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Novel delay-dependent exponential stabilization criteria of a nonlinear system with mixed time-varying delays via hybrid intermittent feedback control

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Abstract

In this paper, we investigate the problem of exponential stabilization criteria for a nonlinear system with mixed time-varying delays, including discrete interval and distributed time-varying delays. The time-varying delays are not necessarily differentiable. The exponential stabilization criteria of the nonlinear system are proposed via hybrid intermittent feedback control. Based on the improved Lyapunov-Krasovskii functionals with Leibniz-Newton's formula, Jensen's inequality and the reciprocal convex combination technique, the novel delay-dependent sufficient condition is derived in terms of linear matrix inequalities (LMIs). The obtained LMIs can be efficiently solved by standard convex optimization algorithms. A numerical example is given to demonstrate the effectiveness of the obtained result. Moreover, the results in this article generalize and improve the corresponding results of the recent works.

Keywords: exponential stabilization; nonlinear system; mixed time-varying delays; hybrid intermittent feedback control

1 Introduction

Nonlinear systems, as an appealing topic, have been thoroughly studied during the past decades. Due to the fact that most systems are inherently nonlinear in nature, these systems are one of the most interesting areas for researchers, including engineers, physicists, mathematicians and other scientists. The exponential stability of nonlinear systems has been widely received and deeply investigated [1–12], and the asymptotical stability of nonlinear systems has been investigated [13–17] as well. Time delay naturally appears in most of the real world systems. It is well known that the existence of time delay in a system may cause instability, poor performances and oscillations in, for instance, chemical engineering systems, biological modeling, electrical networks, physical networks and many natural sciences. Due to these results, the nonlinear system with time-varying delay has become an interesting topic in recent years; the authors have investigated interval time-varying delay [1, 3, 6–9, 18–23], discrete time-varying delay [4, 5, 24, 25], mixed time-varying delays [26–32] and so on.

In practical control designs, due to failure modes, system uncertainty, or systems with various modes of operation, the simultaneous stabilization problem often has to be taken into account. The problem is concerned with designing control which can simultaneously stabilize a set of systems. Among the usual approaches, there are many studies on the stabilization problem of a nonlinear system being reported in the literature [1–4, 9, 10, 14, 33]. In [33], the sufficient conditions for global asymptotic stabilization of nonlinear systems were given, and the corresponding feedback control laws were designed. Different controller design schemes have been proposed to construct feedback controllers, which make the closed-loop system dynamics converge to a fixed point or a periodic orbit. Recently, non-continuous control techniques, such as impulsive control [34–36] and intermittent control [6, 9, 10, 13, 29, 37–40], have attracted much attention. Some engineering researchers have widely focused their attention on intermittent control. Intermittent control is a feedback control method which not only shows some human control systems but also has applications in control engineering. In a periodically intermittent control, every control period consists of two parts: 'work time' for operative control and 'rest time' for inoperative control, which provides a spectrum of possibilities between the extremes of continuous-time and discrete-time control. Hence, intermittent control strategies are more economic and can simulate the real world situations better. Several nonlinear systems with intermittent control have been presented [6, 9, 10, 13, 37, 38, 40]. In [37], the synchronization of chaotic systems was studied by the intermittent feedback method. By using periodically intermittent control and free-matrix-based integral inequality, the exponential stabilization of neural network switch time-varying delay was investigated in [38]. In [9], the exponential stabilization problem for a class of uncertain nonlinear systems with state delay was solved by periodically intermittent control. The problem of delay-independently periodically intermittent stabilization for a class of time delay systems was introduced in [13]. In [40], the problem of exponential stabilization of systems with time-varying delay via periodically intermittent memory state-feedback control was studied by constructing a new Lyapunov-Krasovskii functional and employing the free-matrix-based integral inequality. In [10], the problems of stabilization and synchronization for a class of chaotic systems were discussed via intermittent control with non-fixed both control period and control width. Unfortunately, there have been few papers so far related to the topic of the exponential stabilization criteria of a nonlinear system with hybrid intermittent feedback control. This exponential stabilization of a nonlinear system remains an open problem and it has to be investigated more.

In order to solve the problem of exponential stabilization criteria for a nonlinear system with time-varying delay, most of researchers utilize the improved Lyapunov-Krasovskii functional combined with Newton-Leibniz formula [1–8, 16, 18–22, 24, 26–31, 37, 38, 41–43]; Jensen's inequality [1, 18–20, 23, 29, 42, 43], the inequality technique[18, 20, 31, 37, 42], Wirtinger's integral inequality [19, 23], Razumikhin's technique [2, 3], Gronwall-Bellman's lemma [14], and reciprocally convex combination [1, 18–20, 25, 32, 42, 43]. In [1], H_∞ control for a nonlinear system with interval time-varying delay was studied. The delay function is not necessary to be differentiable. Based on constructing new Lyapunov-Krasovskii functionals and using a new tighter bounding technique, the delay-dependent condition for this system has been established in terms of LMIs by using standard computational algorithms [44]. Thus, in this study, we focus on the reciprocally convex combination in order to solve the problem of exponential stabilization criteria for a nonlinear system with mixed time-varying delays, composed of discrete interval and distributed time-varying delay.

Inspired by the aforementioned discussion, this is the first time that the exponential stabilization criteria for a nonlinear system with mixed time-varying delay via hybrid intermittent feedback control have been studied. The main contributions of this paper lie in the following aspects. Firstly, the time-varying delays are mixture of discrete and distributed time-varying delays in a nonlinear system and hybrid intermittent feedback control. The constraint on the derivative of the time-varying delays is not required. So, this allows the time delay to be a fast time-varying function, which is different from the time delays in [7, 12, 15, 38, 40]. Secondly, for the control method, the exponential stabilization criteria for nonlinear system are studied via hybrid intermittent feedback control, containing state term, interval time-varying delay term and distributed time-varying delay term. It is different from the control method in [6, 9, 10, 13, 37, 38, 40]. From the above discussions, this work is one of the first reports of such investigation to further develop the exponential stabilization criteria for a nonlinear system with mixed time-varying delays via hybrid intermittent feedback control. By constructing the set of improved Lyapunov-Krasovskii functionals with Leibniz-Newton's formula, Jensen's inequality, and the reciprocal convex combination technique, a new delay-dependent sufficient condition of exponential stabilization criteria is established in terms of linear matrix inequalities (LMIs). The obtained LMIs are efficiently solved by standard convex optimization algorithms. A numerical example is included to show the effectiveness of the proposed hybrid intermittent feedback control scheme.

The rest of the paper is organized as follows. Section 2 provides some nonlinear system and mathematical preliminaries. Section 3 presents exponential stabilization criteria for a nonlinear system with mixed time-varying delays via hybrid intermittent feedback control. A numerical example is given in Section 4. Finally, the conclusion is provided in Section 5.

2 Problem formulation and mathematic preliminaries

Let us consider the nonlinear system with mixed time-varying delays as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bx\big(t-h(t)\big) + C\int_{t-k_1(t)}^{t} x(s)\,ds \\ &\quad + f\left(t, x(t), x\big(t-h(t)\big), \int_{t-k_1(t)}^{t} x(s)\,ds, u(t)\right) + \mathcal{U}(t), \quad t \geq 0, \\ x(t) &= \phi(t), \qquad t \in [-\tau_{\max}], \qquad \tau_{\max} = \max\{h_2, d, k_1, k_2\}, \end{aligned} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), x_3(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ is the state vector; A, B and C are known real constant matrices; $\mathcal{U}(t) \in \mathbb{R}^m$ is the control input. Let $x_h := x(t-h(t))$ and $\mathrm{Int}_x := \int_{t-k_1(t)}^{t} x(s)\,ds$, the nonlinear function $f(t, x, x_h, \mathrm{Int}_x, u) : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ satisfies the following condition: $\exists a_1, b_1, c_1, d_1 > 0$ such that

$$\big\| f(t, x, x_h, \mathrm{Int}_x, u) \big\| \leq a_1\|x\| + b_1\|x_h\| + c_1\|\mathrm{Int}_x\| + d_1\|u\|. \tag{2}$$

The time-varying delay functions $h(t)$, $d(t)$, $k_1(t)$ and $k_2(t)$ satisfy the conditions

$$0 \leq h_1 \leq h(t) \leq h_2, \qquad 0 \leq d(t) \leq d, \qquad 0 \leq k_1(t) \leq k_1, \qquad 0 \leq k_2(t) \leq k_2. \tag{3}$$

The initial condition function $\phi(t)$ denotes a continuous vector-valued initial function of $t \in [-\tau_{\max}, 0]$.

