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The asymptotic behavior of the stochastic coupled Kuramoto–Sivashinsky and Ginzburg–Landau equations

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Abstract

The stochastic coupled Kuramoto–Sivashinsky and Ginzburg–Landau equations (KS-GL) perturbed by additive noises is investigated in this paper. By making careful analysis, we first consider the existence and uniqueness of the solution with initial-boundary condition, and then we establish a random attractor for the stochastic KS–GL equations in $X = L^2 \times H^{-1}$.

MSC: Primary 35B40; secondary 60H15

Keywords: Coupled KS–GL equations; Random dynamical systems; Random attractor

1 Introduction

The coupled Kuramoto–Sivashinsky and Ginzburg–Landau (KS–GL) equation comes from the nonlinear evolution of the coupled long-scale oscillatory and monotonic instabilities of a uniformly propagating combustion wave governed by a sequential chemical reaction, having two flame fronts corresponding to two reaction zones with a finite separation distance between them. They describe the interaction between the excited monotonic mode and the excited or damped oscillatory mode (see [1–4]). In fact, the actual chemical kinetics governing the structure of flames can be quite complex and leads to qualitative behavior of flames which cannot be accounted for by a one-stage kinetic model. A number of papers were devoted to the investigation of the propagation of flame fronts [5–11]. A simplified model called the coupled system of KS–GL equations for the Marangoni convection is of the form:

$$\partial_t A - \mu A - \partial_x^2 A + k|A|^2 A = Ah,$$

$$\partial_t h + m\partial_x^2 h + \nu \partial_x^4 h = \alpha \partial_x^2 \bigl(|A|^2\bigr),$$

where $A(x,t)$ is the rescaled complex amplitude for the Marangoni convective mode, the real function $h(x,t)$ is the interface deformation, the constant α is called "Marangoni coefficient", and the parameters k, ν, μ, m are all real-valued constants with $k > 0$, $\nu > 0$.

The periodic initial–boundary problem of the KS–GL equations has been studied by a couple of authors. If taking the coefficient $\mu = \nu = m = 1$, the global unique solvability of the periodic initial–boundary value problem was established in [12–14]. Furthermore, as was shown in [15], the problem has a global solution for any value of the coupling constant and a minimal global attractor in the case of Dirichlet boundary conditions for certain values of the parameters. However, some perturbations may neglect in the derivation of this ideal model, and one often represents the microeffects by random perturbations in the dynamics of the macro observables. Thus, it is interesting to consider stochastic effect in KS–GL equation. So far, we only know that the stochastic version of KS–GL type model for the Marangoni convection with the periodic initial–boundary problem has a unique solution $A \in C([0,T],H^2_{\mathrm{per}})\cap L^2([0,T],H^3_{\mathrm{per}})$, $h \in C([0,T],H^2_{\mathrm{per}})\cap L^2([0,T],H^4_{\mathrm{per}})$ under the condition $0 < \alpha < 2$ in [16].

In this paper, we focus on the existence of random attractor for the following stochastic KS–GL (SKS–GL) equations with additive noises:

$$\partial_t A - \mu A - \partial_x^2 A + k|A|^2 A = Ah + \Phi_1 \dot{W}_1, \tag{1}$$

$$\partial_t h + m\partial_x^2 h + \nu\partial_x^4 h = \alpha\partial_x^2\bigl(|A|^2\bigr) + \Phi_2 \dot{W}_2, \tag{2}$$

under the initial value conditions

$$A(x,0) = A_0(x), \qquad h(x,0) = h_0(x), \quad x \in (0,l) \tag{3}$$

and the boundary value conditions

$$A(0,t) = A(l,t) = 0, \quad t > 0, \tag{4}$$

$$h(0,t) = h(l,t) = \partial_x^2 h(0,t) = \partial_x^2 h(l,t) = 0, \quad t > 0. \tag{5}$$

The random process $W_i = \sum_{j=1}^{\infty} \beta_j^i(t,\omega)e_j(x)$ $(i = 1,2)$ is a two-sided in time cylindrical Wiener process on $L^2(0,l)$ associated to a complete probability space $(\Omega,\mathbb{F},P)$, where $\{\beta_j^i(t,\omega)\}_{j=1}^{\infty}$ is a family of mutually independent, identically distributed standard Brownian motions and $\{e_j(x)\}_{j=1}^{\infty}$ is an orthonormal basis of $L^2(0,l)$. The function Φ_i is a Hilbert–Schmidt operator.

The notion of random attractors for a stochastic dynamical system has been introduced in [17–19]. Random attractors are compact invariant sets, however, they are not fixed, depend on chance and move with time. It seems to be a generalization of the classical concept of global attractors for deterministic dynamical systems and has been successfully applied to many infinite-dimensional stochastic dynamical systems [17, 18, 20]. To investigate the equations (SKS–GL), we first establish the global unique solvability of the problem without the restriction of α as done in [15]. Then we assume that $\nu\lambda_1 - m > 0$ for $\alpha > 0$ and $\nu\lambda_1 - m > \frac{16|\alpha|}{k}$ for $\alpha < 0$ (where $\lambda_1 = (\frac{\pi}{l})^2$ is the first eigenvalue of the problem $-\psi'' = \lambda\psi$ for $x \in (0,l)$ with $\psi(0) = \psi(l) = 0$), and establish the random attractors for the stochastic KS–GL equations in $X = L^2 \times H^{-1}$. So, our results supplement the results of [16].

This paper is organized as follows. In the next section, we briefly give some basic and important concepts related to random dynamical systems (RDS). In Sect. 3, we establish a prior estimates for the solutions to the initial problem (1)–(5). Although the results are similar to the deterministic equations, the proof is more complicated due to the random perturbations. In Sect. 4, we conclude the main result by studying the long time behavior of the studied equation.

2 Notations and preliminaries

2.1 Notations

For the mathematical setting of the problem, we introduce the Sobolev space H^k with the norm $\|\cdot\|_{H^k}$ and the Banach space L^p with the norm $\|\cdot\|_{L^p}$. Especially, we denote $(\cdot,\cdot)$ the inner product in L^2, where

$$(u,v) = \operatorname{Re}\int_0^l u(x)\bar{v}(x)\,dx.$$

Let $\{e_n\}$ be the orthonormal basis in L^2, which themselves are the eigenvectors with the corresponding eigenvalues $\lambda_n > 0$ and $\lambda_n \nearrow \infty$ as $n \mapsto \infty$. For the first eigenvalue, we have the inequality $\lambda_1\|u\|_{L^2}^2 \le \|u\|_{H^1}^2$. Throughout this paper, ϵ_i $(i = 1,2,\ldots,6)$ and C denote positive constants which depend on the coefficients of equations (1)–(2).

