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COMMENTARY

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Notes on the result of solutions of the equilibrium equations

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Abstract

In this short note, we correct some expressions obtained by Wang et al. (Bound. Value Probl. 2015:230, 2015). The corrected expressions will be useful for evaluating the boundary behaviors of solutions of modified equilibrium equations with finite mass subject. Moreover, the correction of Theorem 2.1 is also given.

Keywords: Boundary behavior; Equilibrium equation; Finite mass

1 Introduction

The origin of our work lies in Wang et al. [1]. In [1], they investigated slow equilibrium equations with finite mass subject to a homogeneous Neumann-type boundary condition. As an application, the existence of solutions for Laplace equations with a Neumann-type boundary condition was also proved which has recently been used to study the Cauchy problem of Laplace equation by Wang [2].

However, there exist some misprints and erroneous expressions in [1]. Firstly, we correct some misprints in Sect. 2. Then we correct erroneous expressions in Sect. 3. The corrected versions will be useful for evaluating the boundary behaviors of solutions of the equilibrium equations with finite mass subject. Finally, we correct Theorem 2.1 in Sect. 4. The present notation and terminology is the same as in [1].

2 Some misprints

We are indebted to the anonymous reviewer for pointing out to us that the following should also be corrected in [1].

(I) A correct version of Abstract reads as follows.

The aim of this paper is to study the models of rotating stars with prescribed angular velocity. We prove that it can be formulated as a variational problem. As an application, we are also concerned with the existence of equilibrium solution.

(II) $\mathbb{R}^4$ and x_4 should be written as $\mathbb{R}^3$ and x_3, respectively.

(III) Introduction: instead of "3-D", there should be "4-D".

(IV) Some main references [1, 2, 3] should be corrected as follows:

[1] Auchmuty, G, Beals, R: Variational solutions of some nonlinear free boundary problems. Arch. Ration. Mech. Anal. 43, 255–271 (1971)

[2] Li, Y: On Uniformly Rotating Stars. Arch. Ration. Mech. Anal. 115, 367–393 (1991)

[3] Deng, Y, Yang, T: Multiplicity of stationary solutions to the Euler–Poisson equations. J. Differ. Equ. 231, 252–289 (2006)

(V) On page 2, line 12: instead of "P:", there should be "P_1:".

3 Corrected expressions

We find that [1, inequality (2.4)] is not correct and should be modified as (the sign before the function "$(\frac{M_1}{M})^{5/3}$" should be "+")

$$\begin{aligned} h_M - F(\rho) &\le \left(1 + \left(\frac{M_1}{M}\right)^{5/3} - \left(\frac{M_2}{M}\right)^{5/3} - \left(\frac{M_3}{M}\right)^{5/3}\right) h_M + \frac{C_1}{R_2} \\ &\quad + \frac{C_3}{R_2}\|\nabla\Phi_2\|_2 \\ &\le C_4 h_M M_1 M_3 + \frac{C_5}{R_2}\bigl(1 + \|\nabla\Phi_2\|_2\bigr). \end{aligned} \tag{2.4}$$

Therefore, the expressions in [1] that are derived by using [1, inequality (2.4)] need to be corrected. Specifically, [1, inequality (2.8)] should be modified as

$$\begin{aligned} -C_4 h_M \delta_0 M_{n,3} &\le C_4 h_M M_{n,1} M_{n,3} \\ &\le \frac{C_5}{R_2}\bigl(1 + \|\nabla\Phi_{0,2}\|_2\bigr) + C_5\|\nabla\Phi_{n,2} - \nabla\Phi_{0,2}\|_2 \\ &\quad + \bigl|F(T\rho_n) - h_M\bigr| + \rho_0. \end{aligned} \tag{2.8}$$

These corrections will be useful for the readers who want to use [1, Theorem 2.1] to evaluate the boundary behavior of solutions of the equilibrium equations with finite mass subject.

4 Corrected Theorem 2.1

A correction of Theorem 2.1 in [1] reads as follows.

Theorem 2.1 *Let P_1 hold. Let $(\rho_n)_{n=1}^{\infty} \in \mathcal{A}_M$ be a minimizing sequence of F. Then there exists a subsequence, still denoted by $(\rho_n)_{n=1}^{\infty}$, and a sequence of translations $T\rho_n := \rho_n(\cdot + a_n e_3)$, where a_n are constants, and $e_3 = (0,0,1)$, such that*

$$F(\rho_0) = \inf_{\mathcal{A}_M} F(\rho) = h_M + \rho_0$$

and $T\rho_n \rightharpoonup \rho_0$ weakly in $L^{\frac{4}{3}}(\mathbb{R}^3)$. For the induced potentials, we have $\nabla\Phi_{T\rho_n} \rightharpoonup \nabla\Phi_{\rho_0}$ weakly in $L^2(\mathbb{R}^3)$.

Proof Define

$$I_{lm} := \int\int \frac{\rho_l(x)\rho_m(y)}{x-y}\,dy\,dx$$

for $l, m = 1, 2, 3$.

Let $\rho = \rho_1 + \rho_2 + \rho_3$, where $\rho_1 = \chi_{B_{R_1}}\rho$, $\rho_2 = \chi_{B_{R_1,R_2}}\rho$, and $\rho_3 = \chi_{B_{R_2}}\rho$. So we have

$$F(\rho) = F(\rho_1) + F(\rho_2) + F(\rho_3) - I_{12} - I_{13} - I_{23}.$$

Choosing $R_2 > 2R_1$, we have

$$I_{13} \leq 2 \int_{B_{R_1}} \frac{\rho(x)}{R_1}\, dx \int_{B_{R_2,\infty}} \frac{\rho(y)}{|y|^2}\, dy \leq \frac{C_1}{R_2}.$$

Next, we estimate I_{12} and I_{23}:

$$\begin{aligned} I_{12} + I_{23} &= -\int \rho_1 \Phi_2\, dx - \int \rho_2 \Phi_3\, dx = \frac{1}{4\pi g} \int \nabla(\Phi_1 + \Phi_3) \cdot \nabla \Phi_2\, dx \\ &\leq C_2 \|\rho_1 + \rho_3\|_{\frac{6}{5}} \|\nabla \Phi_2\|_2 \leq C_3 \|\nabla \Phi_2\|_2. \end{aligned}$$

If we define $M_l = \int \rho_l\, dx$, then it is easy to see that $M = M_1 + M_2 + M_3$.

The remaining proofs are carried out in the same way as for Theorem 2.1 in [1], except that instead of the erroneous expressions (2.4) and (2.8), we have to use their corrected versions given in Sect. 2. □

5 Conclusions

In this note, we corrected some expressions obtained by Wang et al. [1]. The corrected expressions will be useful for evaluating the boundary behavior of solutions of the equilibrium equations with finite mass subject. Moreover, the correction of Theorem 2.1 was also given.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
BR drafted the manuscript. [illegible] LW helped to revise the written English and revised the manuscript according to the referee report. All authors read and approved the final manuscript.

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References
1. Wang, J., Pu, J., Huang, B., Shi, G.: Boundary value behaviors for solutions of the equilibrium equations with angular velocity. Bound. Value Probl. **2015**, 230 (2015)
2. Wang, Y.: A regularization method for the Cauchy problem of Laplace equation. Acta Anal. Funct. Appl. **19**(2), 199–205 (2017)