In order to stabilize the origin of nonlinear system (1), we use the state feedback controller $\mathcal{U}(t)$ satisfying

$$\mathcal{U}(t) = \begin{cases} D_1 u(t) + D_2 u(t-d(t)) + D_3 \int_{t-k_2(t)}^{t} u(s)\,ds, & n\omega \leq t \leq n\omega + \delta, \\ 0, & n\omega + \delta < t \leq (n+1)\omega, \end{cases} \tag{4}$$

where $D_i, i = 1,2,3$ are given matrices of appropriate dimensions, $u(t) = Kx(t)$ and K is a constant matrix control gain, $\omega > 0$ is the control period and $\delta > 0$ is the control width (control duration) and n is a non-negative integer. Then, substituting it into nonlinear system (1), it is easy to get the following:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bx\bigl(t-h(t)\bigr) + C\int_{t-k_1(t)}^{t} x(s)\,ds \\ &\quad + f\left(t, x(t), x\bigl(t-h(t)\bigr), \int_{t-k_1(t)}^{t} x(s)\,ds, u(t)\right) \\ &\quad + D_1 u(t) + D_2 u\bigl(t-d(t)\bigr) + D_3 \int_{t-k_2(t)}^{t} u(s)\,ds, \quad n\omega \leq t \leq n\omega + \delta, \\ \dot{x}(t) &= Ax(t) + Bx\bigl(t-h(t)\bigr) + C\int_{t-k_1(t)}^{t} x(s)\,ds \\ &\quad + f\left(t, x(t), x\bigl(t-h(t)\bigr), \int_{t-k_1(t)}^{t} x(s)\,ds, u(t)\right), \quad n\omega + \delta < t \leq (n+1)\omega. \end{aligned} \tag{5}$$

It is clear that if the zero solution of nonlinear system (5) is globally exponentially stable, the exponential stabilization of the controlled nonlinear system (1) is achieved.

The following lemmas and theorem are used in the proof of the main result.

Lemma 1 ([43]) *For any constant symmetric matrix $M \in R^{n\times n}$, $M = M^T > 0$, $0 \leq h_1 \leq h(t) \leq h_2$, $t \geq 0$, and any differentiable vector function $x(t) \in R^n$, we have*

$$\text{(a)} \quad \left[\int_{t-h_1}^{t} \dot{x}(s)\,ds\right]^T M \left[\int_{t-h_1}^{t} \dot{x}(s)\,ds\right] \leq h_1 \int_{t-h_1}^{t} \dot{x}^T(s) M \dot{x}(s)\,ds,$$

$$\begin{aligned} \text{(b)} \quad \left[\int_{t-h(t)}^{t-h_1} \dot{x}(s)\,ds\right]^T M \left[\int_{t-h(t)}^{t-h_1} \dot{x}(s)\,ds\right] &\leq \bigl(h(t) - h_1\bigr) \int_{t-h(t)}^{t-h_1} \dot{x}^T(s) M \dot{x}(s)\,ds \\ &\leq (h_2 - h_1) \int_{t-h(t)}^{t-h_1} \dot{x}^T(s) M \dot{x}(s)\,ds. \end{aligned}$$

Lemma 2 (Cauchy inequality, [45]) *For any symmetric positive definite matrix $N \in M^{n\times n}$ and $x, y \in \mathbb{R}^n$, we have*

$$\pm 2x^T y \leq x^T N x + y^T N^{-1} y.$$

Lemma 3 (Schur complement, [46]) *Given constant symmetric matrices X, Y, Z where $X = X^T$ and $0 < Y = Y^T$, then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0, \quad or \quad \begin{bmatrix} -Y & Z \\ Z^T & X \end{bmatrix} < 0.$$

Theorem 4 (Lower bounds theorem, [42]) *Let $f_1, f_2, \ldots, f_N : R^m \to R$ have positive values in an open subset D of R^m. Then the reciprocally convex combination of f_i over D satisfies*

$$\min_{\rho_i | \rho_i > 0, \sum_i \rho_i = 1} \sum_i \frac{1}{\rho_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j}(t) : R^m \to R, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \right\}.$$

3 Exponential stabilization of a delayed nonlinear system via hybrid intermittent feedback control

In this section, we present delay-dependent exponential stabilization analysis conditions for the nonlinear system with interval discrete and distributed time-varying delays via hybrid intermittent feedback control. Let us denote

$$\begin{aligned}
&\|\phi(t)\| = \|x(0)\|, \qquad \|\varphi(t)\| = \sup_{-\tau_{\max} \leq s \leq 0} \|x(s)\|, \qquad K = -LP^{-1}, \qquad y_h = y\bigl(t - h(t)\bigr), \\
&\mathcal{M}_1 = \lambda_{\max}\bigl(P^{-1}\bigr) + \bigl[2h_2 \lambda_{\max}\bigl(P^{-1} R P^{-1}\bigr) + h_2 \lambda_{\max}\bigl(P^{-1} U P^{-1}\bigr)\bigr]\left(\frac{1 - e^{-2\alpha h_2}}{2\alpha}\right) \\
&\qquad + d\lambda_{\max}\bigl(P^{-1} L^T T^{-1} L P^{-1}\bigr)\left(\frac{1 - e^{-2\alpha d}}{2\alpha}\right) \\
&\qquad + \lambda_{\max}\bigl(P^{-1} Z P^{-1}\bigr)\left[\frac{1}{4\alpha}\left(h_2 - \left(\frac{1 - e^{-4\alpha h_2}}{4\alpha}\right)\right)\right], \\
&\mathcal{M}_2 = \bigl[2\lambda_{\max}\bigl(P^{-1} Q P^{-1}\bigr) + 2h_2 \lambda_{\max}\bigl(P^{-1} R P^{-1}\bigr) + h_2 \lambda_{\max}\bigl(P^{-1} U P^{-1}\bigr)\bigr]\left(\frac{1 - e^{-2\alpha h_2}}{2\alpha}\right) \\
&\qquad + k_1 \lambda_{\max}\bigl(P^{-1} S P^{-1}\bigr)\left(\frac{1 - e^{-2\alpha k_1}}{2\alpha}\right) \\
&\qquad + d\lambda_{\max}\bigl(P^{-1} L^T T^{-1} L P^{-1}\bigr)\left(\frac{1 - e^{-2\alpha d}}{2\alpha}\right) + k_2 \lambda_{\max}\bigl(P^{-1} L^T W^{-1} L P^{-1}\bigr)\left(\frac{1 - e^{-2\alpha k_2}}{2\alpha}\right) \\
&\qquad + \lambda_{\max}\bigl(P^{-1} Z P^{-1}\bigr)\left[\frac{1}{4\alpha}\left(h_2 - \left(\frac{1 - e^{-4\alpha h_2}}{4\alpha}\right)\right)\right], \\
&\mathcal{M} = \mathcal{M}_1 \|\phi(t)\|^2 + \mathcal{M}_2 \|\varphi(t)\|^2.
\end{aligned}$$

Theorem 5 *For some given scalar $0 < \alpha < \varepsilon$, nonlinear system* (1) *with time-varying delay satisfying* (3) *and under the intermittent controller* (4) *is exponentially stabilizable if there exist positive constant ϵ and symmetric positive definite matrices $P > 0$, $Q > 0$, $R > 0$, $S > 0$, $U > 0$, $T > 0$, $W > 0$ and matrices L, S_1 appropriately dimensioned so that the following* *symmetric linear matrix inequalities hold*:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} \\ * & \Pi_{22} & 0 & \Pi_{24} & 0 & 0 \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} & 0 \\ * & * & * & \Pi_{44} & \Pi_{45} & 0 \\ * & * & * & * & \Pi_{55} & 0 \\ * & * & * & * & * & \Pi_{66} \end{bmatrix} < 0, \tag{6}$$

$$\widetilde{\Pi} = \begin{bmatrix} \widetilde{\Pi}_{11} & \widetilde{\Pi}_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} \\ * & \widetilde{\Pi}_{22} & 0 & \Pi_{24} & 0 & 0 \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} & 0 \\ * & * & * & \Pi_{44} & \Pi_{45} & 0 \\ * & * & * & * & \Pi_{55} & 0 \\ * & * & * & * & * & \Pi_{66} \end{bmatrix} < 0, \tag{7}$$

