A scheme for estimating the location and tangential plane of a reflector

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Magmatic reflectors have been detected beneath some volcanic regions by analysing some reflection phases of seismograms. In this letter we consider a problem to determine the location and tangential plane of such a reflector from the ray parameter, back-azimuth and travel time of a reflection phase, and present a scheme to solve it. It is then shown that the problem is essentially a one-dimensional root-finding problem whose independent variable is only the depth of the reflection point, and thus it can be solved by a line search method such as the one-dimensional grid search. While homogeneous velocity structure models have been used in the conventional analyses, the present scheme employs a vertically inhomogeneous one and can also easily treat converted phases at the reflector.

1. Introduction

Last two decades magmatic reflectors have been detected beneath some volcanic regions by analysing reflection phases of seismograms (e.g., Mizoue, 1980; Mizoue and Ishiketa, 1988; Horiuchi *et al.*, 1988; Hori and Hasegawa, 1991; Matsumoto and Hasegawa, 1996). Approaches that have been employed for the detection of these reflectors in the previous studies may be grouped into the following:

(1) Reflector is assumed to be a single horizontal plane, and its depth is estimated from travel time data of a reflection phase observed at all stations for all events (e.g., Sanford *et al.*, 1973; Mizoue, 1980; Iwase *et al.*, 1989; Hartse *et al.*, 1992; Balch *et al.*, 1997).

(2) Reflector's tangential plane at each reflection point is approximated by a horizontal plane to determine the location of a reflection point for the reflection phase for each event-station pair, and the entire geometry of the reflector, whose shape is usually assumed to be a single inclined plane, is estimated from the spatial distribution of the reflection points (e.g., Sanford *et al.*, 1973; Mizoue *et al.*, 1982; Mizoue and Ishiketa, 1988; Inamori *et al.*, 1992).

(3) Reflector is assumed to be a single inclined flat plane, and its location, dip direction and dip angle are estimated by use of all available travel time data of reflection phases in combination with hypocenter coordinates (Sanford *et al.*, 1973; Horiuchi *et al.*, 1988; Hori and Hasegawa, 1991, 1999).

(4) Curved reflector surface is approximated by ensemble of many small inclined planes (like *patchwork*) which are determined from travel time data of reflection phases for different combinations of station and hypocenters (Matsumoto and Hasegawa, 1996, 1997).

In the above four approaches, the main attribute of observed data used for analysis is the travel time of reflection phases. In most studies employing the approaches, velocity structure was assumed to be homogeneous above the reflector in the calculation of ray paths because of simplicity, although it is possible to relax this assumption (e.g., Hartse *et al.*, 1992; Balch *et al.*, 1997).

In this short note we consider a different approach from the previous studies mentioned above. We consider a problem (situation) to determine the location of a reflection point and the tangential plane touching the reflector surface at the reflection point from the ray parameter (inverse of apparent velocity) and back-azimuth of a reflection phase as well as the travel time, and show a simple algorithm to solve it. The ray parameter and back-azimuth can in practice be evaluated, for example, by an array analysis such as the semblance analysis (e.g., Nikolaev and Troitskiy, 1987; Korn, 1988; Kuwahara *et al.*, 1990). In the present algorithm, vertically inhomogeneous (layered) velocity models are employed, and reflected converted phases can be also easily treated. For convenience, we hereafter call the tangential plane of the reflector at the reflection point the "*reflector facet*".

2. Problem Setting

Throughout this note we employ a right handed Cartesian coordinate system (x, y, z), and take coordinate directions $[x, y, z]$ as [North, East, Down]. Further we set the origin (O) of the coordinate system to be at the observation point on the ground where the ray parameter, back-azimuth and travel time of a reflection phase have been measured. In case of array observation, the origin of array can be chosen as the *observation point*.