Given two separable Hilbert space H and K, we define $L_2^0(H,K)$ to be the space of Hilbert operators from H to K with the norm

$$\|\Phi\|_{L_2^0(H,K)}^2 = \sum_{k\in\mathbb{N}} |\Phi e_k|_K^2,$$

where $\{e_k\}_{k\in\mathbb{N}}$ is any orthonormal basis of H. In particular, when $H = L^2$, $K = H^1$, we rewrite the norm in a simpler form:

$$\|\Phi\|_{L_2^0(L^2,H^1)}^2 = \|\Phi\|_{L_2^{0,1}}^2.$$

2.2 Preliminary results on RDS

We now recall some concepts and well-known results related to random attractors for RDS. For further details, the readers are referred to [17] and [18].

Definition 2.1 We call $(\Omega,\mathcal{F},P,(\theta_t)_{t\in\mathbb{R}})$ a metric dynamical system if $\theta_\cdot : \mathbb{R}\times\Omega \to \Omega$ is a $(\mathcal{B}(\mathbb{R})\times\mathcal{F}\to\mathcal{F})$-measurable mapping on a probability space $(\Omega,\mathcal{F},P)$ satisfying $\theta_0 = id$, $P\circ\theta_t^{-1} = P$ and $\theta_{t+s} = \theta_t\circ\theta_s$ for all $s,t\in\mathbb{R}$.

Definition 2.2 A continuous RDS on a Polish space (X,d) with Borel σ-algebra $\mathcal{F}$ over θ on $(\Omega,\mathcal{F},P)$ is a measurable mapping

$$\phi : \mathbb{R}^+\times\Omega\times X \to X, \quad (t,\omega,x) \to \phi(t,\omega)x,$$

which is $(\mathcal{B}(\mathbb{R}^+)\times\mathcal{F}\times\mathcal{B}(X),\mathcal{B}(X))$-measurable and satisfies P-a.s.

(1) $\phi(0,\omega) = id$ on X;

(2) $\phi(t+s,\omega) = \phi(t,\theta_s\omega)\circ\phi(s,\omega)$ for all $t,s\in\mathbb{R}^+$ (cocycle property);

(3) $\phi(t,\omega) : X \to X$ is continuous for all $t\in\mathbb{R}^+$.

Definition 2.3 A random compact set $K(\omega)_{\omega\in\Omega}$ is a family of compact sets indexed by ω such that for each $x\in X$ the map $\omega \to d(x,K(\omega))$ is measurable with respect to $\mathcal{F}$.

Definition 2.4 Let $\mathcal{A}(\omega)$ be a random set and $B \subset X$, one says $\mathcal{A}(\omega)$ attracts B if

$$\lim_{t\to\infty} \operatorname{dist}\bigl(\phi(t,\theta_{-t}\omega)B, \mathcal{A}\bigr) = 0 \quad \text{a.s.}$$

Definition 2.5 A random set $\mathcal{A}(\omega)$ is said to be a random attractor for the RDS ϕ if P-a.s.

(1) $\mathcal{A}(\omega)$ is a random compact set;

(2) $\mathcal{A}(\omega)$ is invariant, i.e., $\phi(t,\omega)\mathcal{A}(\omega) = \mathcal{A}(\theta_t\omega)$, for all $t > 0$;

(3) $\mathcal{A}(\omega)$ attracts all deterministic bounded sets $B \subset X$.

Theorem 2.6 *If there exists a random compact set absorbing every bounded set $B \subset X$, then the RDS ϕ possesses a random attractor $\mathcal{A}(\omega) = \overline{\bigcup_{B\subset X} \Lambda_B(\omega)}$, where $\Lambda_B(\omega) := \bigcap_{s\geq 0} \overline{\bigcup_{t\geq s} \phi(t,\theta_{-t}\omega)B}$ is the omega-limit set of B.*

3 Existence and uniqueness of solutions

In this section, we establish a priori estimates for the solutions of the initial–boundary problem (1)–(5). Firstly, we consider two linear stochastic problems:

$$dW_A = \partial_x^2 W_A\,dt + \Phi_1\,dW_1, \quad W_A(0) = 0, \tag{6}$$

$$dW_h = -\nu\partial_x^4 W_h\,dt + \Phi_2\,dW_2, \quad W_h(0) = 0, \tag{7}$$

under the boundary value conditions:

$$W_A(0,t) = W_A(l,t) = 0, \quad t > 0,$$

and

$$W_h(0,t) = W_h(l,t) = \partial_x^2 W_h(0,t) = \partial_x^2 W_h(l,t) = 0, \quad t > 0.$$

We assume that $\Phi_i \in L_2^{0,1}$ ($i = 1,2$) and recall from [21, 22] that the solutions $W_A(t)$ and $W_h(t)$ given by problems (6)–(7) are well defined. They have paths in $C([0,\infty], H_0^1)$ and $C([0,\infty], H^2 \cap H_0^1)$, respectively. Moreover, for any t, s,

$$W_i(t,\theta_s\omega) = W_i(t+s,\omega), \quad i = A, h, \text{a.s.}$$

Since W_i ($i = A,h$) is Gaussian, it follows that for any $q, r \geq 1$, there exists a constant $M(q,r)$ independent of t such that

$$E\bigl(\| W_i\|_{L^q}^r\bigr) < M(q,r). \tag{8}$$

Set

$$u(t) = A(t) - W_A(t), \qquad v(t) = h(t) - W_h(t), \quad t \geq 0. \tag{9}$$

Note that by (1) and (2), we have

$$dA = \bigl(\mu A + \partial_x^2 A - k|A|^2 A + Ah\bigr)\,dt + \Phi_1\,dW_1, \tag{10}$$

$$dh = \left(-m\partial_x^2 h - \nu\partial_x^4 h + \alpha\partial_x^2\left(|A|^2\right)\right)dt + \Phi_2\, dW_2. \tag{11}$$