$$\Pi_1 = \begin{bmatrix} \Pi_{111} & 4k_1CP & k_2L^T & 2L^T & d_1L & \epsilon P & 4k_1c_1^2P \\ * & -4k_1e^{-2\alpha k_1}S & 0 & 0 & 0 & 0 & 0 \\ * & * & -k_2W & 0 & 0 & 0 & 0 \\ * & * & * & -2e^{2\alpha d}T & 0 & 0 & 0 \\ * & * & * & * & -d_1 & 0 & 0 \\ * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & -4k_1c_1^2e^{-2\alpha k_1}S \end{bmatrix} < 0, \tag{8}$$

$$\Pi_2 = \begin{bmatrix} -0.5P & 4k_1CP & d^2L^T & d_1L & \epsilon P & 4k_1c_1^2P \\ * & -4k_1e^{-2\alpha k_1}S & 0 & 0 & 0 & 0 \\ * & * & -d^2T & 0 & 0 & 0 \\ * & * & * & -d_1 & 0 & 0 \\ * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & -4k_1c_1^2e^{-2\alpha k_1}S \end{bmatrix} < 0, \tag{9}$$

$$\Pi_3 = \begin{bmatrix} \Pi_{111} & 4k_1CP & d_1L & \epsilon P & 4k_1c_1^2P \\ * & -4k_1e^{-2\alpha k_1}S & 0 & 0 & 0 \\ * & * & -d_1 & 0 & 0 \\ * & * & * & -\epsilon I & 0 \\ * & * & * & * & -4k_1c_1^2e^{-2\alpha k_1}S \end{bmatrix} < 0, \tag{10}$$

$$\Pi_4 = \begin{bmatrix} -0.5P & 4k_1CP & d_1L & \epsilon P & 4k_1c_1^2P \\ * & -4k_1e^{-2\alpha k_1}S & 0 & 0 & 0 \\ * & * & -d_1 & 0 & 0 \\ * & * & * & -\epsilon I & 0 \\ * & * & * & * & -4k_1c_1^2e^{-2\alpha k_1}S \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} U & S_1 \\ S_1^T & U \end{bmatrix} \geq 0, \tag{12}$$

and

$$-\alpha\delta + (\varepsilon - \alpha)(\omega - \delta) < 0, \tag{13}$$

where

$$\begin{aligned}
\Pi_{111} &= -0.5\big(e^{-2\alpha h_1} + e^{-2\alpha h_2}\big)R,\\
\Pi_{11} &= P^T(A+\alpha I) + (A+\alpha I)^T P + 2(a_1+d_1)I - D_1L - L^T D_1^T + 3e^{2\alpha d} D_2 T D_2^T\\
&\quad + 2k_2 e^{2\alpha k_2} D_3 W D_3^T + 2Q + k_1 S - 0.5\big(e^{-2\alpha h_1} + e^{-2\alpha h_2}\big)R - 2\frac{(h_2-h_1)^2}{h_2^2-h_1^2} e^{-4\alpha h_2} Z,\\
\widetilde{\Pi}_{11} &= P^T(A+\alpha I) + (A+\alpha I)^T P + 2(a_1+d_1)I + 2Q + k_1 S\\
&\quad - 0.5\big(e^{-2\alpha h_1} + e^{-2\alpha h_2}\big)R - 2\frac{(h_2-h_1)^2}{h_2^2-h_1^2} e^{-4\alpha h_2} Z - 2\varepsilon P,\\
\Pi_{12} &= AP - D_1 L,\\
\widetilde{\Pi}_{12} &= AP,\\
\Pi_{13} &= e^{-2\alpha h_1} R,\\
\Pi_{14} &= BP,\\
\Pi_{15} &= e^{-2\alpha h_2} R,\\
\Pi_{16} &= \frac{2e^{-4\alpha h_2}}{h_2+h_1} Z,\\
\Pi_{22} &= \big(h_1^2 + h_2^2\big)R + (h_2-h_1)^2 U + (h_2-h_1)Z - 1.5P + 3e^{2\alpha d} D_2 T D_2^T\\
&\quad + 2k_2 e^{2\alpha k_2} D_3 W D_3^T,\\
\widetilde{\Pi}_{22} &= \big(h_1^2 + h_2^2\big)R + (h_2-h_1)^2 U + (h_2-h_1)Z - 1.5P,\\
\Pi_{24} &= BP,\\
\Pi_{33} &= -e^{-2\alpha h_1} Q - e^{-2\alpha h_1} R - e^{-2\alpha h_2} U,\\
\Pi_{34} &= e^{-2\alpha h_2} U - e^{-2\alpha h_2} S_1^T,\\
\Pi_{35} &= e^{-2\alpha h_2} S_1^T,\\
\Pi_{44} &= 2b_1 I - 2e^{-2\alpha h_2} U + e^{-2\alpha h_2} S_1 + e^{-2\alpha h_2} S_1^T,\\
\Pi_{45} &= e^{-2\alpha h_2} U - e^{-2\alpha h_2} S_1^T,\\
\Pi_{55} &= -e^{-2\alpha h_1} Q - e^{-2\alpha h_1} R - e^{-2\alpha h_2} U,\\
\Pi_{66} &= -\frac{2e^{-4\alpha h_2}}{h_2^2+h_1^2} Z.
\end{aligned}$$

Moreover, the intermittent feedback control is

$$u(t) = \begin{cases} -LP^{-1}x(t), & n\omega \le t \le n\omega + \delta,\\ 0, & n\omega + \delta < t \le (n+1)\omega, \end{cases} \tag{14}$$

and the solution $x(t,\phi)$ satisfies

$$\left\| \big(x(t,\phi)\big) \right\| \leq \sqrt{\frac{\mathcal{M}}{\lambda_{\min}(P^{-1})} e^{\frac{(-\alpha\delta+(\varepsilon-\alpha)(\omega-\delta))t}{\omega}}}, \quad \forall t \geq 0.$$

Proof Case I: For $n\omega \leq t \leq n\omega + \delta$, let $Y = P^{-1}$ and $y(t) = Yx(t)$. Using the feedback control (14), let us consider the following Lyapunov-Krasovskii functional:

$$V\big(x(t)\big) = \sum_{i=1}^{10} V_i(t), \tag{15}$$

where

$$\begin{aligned}
V_1(t) &= x^T(t)Yx(t), \\
V_2(t) &= \int_{t-h_1}^{t} e^{2\alpha(s-t)} x^T(s)YQYx(s)\,ds, \\
V_3(t) &= \int_{t-h_2}^{t} e^{2\alpha(s-t)} x^T(s)YQYx(s)\,ds, \\
V_4(t) &= h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau)YRY\dot{x}(\tau)\,d\tau\,ds, \\
V_5(t) &= h_2 \int_{-h_2}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau)YRY\dot{x}(\tau)\,d\tau\,ds, \\
V_6(t) &= (h_2-h_1) \int_{-h_2}^{-h_1} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau)YUY\dot{x}(\tau)\,d\tau\,ds, \\
V_7(t) &= \int_{-k_1}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} x^T(\tau)YSYx(\tau)\,d\tau\,ds, \\
V_8(t) &= d \int_{-d}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau)K^T T^{-1} K\dot{x}(\tau)\,d\tau\,ds, \\
V_9(t) &= \int_{-k_2}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} x^T(\tau)K^T W^{-1} Kx(\tau)\,d\tau\,ds, \\
V_{10}(t) &= \int_{-h_1}^{-h_2} \int_{\theta}^{0} \int_{t+s}^{t} e^{2\alpha(\tau+s-t)} \dot{x}^T(\tau)YZY\dot{x}(\tau)\,d\tau\,ds\,d\theta.
\end{aligned}$$

It is easy to check that

$$\lambda_{\min}\big(P^{-1}\big)\left\|x(t)\right\|^2 \leq V\big(x(t)\big). \tag{16}$$

By taking the derivatives of $V_1(t)$ along the trajectories of system (5), we have

$$\begin{aligned}
\dot{V}_1(t) &= y^T(t)\big[PA + A^T P\big]y(t) + 2y^T(t)BPy\big(t-h(t)\big) + 2y^T(t)CP\int_{t-k_1(t)}^{t} y(s)\,ds \\
&\quad + 2y^T(t)f\left(t, x(t), x\big(t-h(t)\big), \int_{t-k_1(t)}^{t} x(s)\,ds, u(t)\right) \\
&\quad - 2y^T(t)D_1Ly(t) + 2y^T(t)D_2u\big(t-d(t)\big) + 2y^T(t)D_3\int_{t-k_2(t)}^{t} u(s)\,ds.
\end{aligned}$$