Figure 1 shows the geometry of the problem to be solved. Points S and E denote the hypocenter and epicenter of a source, respectively, and R and P are the reflection point and its projection onto the plane of $z = 0$ (i.e., ground surface). Vector $\boldsymbol{n}$ is the upward unit normal to the reflector facet. Vector $\boldsymbol{p}$ is the horizontal slowness vector of the reflected

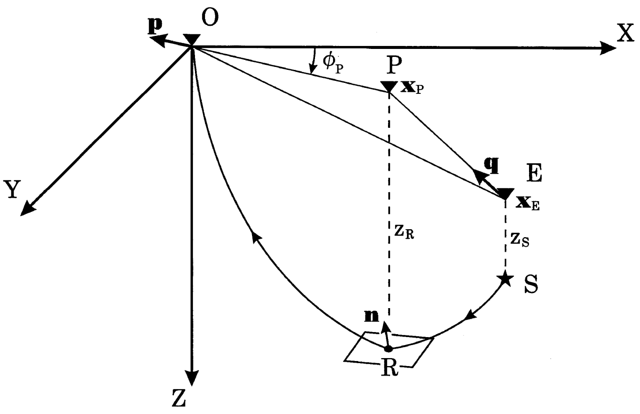

Fig. 1. Configuration of the problem to be solved. Points S and E are the source and its epicenter. R and P are the reflection point and its projection onto the plane of $z = 0$ (ground surface). $\boldsymbol{n}$ is the upward normal vector of the reflector facet. $\boldsymbol{p}$ and $\boldsymbol{q}$ are the horizontal slowness vector at the observation point and at the source, respectively. ϕ_P is the back-azimuth of the reflection phase observed at O (i.e., the back-azimuth of $\boldsymbol{p}$).

ray which is defined by the measured ray parameter (p) and back-azimuth (ϕ_P) of the reflection phase observed at the observation point O as follows:

$$\boldsymbol{p} = (-p\cos\phi_P, -p\sin\phi_P, 0)^T, \tag{1}$$

where T denotes the *transpose* of vector.

In the problem we will find the coordinates of the reflection point R (or the position ($\boldsymbol{x}_P$) of point P and the depth (z_R) of the reflector point) and the normal vector of the reflector facet $\boldsymbol{n}$ when we know the position ($\boldsymbol{x}_E$) of the epicenter E and depth (z_S) of a source, travel time (T_{data}) and horizontal slowness vector $\boldsymbol{p}$ of the reflection phase observed at O as well as the subsurface velocity structure. For convenience, we separate the subsurface velocity structure into the following two: one is the velocity profile $V_{(S)} = V_{(S)}(z)$ of a seismic phase (P- or S-wave velocity) along the ray from the source S to the reflection point R, and the other is the velocity profile $V_{(O)} = V_{(O)}(z)$ along the ray from R to the observation point O. When we consider converted phases at the reflector, $V_{(S)}$ and $V_{(O)}$ are different. In case of SP reflection (SxP phase), for instance, $V_{(S)}$ is a S-wave velocity profile, while $V_{(O)}$ is a P-wave one. Even in case of unconverted reflection (PxP or SxS phase), we can employ different velocity profiles of $V_{(S)}$ and $V_{(O)}$ for shallow parts above R, so that we can take account of different site conditions between around the source and the receiver.

3. Algorithm for Solution

In this section we show that the problem set up in the previous section can be solved by a line search for z_R to describe a solution algorithm. There are several methods for a line search, for example, the one-dimensional grid search, the bisection method and the secant method (e.g., Press *et al.*, 1992). In this section we assume the use of the simplest line search method, the one-dimensional grid search, to make the structure of the solution algorithm show up although any more sophisticated methods for line search are also applicable to the solution algorithm.

Setting a trial value to z_R, which must satisfy

$$1/V_{(O)}(z) > p \quad \text{for} \quad z \le z_R, \tag{2}$$

the distance between the observation point O and point P, $r_P \equiv \overline{OP}$, associated with the trial value of z_R is calculated by

$$r_P = X(p, 0, z_R), \tag{3}$$

where

$$X(p,z,z') = \frac{pV^{(n)}(z_n - z)}{\sqrt{1-(pV^{(n)})^2}} + \sum_{i=n+1}^{n'-1} \frac{pV^{(i)}h^{(i)}}{\sqrt{1-(pV^{(i)})^2}} + \frac{pV^{(n')}(z' - z_{n'-1})}{\sqrt{1-(pV^{(n')})^2}}, \tag{4}$$