Inserting (6)–(7) and (10)–(11) into (9) and noticing that the noise disappears, we obtain that functions $u(t)$, $v(t)$ satisfy

$$\partial_t u - \mu(u + W_A) - \partial_x^2 u + k|u + W_A|^2(u + W_A) = (u + W_A)(v + W_h), \tag{12}$$

$$\partial_t v + m\partial_x^2(v + W_h) + \nu\partial_x^4 v = \alpha\partial_x^2\left(|u + W_A|^2\right), \tag{13}$$

as well as the initial value conditions:

$$u(x,0) = A_0(x), \qquad v(x,0) = h_0(x), \quad x \in (0,l) \tag{14}$$

and the boundary value conditions:

$$u(0,t) = u(l,t) = 0, \quad t > 0, \tag{15}$$

$$v(0,t) = v(l,t) = \partial_x^2 v(0,t) = \partial_x^2 v(l,t) = 0, \quad t > 0. \tag{16}$$

Theorem 3.1 *Assume that* $\Phi_i \in L_2^{0,1}$ $(i = 1,2)$. *For any* $\mathcal{F}_0$*-measurable* $A_0 \in L^2(0,l)$ *and* $h_0 \in H^{-1}(0,l)$, *and for given* $T > 0$, *there exists a unique weak solution*

$$u \in C\left(0,T;L^2(0,l)\right) \cap L^2\left(0,T;H_0^1(0,l)\right),$$
$$v \in C\left(0,T;H^{-1}(0,l)\right) \cap L^2\left(0,T;H_0^1(0,l)\right),$$

satisfying (12)–(16) *in the distributional sense with* $(u,v)|_{t=0} = (A_0,h_0)$. *Moreover, the mapping* $(u_0,v_0) \mapsto (u,v)$ *is continuous in* $E = L^2(0,l) \times H^{-1}(0,l)$.

Proof Let P^2 be the inverse of the operator $L = -\frac{d^2}{dx^2}$ with the domain $D(L) = H^2(0,l) \cap H_0^1(0,l)$. Applying the operator P^2 to both sides of equation (13), we obtain

$$\partial_t u - \mu(u + W_A) - \partial_x^2 u + k|u + W_A|^2(u + W_A) = (u + W_A)(v + W_h), \tag{17}$$

$$P^2\partial_t v - m(v + W_h) - \nu\partial_x^2 v = -\alpha\left(|u + W_A|^2\right), \tag{18}$$

$$u(x,0) = A_0(x), \qquad v(x,0) = h_0(x), \quad x \in (0,l), \tag{19}$$

$$u(0,t) = u(l,t) = 0, \qquad v(0,t) = v(l,t) = 0, \quad t > 0. \tag{20}$$

Multiplying (17) by $\bar{u}$, integrating with respect to x over $(0,l)$, and taking the real part, we obtain

$$\begin{aligned}
&\frac{d}{dt}\|u\|_{L^2}^2 - 2\mu\|u\|_{L^2}^2 + 2\|\partial_x u\|_{L^2}^2 \\
&\quad = 2\,\mathrm{Re}\int_0^l \bar{u}\left[\mu W_A - k|u + W_A|^2(u + W_A) + (u + W_A)(v + W_h)\right]dx \\
&\quad = I_1 + I_2 + I_3.
\end{aligned} \tag{21}$$

To estimate the terms on the RHS of (21), we have

$$I_1 = 2\,\mathrm{Re}\int_0^l \bar{u}\mu W_A\,dx \le 2|\mu|\|u\|_{L^2}\|W_A\|_{L^2} \le \epsilon_1\|u\|_{L^2}^2 + C\bigl(\epsilon_1,|\mu|\bigr)\|W_A\|_{L^2}^2, \tag{22}$$

and

$$\begin{aligned}
I_2 &= -2k\,\mathrm{Re}\int_0^l \bar{u}|u+W_A|^2(u+W_A)\,dx \\
&= -2k\,\mathrm{Re}\int_0^l \bar{u}\bigl[|u|^2 + uW_A + \bar{u}W_A + W_A^2\bigr](u+W_A)\,dx \\
&\le -2k\|u\|_{L^4}^4 + 6k\int_0^l |u|^3|W_A|\,dx + 6k\int_0^l |u|^2|W_A|^2\,dx + 2k\int_0^l |u||W_A|^3\,dx \\
&\le -2k\|u\|_{L^4}^4 + 6k\|u\|_{L^4}^3\|W_A\|_{L^4} + 6k\|u\|_{L^4}^2\|W_A\|_{L^4}^2 + 2k\|u\|_{L^4}\|W_A\|_{L^4}^3 \\
&\le -2k\|u\|_{L^4}^4 + \frac{k}{2}\|u\|_{L^4}^4 + C(k)\|W_A\|_{L^4}^4.
\end{aligned} \tag{23}$$

Furthermore,

$$\begin{aligned}
I_3 &= 2\,\mathrm{Re}\int_0^l \bar{u}(u+W_A)(v+W_h)\,dx \\
&\le 2\int_0^l \bigl(|u|^2 v + |u|^2|W_h| + |u||v||W_A| + |u||W_A||W_h|\bigr)\,dx \\
&\le 2\int_0^l |u|^2 v\,dx + 2\|u\|_{L^4}^2\|W_h\|_{L^2} + \|u\|_{L^4}\|v\|_{L^2}\|W_A\|_{L^4} + \|u\|_{L^2}\|W_A\|_{L^4}\|W_h\|_{L^4} \\
&\le 2\int_0^l |u|^2 v\,dx + \frac{k}{2}\|u\|_{L^4}^4 + \epsilon_1\|u\|_{L^2}^2 + \epsilon_2\|v\|_{L^2}^2 + C(\epsilon_1,\epsilon_2)\bigl(\|W_A\|_{L^4}^4 + \|W_h\|_{L^4}^4\bigr).
\end{aligned} \tag{24}$$

Combining (22)–(24) with (21), we have

$$\begin{aligned}
&\frac{d}{dt}\|u\|_{L^2}^2 - 2(\mu+\epsilon_1)\|u\|_{L^2}^2 + 2\|\partial_x u\|_{L^2}^2 + k\|u\|_{L^4}^4 \\
&\quad\le 2\int_0^l |u|^2 v\,dx + \epsilon_2\|v\|_{L^2}^2 + C\bigl(\epsilon_1,\epsilon_2,|\mu|,k\bigr)\bigl(\|W_A\|_{L^2}^2 + \|W_A\|_{L^4}^4 + \|W_h\|_{L^4}^4\bigr).
\end{aligned} \tag{25}$$