By applying Lemma 2 and Lemma 1, we get

$$\begin{aligned}
2y^T(t)D_2u\big(t-d(t)\big) &\le 3e^{2\alpha d}y^T(t)D_2TD_2^Ty(t) \\
&\quad + \frac{1}{3}e^{-2\alpha d}u^T\big(t-d(t)\big)T^{-1}u\big(t-d(t)\big), \\
2y^T(t)D_3\int_{t-k_2(t)}^t u(s)\,ds &\le 2k_2e^{2\alpha k_2}y^T(t)D_3WD_3^Ty(t) \\
&\quad + \frac{1}{2}e^{-2\alpha k_2}\int_{t-k_2(t)}^t u^T(s)W^{-1}u(s)\,ds, \\
2y^T(t)CP\int_{t-k_1(t)}^t y(s)\,ds &\le 4k_1e^{2\alpha k_1}y^T(t)CPS^{-1}PC^Ty(t) \\
&\quad + \frac{1}{4}e^{-2\alpha k_1}\int_{t-k_1(t)}^t y^T(s)Sy(s)\,ds.
\end{aligned}$$

Let $\epsilon = a_1 + b_1$. By utilizing condition (2) and Lemma 2, we obtain

$$\begin{aligned}
2y^T(t)f(t,x,x_h,\mathrm{Int}_x,u) &\le 2\big\|y(t)\big\|\big(a_1\|x\| + b_1\|x_h\| + c_1\|\,\mathrm{Int}_x\,\| + d_1\|u\|\big) \\
&\le a_1\big\|Py(t)\big\|^2 + a_1\big\|y(t)\big\|^2 + b_1\big\|Py(t)\big\|^2 + b_1\|y_h\|^2 \\
&\quad + 4k_1c_1^2e^{2\alpha k_1}y^T(t)PS^{-1}Py(t) \\
&\quad + \frac{1}{4}e^{-2\alpha k_1}\int_{t-k_1(t)}^t y^T(s)Sy(s)\,ds \\
&\quad + d_1y^T(t)LL^Ty(t) + d_1y^T(t)y(t) \\
&= a_1y^T(t)y(t) + b_1y_h^Ty_h + 4k_1c_1^2e^{2\alpha k_1}y^T(t)PS^{-1}Py(t) \\
&\quad + \frac{1}{4}e^{-2\alpha k_1}\int_{t-k_1(t)}^t y^T(s)Sy(s)\,ds + d_1y^T(t)LL^Ty(t) \\
&\quad + d_1y^T(t)y(t) + \epsilon P^2y^T(t)y(t),
\end{aligned}$$

therefore,

$$\begin{aligned}
\dot{V}_1(t) + 2\alpha V_1(t) &\le y^T(t)\big[PA + A^TP\big]y(t) + 2y^T(t)\alpha Py(t) + a_1y^T(t)y(t) \\
&\quad - 2y^T(t)D_1Ly(t) + d_1y^T(t)LL^Ty(t) + d_1y^T(t)y(t) \\
&\quad + \epsilon P^2y^T(t)y(t) + 4k_1e^{2\alpha k_1}y^T(t)CPS^{-1}PC^Ty(t) \\
&\quad + \frac{1}{2}e^{-2\alpha k_1}\int_{t-k_1(t)}^t y^T(s)Sy(s)\,ds + 2y^T(t)BPy_h + b_1y_h^Ty_h \\
&\quad + 3e^{2\alpha d}y^T(t)D_2TD_2^Ty(t) + \frac{1}{3}e^{-2\alpha d}u^T\big(t-d(t)\big)T^{-1}u\big(t-d(t)\big) \\
&\quad + 2k_2e^{2\alpha k_2}y^T(t)D_3WD_3^Ty(t) + 4k_1c_1^2e^{2\alpha k_1}y^T(t)PS^{-1}Py(t) \\
&\quad + \frac{1}{2}e^{-2\alpha k_2}\int_{t-k_2(t)}^t u^T(s)W^{-1}u(s)\,ds.
\end{aligned} \tag{17}$$

Next, by taking the derivative of $V_i, i = 2, 3, \dots, 9, 10$, along the trajectories of system (5), we have the following:

$$\begin{aligned}
\dot{V}_2(t) &= -2\alpha V_2(t) + y^T(t)Qy(t) - e^{-2\alpha h_1} y^T(t-h_1)Qy(t-h_1),\\
\dot{V}_3(t) &= -2\alpha V_3(t) + y^T(t)Qy(t) - e^{-2\alpha h_2} y^T(t-h_2)Qy(t-h_2),\\
\dot{V}_4(t) &\le -2\alpha V_4(t) + h_1^2 \dot{y}^T(t)R\dot{y}(t) - h_1 e^{-2\alpha h_1}\int_{t-h_1}^{t} \dot{y}^T(s)R\dot{y}(s)\,ds,\\
\dot{V}_5(t) &\le -2\alpha V_5(t) + h_2^2 \dot{y}^T(t)R\dot{y}(t) - h_2 e^{-2\alpha h_2}\int_{t-h_2}^{t} \dot{y}^T(s)R\dot{y}(s)\,ds,\\
\dot{V}_6(t) &\le -2\alpha V_6(t) + (h_2-h_1)^2 \dot{y}^T(t)U\dot{y}(t) - (h_2-h_1)e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1} \dot{y}^T(s)U\dot{y}(s)\,ds,\\
\dot{V}_7(t) &\le -2\alpha V_7(t) + k_1 y^T(t)Sy(t) - e^{-2\alpha k_1}\int_{t-k_1(t)}^{t} y^T(s)Sy(s)\,ds,\\
\dot{V}_8(t) &\le -2\alpha V_8(t) + d^2 \dot{y}^T(t)L^T T^{-1} L\dot{y}(t) - de^{-2\alpha d}\int_{t-d(t)}^{t} \dot{u}^T(s)T^{-1}\dot{u}(s)\,ds,\\
\dot{V}_9(t) &\le -2\alpha V_9(t) + k_2 y^T(t)L^T W^{-1} Ly(t) - e^{-\alpha k_2}\int_{t-k_2(t)}^{t} u^T(s)W^{-1}u(s)\,ds,\\
\dot{V}_{10}(t) &\le -2\alpha V_{10}(t) + (h_2-h_1)\dot{y}^T(t)Z\dot{y}(t) - e^{-4\alpha h_2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{y}^T(s)Z\dot{y}(s)\,ds\,d\theta.
\end{aligned} \tag{18}$$

Applying Lemma 1 and the Leibniz-Newton formula, we get

$$\begin{aligned}
-h_1 e^{-2\alpha h_1}\int_{t-h_1}^{t} \dot{y}^T(s)R\dot{y}(s)\,ds &\le -e^{-2\alpha h_1} y^T(t)Ry(t) + 2e^{-2\alpha h_1} y^T(t)Ry(t-h_1)\\
&\quad - e^{-2\alpha h_1} y^T(t-h_1)Ry(t-h_1),
\end{aligned} \tag{19}$$

and

$$\begin{aligned}
-h_2 e^{-2\alpha h_2}\int_{t-h_2}^{t} \dot{y}^T(s)R\dot{y}(s)\,ds &\le -e^{-2\alpha h_2} y^T(t)Ry(t) + 2e^{-2\alpha h_2} y^T(t)Ry(t-h_2)\\
&\quad - e^{-2\alpha h_2} y^T(t-h_2)Ry(t-h_2).
\end{aligned} \tag{20}$$

Similarly,

$$\begin{aligned}
&-(h_2-h_1)e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1} \dot{y}^T(s)U\dot{y}(s)\,ds\\
&\quad = -(h_2-h_1)e^{-2\alpha h_2}\left(\int_{t-h_2}^{t-h(t)} \dot{y}^T(s)U\dot{y}(s)\,ds + \int_{t-h(t)}^{t-h_1} \dot{y}^T(s)U\dot{y}(s)\,ds\right)\\
&\quad \le -\frac{h_2-h_1}{h_2-h(t)} e^{-2\alpha h_2}\big[y_h - y(t-h_2)\big]^T U\big[y_h - y(t-h_2)\big]\\
&\qquad - \frac{h_2-h_1}{h(t)-h_1} e^{-2\alpha h_2}\big[y(t-h_1) - y_h\big]^T U\big[y(t-h_1) - y_h\big].
\end{aligned}$$