$$z_{n-1} < z < z_n, \qquad z_{n'-1} < z' < z_{n'}.$$

$X(p,z,z')$ is the horizontal distance over which a seismic phase travels along the ray with ray parameter p from a depth level z to a deeper level z' (or from z' to z), where the levels z and z' are in the (n)-th and the (n')-th layer, respectively (Fig. 2). $V^{(i)}$ is the velocity of the considered phase in the (i)-th layer whose thickness is $h^{(i)}$. Equation (3) uses those of the velocity profile $V_{(O)}$ along the reflected (or reflected converted) ray from the reflection point R to the observation point O for $V^{(i)}$ and $h^{(i)}$. Since point P must fall on the line which is through O and parallel to $\boldsymbol{p}$, the position $\boldsymbol{x}_P$ of point P (i.e., the horizontal coordinates of the reflection point R) is derived as

$$\boldsymbol{x}_P = (r_P\cos\phi_P, r_P\sin\phi_P, 0)^T. \tag{5}$$

Note that this derived horizontal position $\boldsymbol{x}_P$ of the reflection point is corresponding to the selected trial value of z_R. The condition that the chosen value of z_R and the derived position $\boldsymbol{x}_P$ is the solution of the problem set up in the previous section, is that the corresponding two-way travel time of the reflection phase calculated assuming them is equal to the measured

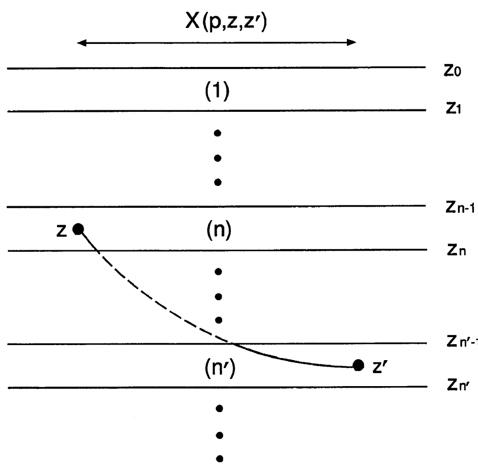

Fig. 2. Configuration of velocity model for Eqs. (4) and (7). Point at a depth level z and point at another level z' ($z < z'$), between which horizontal distance is $X(p,z,z')$, are connected by a ray with ray parameter p. One-way travel time along this ray from level z to level z' (or from level z' to level z) is $T(p,z,z')$.

travel time T_{data}. Next we describe a scheme to calculate the two-way travel time of the reflection phase.

The two-way travel time is the sum of the travel time (T_{SR}) from the source S to the reflection point R and that (T_{RO}) from R to the observation point O. The latter is obtained as

$$T_{\mathrm{RO}} = T(p, z_{\mathrm{O}}, z_{\mathrm{R}}), \tag{6}$$

where

$$\begin{aligned} T(p, z, z') = {} & \frac{z_n - z}{V^{(n)}\sqrt{1-(pV^{(n)})^2}} \\ & + \sum_{i=n+1}^{n'-1} \frac{h^{(i)}}{V^{(i)}\sqrt{1-(pV^{(i)})^2}} \\ & + \frac{z' - z_{n'-1}}{V^{(n')}\sqrt{1-(pV^{(n')})^2}}, \end{aligned} \tag{7}$$

$$z_{n-1} < z < z_n, \qquad z_{n'-1} < z' < z_{n'}.$$

Equation (6) employs the velocity profile $V_{(\mathrm{O})}$ along the reflected (or reflected converted) ray from R to O for $V^{(i)}$ and $h^{(i)}$ in Eq. (7).

To compute T_{SR}, it is necessary to find the ray parameter (q) of the ray at source which is invariant along the path from the source S to the reflection point R. This can be done by solving the following equation for q with the same formulation as Eq. (4):

$$X(q, z_{\mathrm{S}}, z_{\mathrm{R}}) = |\boldsymbol{x}_{\mathrm{P}} - \boldsymbol{x}_{\mathrm{E}}|, \tag{8}$$

where $V^{(i)}$ and $h^{(i)}$ in Eq. (4) for the left hand side of this equation are those of the velocity profile $V_{(\mathrm{S})}$ along the ray path from S to R. Equation (8) can be easily solved numerically by a line search scheme such as the one-dimensional grid search. If Eq. (8) does not have any real root, we select a new trial value of z_{R} which is less than the original one, and restart from Eq. (3).