Similarly, multiplying (18) by v, we get the equality

$$\begin{aligned}
&\frac{d}{dt}\|Pv\|_{L^2}^2 - 2m\|v\|_{L^2}^2 + 2\nu\|\partial_x v\|_{L^2}^2 \\
&\quad = 2m\int_0^l vW_h\,dx - 2\alpha\int_0^l v|u+W_A|^2\,dx = I_4 + I_5.
\end{aligned} \tag{26}$$

For the RHS of (26), we find that

$$I_4 = 2m\int_0^l vW_h\,dx \le \frac{\epsilon_2}{2}\|v\|_{L^2}^2 + C(\epsilon_2)\|W_h\|_{L^2}^2 \tag{27}$$

and

$$
\begin{aligned}
I_5 &= -2\alpha \int_0^l v|u + W_A|^2\,dx = -2\alpha \int_0^l v\bigl(|u|^2 + \bar{u}W_A + uW_A + W_A^2\bigr)\,dx \\
&\le -2\alpha \int_0^l |u|^2 v\,dx + 4|\alpha|\|u\|_{L^4}\|W_A\|_{L^4}\|v\|_{L^2} + 2|\alpha|\|v\|_{L^2}\|W_A\|_{L^4}^2 \\
&\le -2\alpha \int_0^l |u|^2 v\,dx + \epsilon_3\|u\|_{L^4}^4 + \frac{\epsilon_2}{2}\|v\|_{L^2}^2 + C\bigl(\epsilon_2,\epsilon_3,|\alpha|\bigr)\|W_A\|_{L^4}^4. \qquad (28)
\end{aligned}
$$

Taking $\epsilon_3 = \frac{k}{2}$, from (26)–(28), we get

$$
\begin{aligned}
&\frac{d}{dt}\|Pv\|_{L^2}^2 - (2m+\epsilon_2)\|v\|_{L^2}^2 + 2\nu\|\partial_x v\|_{L^2}^2 \\
&\quad \le -2\alpha \int_0^l |u|^2 v\,dx + \frac{k}{2}\|u\|_{L^4}^4 + C\bigl(\epsilon_2,k,|\alpha|\bigr)\bigl(\|W_A\|_{L^4}^4 + \|W_h\|_{L^2}^2\bigr). \qquad (29)
\end{aligned}
$$

Combining (25) with (29), we have

$$
\begin{aligned}
&\frac{d}{dt}\bigl(\|u\|_{L^2}^2 + \|Pv\|_{L^2}^2\bigr) + 2\|\partial_x u\|_{L^2}^2 + \frac{k}{2}\|u\|_{L^4}^4 + 2\nu\|\partial_x v\|_{L^2}^2 \\
&\quad \le 2(\mu+\epsilon_1)\|u\|_{L^2}^2 + (2m+2\epsilon_2)\|v\|_{L^2}^2 + 2(1-\alpha)\int_0^l |u|^2 v\,dx + g_1(t),
\end{aligned}
$$

where

$$
g_1(t) = C\bigl(\epsilon_1,\epsilon_2,k,|\mu|,|\alpha|\bigr)\bigl(\|W_A\|_{L^2}^2 + \|W_A\|_{L^4}^4 + \|W_h\|_{L^2}^2 + \|W_h\|_{L^4}^4\bigr).
$$

On the other hand, we notice that

$$
2(1-\alpha)\int_0^l |u|^2 v\,dx \le \frac{k}{4}\|u\|_{L^4}^4 + \frac{4(1-\alpha)^2}{k}\|v\|_{L^2}^2
$$

and

$$
\|v\|_{L^2}^2 = \bigl(P^{-1}v, Pv\bigr) \le \bigl\|P^{-1}v\bigr\|_{L^2}^2\|Pv\|_{L^2}^2 \le \epsilon_4\|\partial_x v\|_{L^2}^2 + C(\epsilon_4)\|Pv\|_{L^2}^2.
$$

Taking $\epsilon_4 = \nu/[2|m| + 2\epsilon_2 + 4(1-\alpha)^2/k]$, we have

$$
\begin{aligned}
&\frac{d}{dt}\bigl(\|u\|_{L^2}^2 + \|Pv\|_{L^2}^2\bigr) + 2\|\partial_x u\|_{L^2}^2 + \frac{k}{4}\|u\|_{L^4}^4 + \nu\|\partial_x v\|_{L^2}^2 \\
&\quad \le 2\bigl(|\mu|+\epsilon_1\bigr)\|u\|_{L^2}^2 + C\bigl(\nu,\alpha,\epsilon_2,k,|m|\bigr)\|Pv\|_{L^2}^2 + g_1(t). \qquad (30)
\end{aligned}
$$

Applying Gronwall's inequality and (8), we deduce that for $t \in [0,T]$,

$$
E\bigl(\|u\|_{L^2}^2 + \|Pv\|_{L^2}^2\bigr) \le e^{dt}E\biggl(\|u_0\|_{L^2}^2 + \|Pv_0\|_{L^2}^2 + \int_0^t g_1(s)\,ds\biggr), \qquad (31)
$$

where $d = \max\{2|\mu| + 2\epsilon_1, C(\nu,\alpha,\epsilon_2,k,|m|)\}$. Furthermore, integrating and taking the expectation in (30), we can get from (31) that

$$E\left(\int_0^t \|\partial_x u\|_{L^2}^2\,ds\right), E\left(\int_0^t \|u\|_{L^4}^4\,ds\right), E\left(\int_0^t \|\partial_x v\|_{L^2}^2\,ds\right) \leq C\big(\nu,\alpha,k,|m|,|\mu|,\epsilon_1,\epsilon_2,t\big). \tag{32}$$

Based on the estimates (31)–(32), we use the standard argument to prove the existence of a global solution of the problem (12)–(16) on the interval $[0,T]$. As done in [15], the uniqueness of the solution can be obtained, which implies that the solution $(u(t),v(t))$ is continuous with respect to (u_0,v_0) for all $t\in[0,T]$. □

4 Random attractors

By the same proof as Theorem 3.1, we can show that for P-a.s. $\omega\in\Omega$ and linear operator $\Phi_i \in L_2^{0,1}$ $(i=1,2)$, the following statements hold:

(1) For any $t_0\in R$, $u(t_0)\in L^2$, $v(t_0)\in H^{-1}$, there exists a unique solution $u\in C(t_0,T;L^2(0,l))\cap L^2(t_0,T;H_0^1(0,l))$, $v\in C(t_0,T;H^{-1}(0,l))\cap L^2(t_0,T;H^1(0,l))$ of (12)–(13) satisfying the initial value conditions $u(x,t_0)=A(t_0)-W_A(t_0)$, $v(x,t_0)=h(t_0)-W_h(t_0)$ and the boundary value conditions (15)–(16) for $t>t_0$.