Let $\rho_1 = \frac{h_2-h(t)}{h_2-h_1}$ and $\rho_2 = \frac{h(t)-h_1}{h_2-h_1}$, apply Theorem 4, which is

$$\begin{bmatrix} U & S_1 \\ S_1^T & U \end{bmatrix} \geq 0,$$

we have the following inequality:

$$\begin{bmatrix} \sqrt{\frac{\rho_2}{\rho_1}}[y_h - y(t-h_2)] \\ -\sqrt{\frac{\rho_1}{\rho_2}}[y(t-h_1) - y_h] \end{bmatrix}^T \begin{bmatrix} U & S_1 \\ S_1^T & U \end{bmatrix} \begin{bmatrix} \sqrt{\frac{\rho_2}{\rho_1}}[y_h - y(t-h_2)] \\ -\sqrt{\frac{\rho_1}{\rho_2}}[y(t-h_1) - y_h] \end{bmatrix} \geq 0.$$

It follows that

$$\begin{aligned} &-\frac{\rho_2}{\rho_1}\big[y_h - y(t-h_2)\big]^T U\big[y_h - y(t-h_2)\big] - \frac{\rho_1}{\rho_2}\big[y(t-h_1) - y_h\big]^T U\big[y(t-h_1) - y_h\big] \\ &\quad \leq -\big[y_h - y(t-h_2)\big]^T S_1\big[y(t-h_1) - y_h\big] - \big[y(t-h_1) - y_h\big]^T S_1^T\big[y_h - y(t-h_2)\big], \end{aligned}$$

as a result, we have

$$\begin{aligned} &-(h_2-h_1)e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1} \dot{y}^T(s)U\dot{y}(s)\,ds \\ &\quad \leq -\frac{1}{\rho_1}e^{-2\alpha h_2}\big[y_h - y(t-h_2)\big]^T U\big[y_h - y(t-h_2)\big] \\ &\qquad - \frac{1}{\rho_2}e^{-2\alpha h_2}\big[y(t-h_1) - y_h\big]^T U\big[y(t-h_1) - y_h\big] \\ &\quad \leq -e^{-2\alpha h_2}\big[y_h - y(t-h_2)\big]^T U\big[y_h - y(t-h_2)\big] \\ &\qquad - e^{-2\alpha h_2}\big[y(t-h_1) - y_h\big]^T U\big[y(t-h_1) - y_h\big] \\ &\qquad - e^{-2\alpha h_2}\big[y_h - y(t-h_2)\big]^T S_1\big[y(t-h_1) - y_h\big] \\ &\qquad - e^{-2\alpha h_2}\big[y(t-h_1) - y_h\big]^T S_1^T\big[y_h - y(t-h_2)\big]. \end{aligned} \tag{21}$$

From $\dot{V}_8(t)$ and $\dot{V}_{10}(t)$, applying Lemma 1 and the Leibniz-Newton formula, we get

$$\begin{aligned} &-de^{-2\alpha d}\int_{t-d(t)}^{t} \dot{u}^T(s)T^{-1}\dot{u}(s)\,ds \\ &\quad \leq -e^{-2\alpha d}u^T(t)T^{-1}u(t) + 2e^{-2\alpha d}u^T(t)T^{-1}u\big(t-d(t)\big) \\ &\qquad - e^{-2\alpha d}u^T\big(t-d(t)\big)T^{-1}u\big(t-d(t)\big) \\ &\quad = 2e^{-2\alpha d}y^T(t)L^T T^{-1}Ly(t) - \frac{2e^{-2\alpha d}}{3}u^T\big(t-d(t)\big)T^{-1}u\big(t-d(t)\big) \end{aligned} \tag{22}$$

and

$$\begin{aligned} &-e^{-4\alpha h_2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{y}^T(s)Z\dot{y}(s)\,ds\,d\theta \\ &\quad \leq -\frac{2}{h_2^2-h_1^2}e^{-4\alpha h_2}\bigg[\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{y}(s)\,ds\,d\theta\bigg]^T Z\bigg[\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{y}(s)\,ds\,d\theta\bigg] \end{aligned}$$

$$\begin{aligned}
&= -\frac{2}{h_2^2 - h_1^2} e^{-4\alpha h_2} (h_2 - h_1)^2 y^T(t) Z y(t) \\
&\quad + \frac{4}{h_2 + h_1} e^{-4\alpha h_2} y^T(t) Z \left(\int_{t-h_2}^{t-h_1} y(\theta)\, d\theta \right) \\
&\quad - \frac{2}{h_2^2 - h_1^2} e^{-4\alpha h_2} \left(\int_{t-h_2}^{t-h_1} y(\theta)\, d\theta \right)^T Z \left(\int_{t-h_2}^{t-h_1} y(\theta)\, d\theta \right). \qquad (23)
\end{aligned}$$

By using the following identity relation:

$$\begin{aligned}
&-\dot{x}(t) + Ax(t) + Bx\big(t - h(t)\big) + C \int_{t-k_1(t)}^{t} x(s)\, ds + D_1 u(t) + D_2 u\big(t - d(t)\big) \\
&\quad + f\left(t, x(t), x\big(t - h(t)\big), \int_{t-k_1(t)}^{t} x(s)\, ds, u(t) \right) + D_3 \int_{t-k_2(t)}^{t} u(s)\, ds = 0,
\end{aligned}$$

multiplying by $2\dot{y}^T(t)$, we get

$$\begin{aligned}
&-2\dot{y}^T(t) P \dot{y}(t) + 2\dot{y}^T(t) A P y(t) + 2\dot{y}^T(t) B P y\big(t - h(t)\big) - 2\dot{y}^T(t) D_1 L y(t) \\
&\quad + 2\dot{y}^T(t) C P \int_{t-k_1(t)}^{t} y(s)\, ds + 2\dot{y}^T(t) D_2 u\big(t - d(t)\big) + 2\dot{y}^T(t) D_3 \int_{t-k_2(t)}^{t} u(s)\, ds \\
&\quad + 2\dot{y}^T(t) f\left(t, x(t), x\big(t - h(t)\big), \int_{t-k_1(t)}^{t} x(s)\, ds, u(t) \right) = 0. \qquad (24)
\end{aligned}$$

Applying Lemma 2 and Lemma 1, we obtain

$$\begin{aligned}
&2\dot{y}^T(t) C P \int_{t-k_1(t)}^{t} y(s)\, ds \le 4k_1 e^{2\alpha k_1} \dot{y}^T(t) C P S^{-1} P C^T \dot{y}(t) + \frac{1}{4} e^{-2\alpha k_1} \int_{t-k_1(t)}^{t} y^T(s) S y(s)\, ds, \\
&2\dot{y}^T(t) D_2 u\big(t - d(t)\big) \le 3e^{2\alpha d} \dot{y}^T(t) D_2 T D_2^T \dot{y}(t) \\
&\qquad\qquad + \frac{1}{3} e^{-2\alpha d} u^T\big(t - d(t)\big) T^{-1} u\big(t - d(t)\big), \qquad (25) \\
&2\dot{y}^T(t) D_3 \int_{t-k_2(t)}^{t} u(s)\, ds \le 2k_2 e^{2\alpha k_2} \dot{y}^T(t) D_3 W D_3^T \dot{y}(t) \\
&\qquad\qquad + \frac{1}{2} e^{-2\alpha k_2} \int_{t-k_2(t)}^{t} u^T(s) W^{-1} u(s)\, ds.
\end{aligned}$$

By utilizing condition (2) and Lemma 2, we have

$$\begin{aligned}
2\dot{y}^T(t) f(t, x, x_h, \mathrm{Int}_x, u) &\le 2\big\|\dot{y}(t)\big\| \big(a_1 \|x\| + b_1 \|x_h\| + c_1 \| \mathrm{Int}_x \| + d_1 \|u\| \big) \\
&= 2a_1 \big\|\dot{y}(t)\big\| \big\|P y(t)\big\| + 2b_1 \big\|\dot{y}(t)\big\| \|P y_h\| \\
&\quad + 2c_1 \big\|\dot{y}(t)\big\| \|P\, \mathrm{Int}_y \| + d_1 \big\|\dot{y}(t)\big\| \|u\| \\
&\le a_1 y^T(t) y(t) + b_1 y_h^T y_h + 4k_1 c_1^2 e^{2\alpha k_1} \dot{y}^T(t) P S^{-1} P \dot{y}(t) \\
&\quad + \frac{1}{4} e^{-2\alpha k_1} \int_{t-k_1(t)}^{t} y^T(s) S y(s)\, ds + d_1 \dot{y}^T(t) L L^T \dot{y}(t) \\
&\quad + d_1 y^T(t) y(t) + \epsilon P^2 \dot{y}^T(t) \dot{y}(t). \qquad (26)
\end{aligned}$$