Travel time T_{SR} from the source S to the reflection point R is calculated with Eq. (7) by

$$T_{\mathrm{SR}} = T(q, z_{\mathrm{S}}, z_{\mathrm{R}}), \tag{9}$$

where the velocity profile $V_{(\mathrm{S})}$ is employed. The two-way travel time ($T_{\mathrm{SRO}}(z_{\mathrm{R}})$) of the reflection phase associated with the selected trial value of z_{R} is then

$$T_{\mathrm{SRO}}(z_{\mathrm{R}}) = T_{\mathrm{SR}}(z_{\mathrm{R}}) + T_{\mathrm{RO}}(z_{\mathrm{R}}). \tag{10}$$

If this calculated value of T_{SRO} is equal to the measured travel time T_{data} (in practice the difference between them is less than a pre-specified tolerance), we regard the chosen trial value of z_{R} and the calculated position $\boldsymbol{x}_{\mathrm{P}}$ as the solution, and use them to derive the last unknown quantity of the problem, the normal vector of the reflector facet $\boldsymbol{n}$. Otherwise, we select a next trial value of z_{R} along a rule of the grid search to restart from Eq. (3), and iterate the process described above until we find a value of z_{R} which we can regard as the solution.

Upward normal vector $\boldsymbol{n}$ of the reflector facet is obtained from *Snell's law* for reflection, that is, the tangential component of the slowness vector of the reflected (or reflected converted) ray at the reflection point is equal to that of the incident ray on the reflector facet just before the reflection:

$$\boldsymbol{n} = \frac{\boldsymbol{P} - \boldsymbol{Q}}{|\boldsymbol{P} - \boldsymbol{Q}|}, \tag{11}$$

where $\boldsymbol{P}$ and $\boldsymbol{Q}$ are the slowness vector of the reflected (or reflected converted) ray (upgoing ray) at the reflection point and that of the incident ray (downgoing ray) on the reflector facet just before the reflection (reflection conversion), respectively (Fig. 3). $\boldsymbol{P}$ and $\boldsymbol{Q}$ can be derived from the quantities known so far by

$$\boldsymbol{P} = (p_x, p_y, -\nu)^{\mathrm{T}}, \qquad \boldsymbol{Q} = (q_x, q_y, \xi)^{\mathrm{T}}, \tag{12}$$

where $p_{x,y}$ and $q_{x,y}$ are x- and y-component of $\boldsymbol{p}$ and the following vector $\boldsymbol{q}$:

$$\boldsymbol{q} = q(\boldsymbol{x}_{\mathrm{P}} - \boldsymbol{x}_{\mathrm{E}})/|\boldsymbol{x}_{\mathrm{P}} - \boldsymbol{x}_{\mathrm{E}}|. \tag{13}$$

This is the horizontal slowness vector of the ray at source (see Fig. 1). The vertical component of $\boldsymbol{P}$ and $\boldsymbol{Q}$, $-\nu$ and ξ are calculated as

$$\nu = \sqrt{[1/V_{(\mathrm{O})}(z_{\mathrm{R}})]^2 - p^2}, \qquad \xi = \sqrt{[1/V_{(\mathrm{S})}(z_{\mathrm{R}})]^2 - q^2}. \tag{14}$$

4. Discussion and Summary

In the previous sections we have set up a problem to determine the position of the reflection point and the normal vector of the reflection facet of a reflected (or reflected converted) phase from the travel time, ray parameter and back-azimuth of the reflection phase, and have shown an algorithm to find the solution. The algorithm is summarised as follows:

[1] Select a trial value of z_{R}.

[2] Calculate the horizontal position $\boldsymbol{x}_{\mathrm{P}}$ by Eqs. (3) to (5).