(2) The mapping $\phi(t_0)\mapsto\phi(t)$ from $L^2\times H^{-1}$ to $L^2\times H^{-1}$ is continuous for all $t\geq t_0$.

Hence, the solution mapping $\phi(t_0)\mapsto\phi(t,\omega,\phi(t_0))=(u(t,\omega,u(t_0)),v(t,\omega,v(t_0)))$ generates a continuous RDS, where $\phi(t_0)=(u(t_0),v(t_0))$.

In order to prove the existence of a compact absorbing set at time $t=-1$ in $L^2\times H^{-1}$, we give the following estimates.

Lemma 4.1 *There exists a random radius $r(\omega)$ such that for all $\rho>0$ there exists $\bar{t}\leq -1$ such that for all $t_0\leq\bar{t}$ with $|\alpha|\|A(t_0)\|_{L^2}^2+\|Ph(t_0)\|_{L^2}^2\leq\rho^2$, the solutions of (17)–(20) over $[t_0,\infty)$ satisfy the inequality*

$$|\alpha|\big\|u\big(-1,\omega,A(t_0)-W_A(t_0)\big)\big\|_{L^2}^2+\big\|Pv\big(-1,\omega,Ph(t_0)-PW_h(t_0)\big)\big\|_{L^2}^2\leq r^2(\omega)\quad\text{a.s.}$$

Proof We consider two cases and begin with the case $\alpha>0$. Taking $\epsilon_3=\frac{k\alpha}{2}$ in (28), we find that (29) becomes

$$\frac{d}{dt}\|Pv\|_{L^2}^2-(2m+\epsilon_2)\|v\|_{L^2}^2+2\nu\|\partial_x v\|_{L^2}^2 \leq -2\alpha\int_0^l |u|^2 v\,dx+\frac{k\alpha}{2}\|u\|_{L^4}^4+C(\epsilon_2,k,\alpha)\big(\|W_A\|_{L^4}^4+\|W_h\|_{L^2}^2\big). \tag{33}$$

Multiplying (25) by α and putting it into (33), we have

$$\frac{d}{dt}\big(\alpha\|u\|_{L^2}^2+\|Pv\|_{L^2}^2\big)+2\alpha\|\partial_x u\|_{L^2}^2+\frac{k\alpha}{2}\|u\|_{L^4}^4+2\nu\|\partial_x v\|_{L^2}^2 \leq 2\alpha(\mu+\epsilon_1)\|u\|_{L^2}^2+\big(2m+(\alpha+1)\epsilon_2\big)\|v\|_{L^2}^2+g_2(t), \tag{34}$$

where

$$g_2(t)=\alpha C\big(\epsilon_1,\epsilon_2,|\mu|,k\big)\big(\|W_A\|_{L^2}^2+\|W_A\|_{L^4}^4+\|W_h\|_{L^4}^4\big)+C(\epsilon_2,k,\alpha)\big(\|W_A\|_{L^4}^4+\|W_h\|_{L^2}^2\big),$$

and $h(t)$ has at most polynomial growth as $t\mapsto\infty$, for P-a.s. $\omega\in\Omega$.

By Poincaré inequality, we get

$$2\alpha\|\partial_x u\|_{L^2}^2 \geq 2\alpha\lambda_1\|u\|_{L^2}^2, \tag{35}$$

$$\begin{aligned} 2\nu\|\partial_x v\|_{L^2}^2 - \bigl(2m + (\alpha+1)\epsilon_2\bigr)\|v\|_{L^2}^2 &\geq \bigl(2\nu\lambda_1 - 2m - (\alpha+1)\epsilon_2\bigr)\|v\|_{L^2}^2 \\ &\geq \lambda_1\gamma_0\|Pv\|_{L^2}^2 + \gamma_0\|v\|_{L^2}^2. \end{aligned} \tag{36}$$

We take $\epsilon_2 = \frac{\nu\lambda_1 - m}{\alpha+1}$ and $2\gamma_0 = 2\nu\lambda_1 - 2m - (\alpha+1)\epsilon_2$ provided $\nu\lambda_1 - m > 0$. Using Cauchy inequality, we find

$$2\alpha(\mu+\epsilon_1)\|u\|_{L^2}^2 \leq \frac{k\alpha}{4}\|u\|_{L^4}^4 + \frac{4\alpha l(\mu+\epsilon_1)^2}{k}. \tag{37}$$

From (34)–(37), we obtain the following inequality:

$$\begin{aligned} &\frac{d}{dt}\bigl(\alpha\|u\|_{L^2}^2 + \|Pv\|_{L^2}^2\bigr) + 2\alpha\lambda_1\|u\|_{L^2}^2 + \lambda_1\gamma_0\|Pv\|_{L^2}^2 + \frac{k\alpha}{4}\|u\|_{L^4}^4 + \gamma_0\|v\|_{L^2}^2 \\ &\quad\leq \frac{4\alpha l(\mu+\epsilon_1)^2}{k} + g_2(t) = g_3(t). \end{aligned} \tag{38}$$