Hence, according to (17)-(23) and adding the zero items of (24)-(26), it follows that

$$\dot{V}\big(x(t)\big) + 2\alpha V\big(x(t)\big) \leq \xi^T(t)\Pi\xi(t) + y^T(t)\mathcal{N}_1 y(t) + \dot{y}^T(t)\mathcal{N}_2\dot{y}(t), \tag{27}$$

where Π is defined as in (6) and

$$\begin{aligned}
\xi^T(t) &= \left[y^T(t)\dot{y}^T(t)y^T(t-h_1)y_h^T y^T(t-h_2)\int_{t-h_2}^{t-h_1} y(\theta)\,d\theta\right],\\
\mathcal{N}_1 &= -0.5\big(e^{-2\alpha h_1} + e^{-2\alpha h_2}\big)R + 4k_1e^{2\alpha k_1}CPS^{-1}PC^T + k_2L^TW^{-1}L\\
&\quad + 2e^{-2\alpha d}L^TT^{-1}L + d_1LL^T + \epsilon P^2 + 4k_1c_1^2e^{2\alpha k_1}PS^{-1}P,\\
\mathcal{N}_2 &= -0.5P + 4k_1e^{2\alpha k_1}CPS^{-1}PC^T + d^2L^TT^{-1}L + d_1LL^T + \epsilon P^2 + 4k_1c_1^2e^{2\alpha k_1}PS^{-1}P.
\end{aligned}$$

Applying Lemma 3, the inequalities $\mathcal{N}_1 < 0$ and $\mathcal{N}_2 < 0$ are equivalent to $\Pi_1 < 0$ and $\Pi_2 < 0$, respectively. Therefore, it follows from (6), (8)-(9) and (27) that

$$\dot{V}\big(x(t)\big) + 2\alpha V\big(x(t)\big) \leq 0, \quad n\omega \leq t \leq n\omega + \delta. \tag{28}$$

Thus, equation (28) can be reduced to the following form:

$$V\big(x(t)\big) \leq V\big(x(n\omega)\big)e^{-2\alpha(t-n\omega)}, \quad n\omega \leq t \leq n\omega + \delta. \tag{29}$$

Case II: For $n\omega + \delta < t \leq (n+1)\omega$, we choose the Lyapunov-Krasovskii functional having the following form:

$$V\big(x(t)\big) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) + V_7(t) + V_{10}(t),$$

where $V_i(t), i = 1, 2, \ldots, 7$ and 10 are defined similarly as in (15). Using a method similar to that of Case I, we get

$$\begin{aligned}
&\dot{V}\big(x(t)\big) + 2\alpha V\big(x(t)\big)\\
&\quad \leq \xi^T(t)\widetilde{\Pi}\xi(t) + y^T(t)\mathcal{N}_3 y(t) + \dot{y}^T(t)\mathcal{N}_4\dot{y}(t)\\
&\quad \leq \xi^T(t)\widetilde{\Pi}\xi(t) + y^T(t)\mathcal{N}_3 y(t)\\
&\qquad + \dot{y}^T(t)\mathcal{N}_4\dot{y}(t) + 2\varepsilon V\big(x(t)\big) - 2\varepsilon V_1\big(x(t)\big)\\
&\quad = \xi^T(t)\widetilde{\Pi}\xi(t) + y^T(t)\mathcal{N}_3 y(t)\\
&\qquad + \dot{y}^T(t)\mathcal{N}_4\dot{y}(t) + 2\varepsilon V\big(x(t)\big) - 2\varepsilon y^T(t)Py(t),\\
&\dot{V}\big(x(t)\big) - 2(\varepsilon - \alpha)V\big(x(t)\big)\\
&\quad \leq \xi^T(t)\widetilde{\Pi}\xi(t) + y^T(t)(\mathcal{N}_3 - 2\varepsilon P)y(t) + \dot{y}^T(t)\mathcal{N}_4\dot{y}(t),
\end{aligned} \tag{30}$$

where $\widetilde{\Pi}$ is defined as in (7) and

$$\begin{aligned}
\mathcal{N}_3 &= -0.5\big(e^{-2\alpha h_1} + e^{-2\alpha h_2}\big)R + 4k_1e^{2\alpha k_1}CPS^{-1}PC^T + d_1LL^T + \epsilon P^2 + 4k_1c_1^2e^{2\alpha k_1}PS^{-1}P,\\
\mathcal{N}_4 &= -0.5P + 4k_1e^{2\alpha k_1}CPS^{-1}PC^T + d_1LL^T + \epsilon P^2 + 4k_1c_1^2e^{2\alpha k_1}PS^{-1}P.
\end{aligned}$$

Applying Lemma 3, the inequalities $(\mathcal{N}_3 - 2\varepsilon P) < 0$ and $\mathcal{N}_4 < 0$ are equivalent to $\Pi_3 < 0$ and $\Pi_4 < 0$, respectively. Therefore, it follows from (7), (10)-(11) and (30) that

$$\dot{V}(x(t)) - 2(\varepsilon - \alpha)V(x(t)) \leq 0, \quad n\omega + \delta < t \leq (n+1)\omega. \tag{31}$$

From the above differential inequality (31), we have

$$V(x(t)) \leq V(x(n\omega + \delta))e^{2(\varepsilon-\alpha)(t-n\omega-\delta)}, \quad n\omega + \delta < t \leq (n+1)\omega. \tag{32}$$

From (29) and (32), it follows that

$$\begin{aligned}
V(x(n+1)\omega) &\leq V(x(n\omega+\delta))e^{2(\varepsilon-\alpha)(\omega-\delta)} \\
&\leq V(x(n\omega))e^{-2\alpha\delta}e^{2(\varepsilon-\alpha)(\omega-\delta)} \\
&= V(x(n\omega))e^{-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta)} \\
&\leq V(x((n-1)\omega))e^{2(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))} \\
&\ \vdots \\
&\leq V(x((0)\omega))e^{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))(n+1)}.
\end{aligned}$$

For any $t > 0$, there is $n_0 \geq 0$ such that $n_0\omega + \delta < t \leq (n_0+1)\omega$.

Case 1. For $n_0\omega \leq t \leq n_0\omega + \delta$, using condition (13), we have

$$\begin{aligned}
V(x(t)) &\leq V(x(n_0\omega))e^{-2\alpha(t-(n_0\omega+\delta))} \\
&\leq V(x((0)\omega))e^{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))n_0}e^{-2\alpha(t-(n_0\omega+\delta))} \\
&\leq V(x((0)\omega))e^{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))n_0} \\
&= V(x((0)\omega))e^{-(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))n_0}e^{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))(n_0+1)} \\
&= V(x((0)\omega))e^{-(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))}e^{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))(n_0+1)} \\
&= V(x((0)\omega))e^{-(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))}e^{\frac{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))(n_0+1)\omega}{\omega}} \\
&\leq V(x((0)\omega))e^{-(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))}e^{\frac{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))t}{\omega}}.
\end{aligned} \tag{33}$$

Case 2. For $n_0\omega + \delta < t \leq (n_0+1)\omega$, using condition (13), we get

$$\begin{aligned}
V(x(t)) &\leq V(x(n_0\omega+\delta))e^{2(\varepsilon-\alpha)(t-(n_0\omega+\delta))} \\
&\leq V(x(n_0\omega))e^{-2\alpha\delta}e^{2(\varepsilon-\alpha)(t-(n_0\omega+\delta))} \\
&\leq V(x(0))e^{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))n_0}e^{-2\alpha\delta+2(\varepsilon-\alpha)(t-(n_0\omega+\delta))} \\
&\leq V(x(0))e^{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))n_0}e^{-2\alpha\delta+2(\varepsilon-\alpha)((n_0+1)\omega-(n_0\omega+\delta))} \\
&= V(x(0))e^{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))(n_0+1)} \\
&= V(x(0))e^{\frac{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))(n_0+1)\omega}{\omega}} \\
&\leq V(x(0))e^{\frac{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))t}{\omega}}.
\end{aligned} \tag{34}$$

Let $\xi = e^{-(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))}$, from (33) and (34) it follows that

$$V\big(x(t)\big) \le \xi V\big(x(0)\big)e^{\frac{(-2\alpha\delta+2(\varepsilon-\alpha)(\omega-\delta))t}{\omega}}.$$