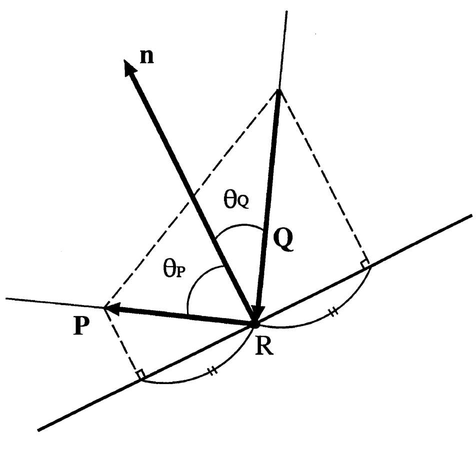

Fig. 3. Reflection (reflection conversion) at a reflector facet. $\boldsymbol{Q}$ is the slowness vector of the incident ray at the reflection point R ($|\boldsymbol{Q}| = 1/V_{(\mathrm{S})}(z_{\mathrm{R}})$), while $\boldsymbol{P}$ is that of the reflected (or reflected converted) ray at R ($|\boldsymbol{P}| = 1/V_{(\mathrm{O})}(z_{\mathrm{R}})$). Angles θ_Q and θ_P that the incident and the reflected rays make with the upward normal vector ($\boldsymbol{n}$) of the reflection facet are governed by Snell's law: $|\boldsymbol{P}|\sin\theta_P = |\boldsymbol{Q}|\sin\theta_Q$. In case of the reflection without conversion, $\theta_P = \theta_Q$; while in case of the reflection with conversion, $\theta_P \neq \theta_Q$. Note that in both cases $\boldsymbol{n}$ is parallel to the composite vector $\boldsymbol{P} - \boldsymbol{Q}$.

[3] Find the ray parameter q of the ray at source by solving Eq. (8). If Eq. (8) does not have any real root, select a new trial value of z_R which is less than the original one, and return to step [2].

[4] Calculate the two-way travel time of the reflection phase T_{SRO} using Eqs. (6), (7), (9) and (10).

[5] Compare the calculated travel time T_{SRO} with the measured one T_{data} to determine whether the trial value of z_R and the $\boldsymbol{x}_P$ calculated in step [2] are the desired solution or not. If they are regarded as the solution, proceed to the next step [6]. Otherwise, select a next trial value of z_R along the rule of a line search method, and return to step [2].

[6] Calculate the normal vector of the reflection facet $\boldsymbol{n}$ by Eqs. (11) to (14).

This algorithm clearly shows that the depth of the reflection point z_R is only an independent variable of the problem considered here, and therefore it is essentially a one-dimensional root finding problem (e.g., Press *et al.*, 1992).

The derived position of the reflection point and normal vector of the reflector facet gives the equation of the reflector facet, which is represented as

$$n_x(x - r_P\cos\phi_P) + n_y(y - r_P\sin\phi_P) + n_z(z - z_R) = 0, \quad (15)$$

where $n_{x,y,z}$ are x-, y- and z-component of the normal vector $\boldsymbol{n}$. The dip angle δ and dip direction φ of the facet are also obtained as

$$\delta = \cos^{-1}(|n_z|), \quad (16)$$

and

$$\varphi = \mathrm{atan}(n_y, n_x), \quad (17)$$

where the definition of the dip angle and dip direction of the reflector facet is the same as that for fault plane employed by Aki and Rechards (1980) (see figure 4.13 in Aki and Rechards (1980)). Further, the equation of intersection line of the facet and x-z plane (North-South cross section) at the reflection point is expressed as

$$\tan\delta\cos\varphi(x - r_P\cos\phi_P) - (z - z_R) = 0, \quad (18)$$

and the equation of intersection line of the facet and y-z plane (East-West cross section) at the reflection point is

$$\tan\delta\sin\varphi(y - r_P\sin\phi_P) - (z - z_R) = 0. \quad (19)$$

The present new scheme for detecting reflector facets has been successfully applied to map a magmatic reflector beneath Unzen volcanic area, Shimabara Peninsula, Kyushu, Japan (Yamaguchi *et al.*, 1999). Yamaguchi *et al.* (1999) analysed the PxP reflection phase appearing on the records of four small arrays for the explosion seismic experiment conducted at Unzen volcano in 1995 to investigate the structure of the volcano. They performed the semblance analysis for each combination of array and shot to estimate the ray parameter and back-azimuth of the reflection phase, and then applied the present scheme to the estimated values with the travel times. Using the same velocity model as employed for earthquake hypocenter determination routine around Unzen area, they found a reflector exists at a depth of about 15 km beneath the western shore of the Shimabara Peninsula and it has a slight dip down to the southwest, which is reasonable for the magma system model of Unzen volcano inferred by Umakoshi *et al.* (1994) from the seismicity. As this successful example shows, the present scheme is also available for seismic experiment data as well as natural earthquake data.

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