\lambda) > \max\{0, \Re(-\sigma + \rho), \Re(-\sigma - \sigma' - \rho' + \eta)\}$. *Then, for* $\mathfrak{X} > 0$,

$$
\left( \mathcal{D}_{0+}^{\sigma,\sigma',\rho,\rho',\eta} \left( \mathfrak{t}^{\lambda-1}{}_p \Psi_q^{(\Gamma)} \left[ \mathfrak{t}^\nu \right] \right) \right)(\mathfrak{X})
$$

$$
= \mathfrak{X}^{\lambda+\sigma+\sigma'-\eta-1}{}_{p+3} \Psi_{q+3}^{(\Gamma)} \left[ \begin{matrix} (a_1, A_1, x), (\lambda, \nu), (\sigma - \rho + \lambda, \nu), \\ (-\rho + \lambda, \nu), (\sigma + \sigma' - \eta + \lambda, \nu), \end{matrix} \right.
$$

$$
\left. \begin{matrix} (\sigma + \sigma' + \rho' - \eta + \lambda, \nu), (a_j, A_j)_{2,p}; \\ (\sigma + \rho' - \eta + \lambda, \nu), (b_j, B_j)_{1,q}; \end{matrix} \quad \mathfrak{X}^\nu \right].
$$

$$\tag{4.2}$$

**Corollary 7.3** *The coming Jacobi transform formula holds true under the condition stated in* (1.2):

$$
\mathbb{J}^{(\mathfrak{h},\mathfrak{g})} \left\{ \mathfrak{t}^{\rho-1}{}_p \Psi_q^{(\Gamma)} [\omega \mathfrak{t}]; n \right\}
$$

$$
= 2^{\mathfrak{h}+\mathfrak{g}+1} \binom{\mathfrak{h}+n}{n} B(\mathfrak{h}+1, \mathfrak{g}+1) \sum_{\ell=0}^{\infty} \frac{\Gamma(a_1 + A_1 \ell, x) \prod_{j=2}^{p} \Gamma(a_j + A_j \ell)}{\prod_{j=1}^{q} \Gamma(b_j + B_j \ell)}
$$

$$
\times F_{1:1;0}^{1:2;1} \left[ \begin{matrix} \mathfrak{h}+1 : -n, \mathfrak{h}+\mathfrak{g}+n+1; 1-\rho-\ell; \\ \mathfrak{h}+\mathfrak{g}+2 : \quad \mathfrak{h}+1; \quad -; \end{matrix} \quad 1, 2 \right] \frac{\omega^\ell}{\ell!}
$$

$$\tag{4.3}$$

$$
\left( x \geqq 0; n \in \mathbb{N}_0; \min\{\Re(\mathfrak{h}), \Re(\mathfrak{g})\} > -1; \rho \in \mathbb{C}; p, q \in \mathbb{N}_0 \right),
$$

*where it is considered that the Jacobi transform in* (3.4) *exists.*

*Remark* 1 It is important to note that the particular cases of the results obtained in this paper for $x = 0$ would give the corresponding new results for the product of a more general class of polynomials and Fox–Wright hypergeometric functions $_p \Psi_q(\mathfrak{t})$.

**Availability of data and materials**
Not applicable.

**Competing interests**
The authors declare that they have no competing interests.

**Authors' contributions**
The authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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