Denoting $d = \lambda_1 \min\{2, \gamma_0\}$ and applying Gronwall's inequality, we have

$$\alpha\bigl\|u(t)\bigr\|_{L^2}^2 + \bigl\|Pv(t)\bigr\|_{L^2}^2 \leq \bigl(\alpha\bigl\|u(s)\bigr\|_{L^2}^2 + \bigl\|Pv(s)\bigr\|_{L^2}^2\bigr)e^{-d(t-s)} + \int_s^t g_3(\tau)e^{-d(t-\tau)}\,d\tau,$$

for all $t_0 \leq s \leq t$. For $t = -1$, $s = t_0$, we get

$$\begin{aligned} &\alpha\bigl\|u(-1)\bigr\|_{L^2}^2 + \bigl\|Pv(-1)\bigr\|_{L^2}^2 \\ &\quad\leq \bigl(\alpha\bigl\|u(t_0)\bigr\|_{L^2}^2 + \bigl\|Pv(t_0)\bigr\|_{L^2}^2\bigr)e^{d(1+t_0)} + \int_{t_0}^{-1} g_3(\tau)e^{d(1+\tau)}\,d\tau \\ &\quad\leq \bigl(\alpha\bigl\|A(t_0)\bigr\|_{L^2}^2 + \alpha\bigl\|W_A(t_0)\bigr\|_{L^2}^2 + \bigl\|Ph(t_0)\bigr\|_{L^2}^2 + \bigl\|PW_h(t_0)\bigr\|_{L^2}^2\bigr)e^{d(1+t_0)} \\ &\qquad + \int_{-\infty}^{-1} g_3(\tau)e^{d(1+\tau)}\,d\tau. \end{aligned}$$

Set

$$r_1^2(\omega) = 1 + \sup_{t_0\leq -1}\bigl(\alpha\bigl\|W_A(t_0)\bigr\|_{L^2}^2 + \bigl\|PW_h(t_0)\bigr\|_{L^2}^2\bigr)e^{d(1+t_0)} + \int_{-\infty}^{-1} g_3(\tau)e^{d(1+\tau)}\,d\tau,$$

which is finite P-a.s. since $\|W_A(t_0)\|_{L^2}^2$, $\|PW_h(t_0)\|_{L^2}^2$ and $g_3(\tau,\omega)$ have at most polynomial growth for t_0 and τ, respectively, as they tend to $-\infty$. Given $\rho > 0$ such that $\alpha\|A(t_0)\|_{L^2}^2 + \|Ph(t_0)\|_{L^2}^2 \leq \rho^2$, there exists $\bar{t}$ such that $e^{d(1+t_0)}\rho^2 \leq 1$.

For the second case $\alpha < 0$, we take $\alpha_1 = -\alpha > 0$. Letting $\epsilon_3 = \frac{k\alpha_1}{2}$, we observe that (29) becomes

$$\begin{aligned} &\frac{d}{dt}\|Pv\|_{L^2}^2 - (2m+\epsilon_2)\|v\|_{L^2}^2 + 2\nu\|\partial_x v\|_{L^2}^2 \\ &\quad\leq -2\alpha\int_0^l |u|^2 v\,dx + \frac{k\alpha_1}{2}\|u\|_{L^4}^4 + C(\epsilon_2,k,\alpha_1)\bigl(\|W_A\|_{L^4}^4 + \|W_h\|_{L^2}^2\bigr). \end{aligned} \tag{39}$$

Multiplying (25) by α_1 and putting it into (39), we get

$$\frac{d}{dt}\left(\alpha_1\|u\|_{L^2}^2+\|Pv\|_{L^2}^2\right)+2\alpha_1\|\partial_x u\|_{L^2}^2+\frac{k\alpha_1}{2}\|u\|_{L^4}^4+2\nu\|\partial_x v\|_{L^2}^2$$
$$\leq 2\alpha_1(\mu+\epsilon_1)\|u\|_{L^2}^2+(2m+\epsilon_2+\epsilon_2\alpha_1)\|v\|_{L^2}^2+4\alpha_1\int_0^l |u|^2 v\,dx+g_4(t), \tag{40}$$

where

$$g_4(t)=\alpha_1 C\big(\epsilon_1,\epsilon_2,|\mu|\big)\big(\|W_A\|_{L^2}^2+\|W_A\|_{L^4}^4+\|W_h\|_{L^4}^4\big)+C(\epsilon_2,k,\alpha_1)\big(\|W_A\|_{L^4}^4+\|W_h\|_{L^2}^2\big).$$

We choose a positive number $k_1<k$ such that the following inequalities hold:

$$2\alpha_1(\mu+\epsilon_1)\|u\|_{L^2}^2\leq\frac{k_1\alpha_1}{4}\|u\|_{L^4}^4+\frac{4\alpha_1 l(\mu+\epsilon_1)^2}{k_1} \tag{41}$$

and

$$4\alpha_1\int_0^l |u|^2 v\,dx\leq\frac{k_1\alpha_1}{4}\|u\|_{L^4}^4+\frac{16\alpha_1}{k_1}\|v\|_{L^2}^2. \tag{42}$$

Applying Poincaré inequality once again, we have

$$2\nu\|\partial_x v\|_{L^2}^2-\left(2m+\epsilon_2+\epsilon_2\alpha_1+\frac{16\alpha_1}{k_1}\right)\|v\|_{L^2}^2$$
$$\geq\left(2\nu\lambda_1-2m-\epsilon_2-\epsilon_2\alpha_1-\frac{16\alpha_1}{k_1}\right)\|v\|_{L^2}^2$$
$$\geq\lambda_1\gamma_1\|Pv\|_{L^2}^2+\gamma_1\|v\|_{L^2}^2. \tag{43}$$

We take $2\epsilon_2(1+\alpha_1)=2\nu\lambda_1-2m-\frac{16\alpha_1}{k_1}$ and $2\gamma_1=2\nu\lambda_1-2m-\epsilon_2(1+\alpha_1)-\frac{16\alpha_1}{k_1}$, provided $2\nu\lambda_1-2m-\frac{16\alpha_1}{k_1}>0$. Combining (41)–(43) with (40) yields

$$\frac{d}{dt}\left(\alpha_1\|u\|_{L^2}^2+\|Pv\|_{L^2}^2\right)+\alpha_1\lambda_1\|u\|_{L^2}^2+\lambda_1\gamma_1\|Pv\|_{L^2}^2+\frac{(k-k_1)\alpha_1}{2}\|u\|_{L^4}^4+\gamma_1\|v\|_{L^2}^2$$
$$\leq\frac{4\alpha_1 l(\mu+\epsilon_1)^2}{k_1}+g_4(t)\triangleq g_5(t).$$