Estimating $V(x(0))$, we get

$$\begin{aligned}
V_1\big(x(0)\big) &\le \lambda_{\max}\big(P^{-1}\big)\big\|\phi(t)\big\|^2, \\
V_2\big(x(0)\big) &\le \lambda_{\max}\big(P^{-1}QP^{-1}\big)\left(\frac{1-e^{-2\alpha h_2}}{2\alpha}\right)\big\|\varphi(t)\big\|^2, \\
V_3\big(x(0)\big) &\le \lambda_{\max}\big(P^{-1}QP^{-1}\big)\left(\frac{1-e^{-2\alpha h_2}}{2\alpha}\right)\big\|\varphi(t)\big\|^2, \\
V_4\big(x(0)\big) &= h_1\int_{-h_1}^{0}\int_{s}^{0} e^{2\alpha\tau}\dot{x}^T(\tau)YRY\dot{x}(\tau)\,d\tau\,ds \\
&\le h_2\lambda_{\max}\big(P^{-1}RP^{-1}\big)\left(\frac{1-e^{-2\alpha h_2}}{2\alpha}\right)\big\|\phi(t)\big\|^2 \\
&\quad + h_2\lambda_{\max}\big(P^{-1}RP^{-1}\big)\left(\frac{1-e^{-2\alpha h_2}}{2\alpha}\right)\big\|\varphi(t)\big\|^2, \\
V_5\big(x(0)\big) &\le h_2\lambda_{\max}\big(P^{-1}RP^{-1}\big)\left(\frac{1-e^{-2\alpha h_2}}{2\alpha}\right)\big\|\phi(t)\big\|^2 \\
&\quad + h_2\lambda_{\max}\big(P^{-1}RP^{-1}\big)\left(\frac{1-e^{-2\alpha h_2}}{2\alpha}\right)\big\|\varphi(t)\big\|^2, \\
V_6\big(x(0)\big) &\le h_2\lambda_{\max}\big(P^{-1}UP^{-1}\big)\left(\frac{1-e^{-2\alpha h_2}}{2\alpha}\right)\big\|\phi(t)\big\|^2 \\
&\quad + h_2\lambda_{\max}\big(P^{-1}UP^{-1}\big)\left(\frac{1-e^{-2\alpha h_2}}{2\alpha}\right)\big\|\varphi(t)\big\|^2, \\
V_7\big(x(0)\big) &\le k_1\lambda_{\max}\big(P^{-1}SP^{-1}\big)\left(\frac{1-e^{-2\alpha k_1}}{2\alpha}\right)\big\|\varphi(t)\big\|^2, \\
V_8\big(x(0)\big) &\le d\lambda_{\max}\big(P^{-1}L^TT^{-1}LP^{-1}\big)\left(\frac{1-e^{-2\alpha d}}{2\alpha}\right)\big\|\phi(t)\big\|^2 \\
&\quad + d\lambda_{\max}\big(P^{-1}L^TT^{-1}LP^{-1}\big)\left(\frac{1-e^{-2\alpha d}}{2\alpha}\right)\big\|\varphi(t)\big\|^2, \\
V_9\big(x(0)\big) &\le k_2\lambda_{\max}\big(P^{-1}L^TW^{-1}LP^{-1}\big)\left(\frac{1-e^{2\alpha k_2}}{2\alpha}\right)\big\|\varphi(t)\big\|^2, \\
V_{10}\big(x(0)\big) &\le \lambda_{\max}\big(P^{-1}ZP^{-1}\big)\frac{1}{4\alpha}\left(h_2-\left(\frac{1-e^{-4\alpha h_2}}{4\alpha}\right)\right)\big\|\phi(t)\big\|^2 \\
&\quad + \lambda_{\max}\big(P^{-1}ZP^{-1}\big)\frac{1}{4\alpha}\left(h_2-\left(\frac{1-e^{-4\alpha h_2}}{4\alpha}\right)\right)\big\|\varphi(t)\big\|^2.
\end{aligned}$$

We have obtained the following:

$$\big\|\big(x(t,\phi)\big)\big\| \le \sqrt{\frac{\mathcal{M}}{\lambda_{\min}(P^{-1})}}e^{\frac{(-\alpha\delta+(\varepsilon-\alpha)(\omega-\delta))t}{\omega}}, \quad \forall t \ge 0,$$

which implies that nonlinear system (1) is exponentially stable under controller (4). This completes the proof. □

Remark 6 In our main results, the exponential stabilization problems are considered for a class of nonlinear systems with non-differentiable time-varying delays, including interval time-varying delay and distributed time-varying delay. We construct the improved Lyapunov-Krasovskii functionals $V(x(t))$ as shown in (15). The exponential stabilizability conditions are independent of the derivatives of the time-varying delays and then the methods used in [7, 12, 15, 38, 40] are not applicable to this system.

4 A numerical example

In this section, we present an example to show the effectiveness of the result in Theorem 5.

Example 4.1 In this example, the nonlinear systems with mixed time-varying delays proposed by [10] can be described by

$$\dot{x}(t) = A_1x(t) + B_1f\big(x(t)\big) + C_1f\big(x\big(t-h(t)\big)\big) + D_1\int_{t-k_1(t)}^{t} f\big(x(s)\big)\,ds + \mathcal{U}(t), \tag{35}$$

where

$$A_1 = \begin{bmatrix} -1.2 & -0.1 \\ 0.1 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 & -0.3 \\ 1 & 5 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -1.4 & 0.1 \\ 0.3 & -8 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} -1.2 & -0.1 \\ -2.8 & -0.9 \end{bmatrix},$$
$$f\big(x(t)\big) = \big[\tanh\big(x_1(t)\big), \tanh\big(x_2(t)\big)\big]^T.$$

Model (35) turns into the following model (1) with parameters:

$$A = \begin{bmatrix} -1.2 & -0.1 \\ 0.1 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$f(\cdot) = \begin{bmatrix} -0.3\tanh(x_2(t)) - 1.4\tanh(x_1(t-h(t))) + 0.1\tanh(x_2(t-h(t))) \\ -1.2\int_{t-k_1(t)}^{t}\tanh(x_1(s))\,ds - 0.1\int_{t-k_1(t)}^{t}\tanh(x_2(s))\,ds \\ \\ \tanh(x_1(t)) + 5\tanh(x_2(t)) + 0.3\tanh(x_1(t-h(t))) - 8\tanh(x_2(t-h(t))) \\ -2.8\int_{t-k_1(t)}^{t}\tanh(x_1(s))\,ds - 0.9\int_{t-k_1(t)}^{t}\tanh(x_2(s))\,ds, \end{bmatrix}$$

and we give

$$D_1 = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad D_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Solution: From conditions (6)-(13) of Theorem 5 with parameters $a_1 = 4.7249$, $b_1 = 8.0045$, $c_1 = 1.6$, $d_1 = 0$ $h_1 = 0, h_2 = 0.9, d = 0.2, k_1 = 0.1, k_2 = 0.1$, $\alpha = 0.3$, $\varepsilon = 1.5$, $\omega = 4$,

$\delta = 3.25$. By using the LMI Toolbox in MATLAB, we obtain

$$P = \begin{bmatrix} 4.2978 & -0.0331 \\ -0.0331 & 4.2957 \end{bmatrix}, \qquad T = \begin{bmatrix} 0.0159 & 0.0002 \\ 0.0002 & 0.0183 \end{bmatrix},$$
$$W = \begin{bmatrix} 0.2145 & 0.0033 \\ 0.0033 & 0.2516 \end{bmatrix}, \qquad U = \begin{bmatrix} 2.8160 & -0.0046 \\ -0.0046 & 3.0519 \end{bmatrix},$$
$$Q = \begin{bmatrix} 0.0360 & 0.0011 \\ 0.0011 & 0.0368 \end{bmatrix}, \qquad R = \begin{bmatrix} 1.3020 & 0.0013 \\ 0.0013 & 1.4236 \end{bmatrix},$$
$$S = \begin{bmatrix} 0.0756 & 0.0020 \\ 0.0020 & 0.0797 \end{bmatrix}, \qquad Z = \begin{bmatrix} 0.1074 & 0.0024 \\ 0.0024 & 0.1168 \end{bmatrix},$$
$$L = \begin{bmatrix} 0.2012 & -0.0084 \\ 0.0076 & 0.1553 \end{bmatrix}, \qquad S_1 = \begin{bmatrix} -0.5735 & 0.0097 \\ 0.0095 & -0.7085 \end{bmatrix},$$
$$K = \begin{bmatrix} -0.0468 & 0.0016 \\ -0.0020 & -0.0362 \end{bmatrix}, \qquad \epsilon = 1.2315,$$

with a stabilizing controller

$$\begin{aligned} \mathcal{U}(t) &= \begin{bmatrix} -0.14040.0048 \\ -0.0082 - 0.1447 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} -0.04680.0016 \\ -0.0020 - 0.0362 \end{bmatrix} \begin{bmatrix} x_1(t-d(t)) \\ x_2(t-d(t)) \end{bmatrix} \\ &\quad \times \begin{bmatrix} -0.04680.0016 \\ -0.0020 - 0.0362 \end{bmatrix} \begin{bmatrix} \int_{t-k_2(t)}^{t} x_1(s)\,ds \\ \int_{t-k_2(t)}^{t} x_2(s)\,ds \end{bmatrix}, \qquad 4n \le t \le 4n + 3.25, \\ \mathcal{U}(t) &= 0, \qquad 4n + 3.25 < t \le 4(n+1), \quad n = 0, 1, 2, \ldots. \end{aligned} \tag{36}$$