In a similar way to the first case, we have

$$r_2^2(\omega)=1+\sup_{t_0\leq -1}\left(\alpha_1\|W_A(t_0)\|_{L^2}^2+\|PW_h(t_0)\|_{L^2}^2\right)e^{\tilde{d}(1+t_0)}+\int_{-\infty}^{-1}g_5(\tau)e^{\tilde{d}(1+\tau)}\,d\tau,$$

where $\tilde{d}=\lambda_1\min\{1,\gamma_1\}$ is such that $\alpha_1\|u(-1)\|_{L^2}^2+\|Pv(-1)\|_{L^2}^2\leq r_2^2(\omega)$. Therefore, we can take $r(\omega)=\max\{r_1(\omega),r_2(\omega)\}$ as a desired random radius. □

Lemma 4.2 *There exist random variables $c_i(\omega)$ ($i=1,2,3,4$) and a fixed time $\bar{t}\leq -1$ such that for all $\rho>0$ and $t_0\leq\bar{t}$ with $|\alpha|\|A(t_0)\|_{L^2}^2+\|Ph(t_0)\|_{L^2}^2\leq\rho^2$, the solutions of (17)–(20) satisfy*

$$\int_{-1}^0\|u(s)\|_{L^4}^4\,ds\leq c_1(\omega),\qquad\int_{-1}^0\|v(s)\|_{L^2}^2\,ds\leq c_2(\omega),$$

and

$$\int_{-1}^{0}\|\partial_x u(s)\|_{L^2}^2\,ds \le c_3(\omega), \qquad \int_{-1}^{0}\|\partial_x v(s)\|_{L^2}^2\,ds \le c_4(\omega).$$

Proof We only consider the case $\alpha > 0$, and the other case $\alpha < 0$ can be proved similarly. Integrating (38) over $[-1,0]$, we have

$$\begin{aligned}
&\int_{-1}^{0}\left[\frac{k\alpha}{4}\|u(s)\|_{L^4}^4 + \gamma_0\|v(s)\|_{L^2}^2 + \alpha\|\partial_x u(s)\|_{L^2}^2 + \nu\|\partial_x v(s)\|_{L^2}^2\right]ds \\
&\quad \le \alpha\|u(-1)\|_{L^2}^2 + \|Pv(-1)\|_{L^2}^2 + \int_{-1}^{0} g_3(\tau)\,d\tau. \qquad (44)
\end{aligned}$$

Then the desired estimates follow immediately from Lemma 4.1. □

Next we consider the absorption in $H^1 \times L^2$ at time $t = 0$.

Lemma 4.3 *There exist a random radius $r_3(\omega)$ and a fixed time $\bar{t} \le -1$ such that for all $\rho > 0$ and $t_0 \le \bar{t}$ with $|\alpha|\|A(t_0)\|_{L^2}^2 + \|Ph(t_0)\|_{L^2}^2 \le \rho^2$, the solutions of (17)–(20) satisfy the inequality*

$$\|\partial_x u(0,\omega,A(t_0) - W_A(t_0))\|_{L^2}^2 + \|v(0,\omega,h(t_0) - W_h(t_0))\|_{L^2}^2 \le r_3^2(\omega).$$

Proof Multiplying (17) by $-\partial_x^2 u$, integrating with respect to x over $(0,l)$, and taking the real part, we obtain

$$\begin{aligned}
&\frac{d}{dt}\|\partial_x u\|_{L^2}^2 - 2\mu\|\partial_x u\|_{L^2}^2 + 2\|\partial_x^2 u\|_{L^2}^2 \\
&\quad = -2\,\mathrm{Re}\int_0^l \partial_x^2\bar{u}\big[\mu W_A - k|u + W_A|^2(u + W_A) + (u + W_A)(v + W_h)\big]\,dx = I_6 + I_7. \qquad (45)
\end{aligned}$$

Then we can estimate the terms on the RHS of (44) as follows:

$$\begin{aligned}
I_6 &= -2\,\mathrm{Re}\int_0^l \partial_x^2\bar{u}\big[\mu W_A - k|u + W_A|^2(u + W_A)\big]\,dx \\
&\le 2\int_0^l |\partial_x^2\bar{u}|\big[|\mu||W_A| + k\big(|u|^3 + 3|u|^2|W_A| + 3|u||W_A|^2 + |W_A|^3\big)\big]\,dx \\
&\le 2|\mu|\|\partial_x^2 u\|_{L^2}\|W_A\|_{L^2} + 2k\|\partial_x^2 u\|_{L^2}\|u\|_{L^6}^3 + 2k\|\partial_x^2 u\|_{L^2}\|u\|_{L^6}^2\|W_A\|_{L^6} \\
&\quad + 2k\|\partial_x^2 u\|_{L^2}\|u\|_{L^6}\|W_A\|_{L^6}^2 + 2k\|\partial_x^2 u\|_{L^2}\|W_A\|_{L^6}^3 \\
&\le \epsilon_5\|\partial_x^2 u\|_{L^2}^2 + C(\epsilon_5,|\mu|)\big(\|u\|_{L^6}^6 + \|W_A\|_{L^2}^2 + \|W_A\|_{L^6}^6\big) \\
&\le \epsilon_5\|\partial_x^2 u\|_{L^2}^2 + C(\epsilon_5,|\mu|,\tilde{c})\big(\|\partial_x u\|_{L^2}^2 + \|W_A\|_{L^2}^2 + \|W_A\|_{L^6}^6\big), \qquad (46)
\end{aligned}$$

where the last inequality in (45) can be obtained by the interpolation inequality below and Theorem 3.1, while

$$\|u\|_{L^6}^6 \le c\|\partial_x u\|_{L^2}^2\|u\|_{L^2}^4 \le \tilde{c}\|\partial_x u\|_{L^2}^2. \qquad (47)$$

Furthermore, for the last term of (44), we obtain

$$
\begin{aligned}
I_7 &= -2\,\mathrm{Re}\int_0^l \partial_x^2\bar{u}(u+W_A)(v+W_h)\,dx \\
&\le 2\int_0^l \left|\partial_x^2\bar{u}\right|\bigl(|u||v|+|u||W_h|+|v||W_A|+|W_A||W_h|\bigr)\,dx \\
&\le 2\|v\|_{L^\infty}\left\|\partial_x^2 u\right\|_{L^2}\|u\|_{L^2}+2\left\|\partial_x^2 u\right\|_{L^2}\|u\|_{L^4}\|W_h\|_{L^4} \\
&\quad + 2\|v\|_{L^\infty}\left\|\partial_x^2 u\right\|_{L^2}\|W_A\|_{L^2}+2\left\|\partial_x^2 u\right\|_{L^2}\|W_A\|_{L^4}\|W_h\|_{L^4} \\
&\le \epsilon_5\left\|\partial_x^2 u\right\|_{L^2}^2+C(\epsilon_5)\bigl[\|v\|_{L^\infty}^2\bigl(\|u\|_{L^2}^2+\|W_A\|_{L^2}^2\bigr)+\|u\|_{L^4}^4+\|W_A\|_{L^4}^4+\|W_h\|_{L^4}^4\bigr] \\
&\le \epsilon_5\left\|\partial_x^2 u\right\|_{L^2}^2+C(\epsilon_5,\tilde{c})\bigl(\|\partial_x v\|_{L^2}^2+\|u\|_{L^4}^4+\|W_A\|_{L^4}^4+\|W_h\|_{L^4}^4\bigr).
\end{aligned}
\tag{48}
$$

Taking $\epsilon_5 = 1/2$, from (45)–(47), we observe that (44) becomes

$$
\frac{d}{dt}\|\partial_x u\|_{L^2}^2+\left\|\partial_x^2 u\right\|_{L^2}^2 \le C\bigl(|\mu|,\tilde{c}\bigr)\bigl(\|\partial_x u\|_{L^2}^2+\|\partial_x v\|_{L^2}^2+\|u\|_{L^4}^4+g_6(t)\bigr),
\tag{49}
$$

where

$$
g_6(t) = \|W_A\|_{L^2}^2+\|W_A\|_{L^4}^4+\|W_A\|_{L^6}^6+\|W_h\|_{L^4}^4.
$$

On the other hand, multiplying (18) by $-\partial_x^2 v$ and integrating with respect to x over $(0,l)$, we get

$$
\frac{d}{dt}\|v\|_{L^2}^2-2m\|\partial_x v\|_{L^2}^2+2\nu\left\|\partial_x^2 v\right\|_{L^2}^2 = -2\int_0^l\bigl(mW_h-\alpha|u+W_A|^2\bigr)\partial_x^2 v\,dx = I_8+I_9.
$$

By Hölder and Cauchy inequalities, the RHS of (49) can be estimated as

$$
I_8 = -2m\int_0^l W_h\partial_x^2 v\,dx \le \epsilon_6\left\|\partial_x^2 v\right\|_{L^2}^2+C\bigl(\epsilon_6,|m|\bigr)\|W_h\|_{L^2}^2,
$$

and

$$
\begin{aligned}
I_9 &= 2\alpha\int_0^l |u+W_A|^2\partial_x^2 v\,dx \le 2|\alpha|\int_0^l\bigl(|u|^2+2|u||W_A|+|W_A|^2\bigr)\left|\partial_x^2 v\right|\,dx \\
&\le \epsilon_6\left\|\partial_x^2 v\right\|_{L^2}^2+C\bigl(\epsilon_6,|\alpha|\bigr)\bigl(\|u\|_{L^4}^4+\|W_A\|_{L^4}^4\bigr).
\end{aligned}
\tag{50}
$$

Taking $\epsilon_6 = \nu/2$ and putting above estimates into (49), we have

$$
\frac{d}{dt}\|v\|_{L^2}^2+\nu\left\|\partial_x^2 v\right\|_{L^2}^2 \le C\bigl(\nu,|\alpha|,|m|\bigr)\bigl(\|u\|_{L^4}^4+\|\partial_x v\|_{L^2}^2+g_7(t)\bigr),
$$

where

$$
g_7(t) = \|W_A\|_{L^4}^4+\|W_h\|_{L^2}^2.
$$

Combining (48) with (50), we obtain

$$\frac{d}{dt}\left(\|\partial_x u\|_{L^2}^2 + \|v\|_{L^2}^2\right) \le C\left(\|u\|_{L^4}^4 + \|\partial_x u\|_{L^2}^2 + \|\partial_x v\|_{L^2}^2 + g_6(t) + g_7(t)\right),$$

where $C = C(|\mu|, \tilde{c}, \nu, |\alpha|, |m|)$. Integrating over an arbitrary interval $[s, 0]$, we get

$$\begin{aligned}\left\|\partial_x u(0)\right\|_{L^2}^2 + \left\|v(0)\right\|_{L^2}^2 \le \left\|\partial_x u(s)\right\|_{L^2}^2 + \left\|v(s)\right\|_{L^2}^2 + C\int_s^0 \Big(\left\|u(\tau)\right\|_{L^4}^4 + \left\|\partial_x u(\tau)\right\|_{L^2}^2 \\ + \left\|\partial_x v(\tau)\right\|_{L^2}^2 + g_6(\tau) + g_7(\tau)\Big)\, d\tau.\end{aligned}$$

Integrating again with respect to s over $[-1, 0]$, we finally have

$$\begin{aligned}\left\|\partial_x u(0)\right\|_{L^2}^2 + \left\|v(0)\right\|_{L^2}^2 \le \int_{-1}^0 \left(\left\|\partial_x u(s)\right\|_{L^2}^2 + \left\|v(s)\right\|_{L^2}^2\right) ds + C\int_{-1}^0 \Big(\left\|u(\tau)\right\|_{L^4}^4 \\ + \left\|\partial_x u(\tau)\right\|_{L^2}^2 + \left\|\partial_x v(\tau)\right\|_{L^2}^2 + g_6(\tau) + g_7(\tau)\Big)\, d\tau.\end{aligned}$$

Then Lemma 4.3 follows from Lemma 4.2. □

In the end, using Lemma 4.3 and applying Theorem 2.6, we have the following claim.

Theorem 4.4 *Assume that* $\nu(\frac{\pi}{l})^2 - m > 0$ *for* $\alpha > 0$ *and* $\nu(\frac{\pi}{l})^2 - m > \frac{16|\alpha|}{k}$ *for* $\alpha < 0$. *Then the RDS generated by the stochastic KS–GL equations* (1)–(2) *with initial–boundary conditions possesses a global random attractor* $\mathcal{A}(\omega)$ *in* $L^2(0, l) \times H^{-1}(0, l)$.

Remark 4.5 We would like to indicate that ϵ_3 is specified in (28) to simplify the calculus, and the condition in Theorem 4.4 for $\alpha < 0$ may be improved without the specification of ϵ_3 there.

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