For the purpose of comparison, we also tested the method proposed in [10]. Table 1 compares the feedback controller gains obtained from those two methods. The numerical simulation of nonlinear system (35) with time-varying delays $h(t) = 0.3 + 0.4|\cos t|$, $k_1(t) = 0.2|\sin t|$, the initial condition $\phi(t) = [7\cos(8s), -8\cos(7s)]$, $\forall s \in [-0.7, 0]$ and without hybrid intermittent feedback control is represented in Figure 1, which shows that system (35) is stable. Figure 2 shows the trajectories of $x_1(t)$ and $x_2(t)$ of nonlinear system (35) with time-varying delays $d(t) = 0.1 + 0.1|\cos t|$, $k_2(t) = 0.1|\sin t|$ and hybrid intermittent feedback control (36). Furthermore, because of the lower bounds $h_1 \neq 0$ of the delay function, the method proposed in [10] is not applicable in this case.

Remark 7 In Example 4.1, we see that every state variable of nonlinear system (35) is stable without control. After applying controller (36), all the state variables of nonlinear system (35) converge to 0. That shows the effectiveness of the controller.

5 Conclusion

In this paper, the exponential stabilization criterion of a nonlinear system with mixed time-varying delays via hybrid intermittent feedback control was investigated. The time-

Table 1 Comparison of the feedback controller gain matrix

Delay	Method	Controller gain matrix		
		K_1	K_2	K_3
$h_1 = 0, h_2 = 0.9$	Method of [10]	$\begin{bmatrix} -1.9212 & 1.7480 \\ -2.0908 & -49.3170 \end{bmatrix}$	$\begin{bmatrix} -0.0034 & 0.0023 \\ 0.0055 & -0.00640 \end{bmatrix}$	$\begin{bmatrix} -0.0927 & 0.0493 \\ 0.1343 & -1.4377 \end{bmatrix}$
	Proposed method	$\begin{bmatrix} -0.0468 & 0.0016 \\ -0.0020 & -0.0362 \end{bmatrix}$	$\begin{bmatrix} -0.0468 & 0.0016 \\ -0.0020 & -0.0362 \end{bmatrix}$	$\begin{bmatrix} -0.0468 & 0.0016 \\ -0.0020 & -0.0362 \end{bmatrix}$
$h_1 = 0.3, h_2 = 1$	Method of [10]	Infeasible	Infeasible	Infeasible
	Proposed method	$\begin{bmatrix} -0.0411 & -0.0004 \\ 0.0003 & -0.0515 \end{bmatrix}$	$\begin{bmatrix} -0.0411 & -0.0004 \\ 0.0003 & -0.0515 \end{bmatrix}$	$\begin{bmatrix} -0.0411 & -0.0004 \\ 0.0003 & -0.0515 \end{bmatrix}$

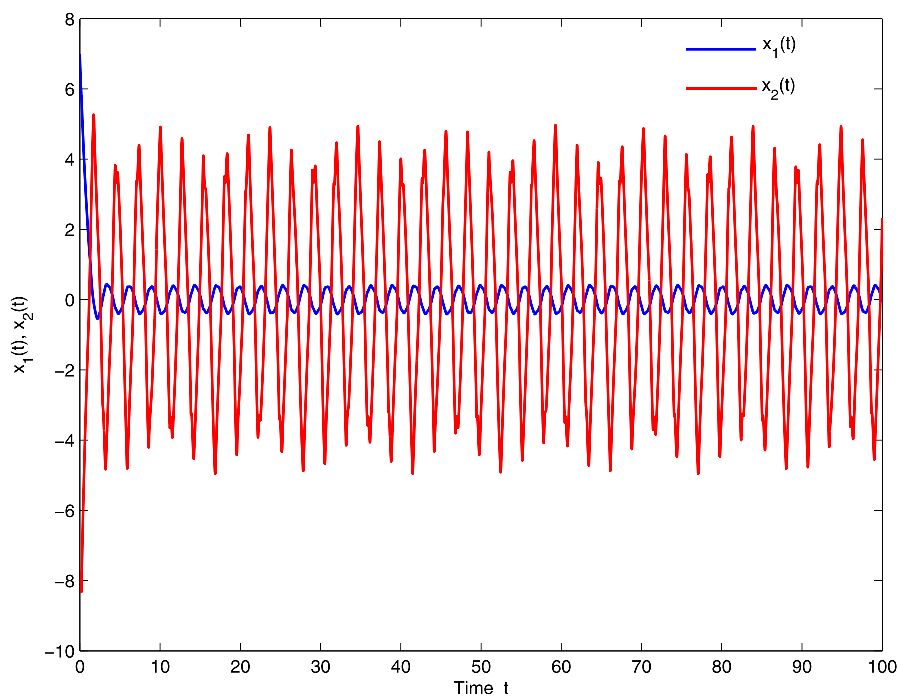

Figure 1 The trajectories of $x_1(t)$ and $x_2(t)$ of nonlinear system (35) with mixed time-varying delay and hybrid intermittent feedback control deactivated.

varying delay functions are not necessary to be differentiable, which allows time delay functions to be fast time-varying functions. Moreover, hybrid intermittent feedback control, including state term, interval time-varying delay term and distributed time-varying delay term, was considered for the exponential stabilization of the nonlinear system. Based on constructing an improved Lyapunov-Krasovskii functional, Leibniz-Newton's formula, Jensen's inequality, reciprocal convex and novel delay-dependent sufficient condition for the exponential stabilization of the system are first achieved in terms of LMIs. Finally, a numerical example is included to show the effectiveness of the proposed hybrid intermittent feedback control scheme. The results in this paper generalize and improve the corresponding results of the recent works.

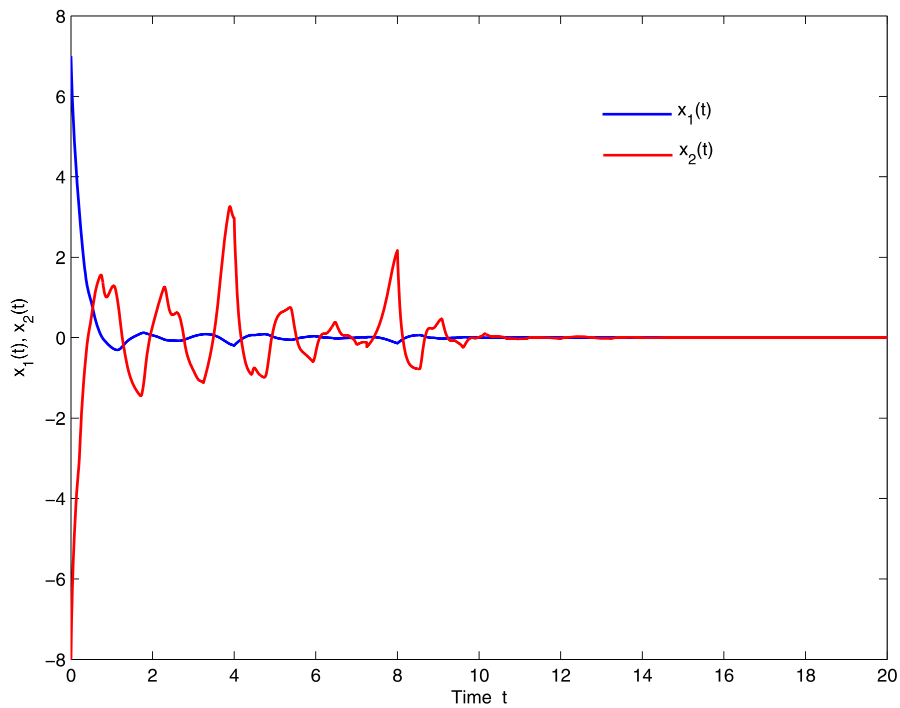

Figure 2 The trajectories of $x_1(t)$ and $x_2(t)$ of nonlinear system (35) with mixed time-varying delay and intermittent hybrid feedback control (36) activated.

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Competing interests
The authors declare that they have no competing interests.

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All authors contributed equally to the writing of this paper. Moreover, all authors also read and carefully approved the final manuscript.

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