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Adaptive power allocation and outage performance of cognitive best relay cooperation systems with multiple primary transceiver pairs and direct path between cognitive source and destination

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Abstract

Based on decode-and-forward (DF) protocol, this work focuses on the adaptive power allocation and outage performance of underlay cognitive radio and opportunistic relaying (UCR-OR) systems with direct path between cognitive source and destination. The UCR-OR systems suffer from the interference of multiple primary user (PU) pairs. Under the outage constraint of PUs and the cognitive peak transmit power limit, we first obtain the adaptive power allocation schemes for secondary transmitters. Secondly, we obtain the exact closed-form expression to the outage probability of UCR-OR systems by using appropriate mathematical proof. Finally, to obtain a clear insight and to highlight the effect of system parameters on the performance of UCR-OR systems, the asymptotic closed-form expression of outage probability is achieved with the assumption of high cognitive transmit power. The presented simulations show that, due to the adaptive power allocation employed, the outage probability of UCR-OR systems is decreasing with PUs' transmit power P_p when P_p is less than a specific value P_P^*. Only when the value of P_p is greater than P_P^* the outage probability is increasing gradually with the increase of P_P. When the transmit power of PUs is very high, the outage probability of UCR-OR systems tends to one. That is to say, in this case, the increase of PUs' transmit power degrades severely the performance of UCR-OR systems. Besides this, it is also found that the diversity gain of UCR-OR systems is proportional to the number of cognitive relays. The parameters of PUs only affect the coding gain of UCR-OR but not the diversity gain.

Keywords: Cognitive radio; Opportunistic relaying; Direct path; Multiple primary user pairs

1 Introduction

Since the electromagnetic spectrum is becoming more and more scarce, improving spectrum efficiency is becoming extremely important for the sustainable development of wireless communication systems and service. However, under the current command-and-control spectrum management policy, spectrum resource is not utilized sufficiently as reported by the Federal Communications Commission [1] and becomes crowded due to the increasing number of various bandwidth-consuming wireless applications. Recently, cognitive radio (CR) has been proposed as an effective solution to deal with these problems by allowing the access of unlicensed secondary users (SUs) to the frequency band that is allowed to licensed primary users (PUs), in a way that does not affect the quality of service (QoS) of the licensed primary systems [2,3]. In general, there are three main CR paradigms: interweave, overlay, and underlay [4]. Among the three paradigms, the underlay paradigm has been considered as a promising solution due to the high spectrum

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efficiency and has become the hot topic of wireless communications [5-7]. The basic idea of underlay CR is that SUs are allowed to share the spectrum with PUs so long as the interference they create on PUs remains below a specific threshold. This results in the improvement on spectrum efficiency. The underlay paradigm is also called spectrum-sharing paradigm [8,9].

However, due to the stringent interference constraint, very low transmit power level is often allowed for the secondary transmitters, and this would significantly degrade the QoS of cognitive systems and reduce the coverage of cognitive networks. One efficient method to improve the performance of cognitive systems is to employ the cooperative communication techniques [10,11]. Cooperative communications allow different users in a wireless network to collaborate and share each other's resource; thus, a particular user may transmit data of its own or assist another user through forwarding the received message by acting as a relay. Cooperation among the users helps in generating diversity and enhancing communication coverage [12-14]. Aiming at such improvements, so far, various cooperation schemes have been proposed in literature. Among them, opportunistic relaying (OR) has been shown simple but achieving near optimal outage performance with full diversity. In [15], authors have found that OR schemes can obtain the same diversity order as obtained by the complex distributed space-time code ones. The multi-user multi-relay scenarios have been considered in [16]. The results in [16] showed that the employment of multi-relay can enhance the diversity gain such that the system performance can be improved greatly. Thus, the combination of underlay CR and OR (UCR-OR) can not only undoubtedly inherit the advantages of the two techniques but also shed new light on high performance. For example, [17] is a very important work about cognitive radio with relay cooperation. In this work, authors have presented an exact outage performance analysis for the rates of a decode-and-forward cooperative network where a source communicates with its destination using the well-known repetition-based relaying scheme or using the single best relay, i.e., selection cooperation. Closed-form expressions have been obtained for independent Rayleigh fading channels. The obtained results in [17] indicated that selection cooperation exhibits lower outage probabilities compared to the repetition-based scheme.

Currently, the cognitive radio relay cooperation systems have been investigated widely in literature, see, e.g., [18-23] and references therein. A typical CR relay system consists of a secondary system and a primary system. The primary system includes a pair of primary source and primary destination, while the secondary system includes a secondary source, a secondary destination, and a secondary relay. In such scheme, besides the interference at primary receiver created by SUs, the primary transmitter's interference to the secondary relay and destination cannot be neglected, too. From the viewpoint of SUs, the interference from PUs has severe impact on system performance; thus, it is not ignored and must be considered. With the consideration, based on such system schemes, in [18,19], authors have investigated the outage performance of underlay CR systems with interference from PUs, where the conventional amplify-and-forward (AF) and decode-and-forward (DF) relaying protocols have been employed, respectively. Though in [18,19] the impact of the primary user's interference on the secondary users for the cognitive systems with single relay has been investigated, the corresponding results for the underlay CR systems with multiple relays have not been presented. That is to say, in the two works, the outage performance of cognitive best relay selection systems has not been studied. Moreover, in [18,19], the direct path transmission between secondary source and destination has been neglected. Thus, in [20], the outage performance of cognitive best relay selection system with primary user interference has been investigated. In particular, authors have obtained the closed-form expression for outage probability. The obtained results show that, though the interference from PUs badly degrades the performance of SUs, an increase of relays can compensate the loss. However, the drawback of the work is that the direct path between secondary source and destination has been ignored. In practice, for such scheme investigated in [20], the performance can be further improved by exploiting the direct path between cognitive source and destination. With this consideration, in [21], the outage performance of UCR-OR systems with direct path has been achieved. Though the schemes considered in [20,21] outperform the one in [18,19], the drawback of the schemes in [20,21] is that the maximal transmit power limits at secondary users have been ignored. That is to say, in [20,21], the transmit powers of secondary transmitters were determined only by the interference power constraint at PUs. The available maximal transmit power was assumed to be large enough. However, in practical implementation, the transmitters are maximal power-limited.

In [22], a more general cognitive relay system with primary users' interference has been investigated. In [22], authors have considered a system where the primary system consists of multiple transceiver pairs. This is a realistic consideration in large-scale cognitive systems where the SUs transmit over long distance and may suffer from the interference signals created by multiple primary users. For the scheme, the exact and asymptotic expressions to outage probability were obtained [22].

Obviously, in the systems with multiple primary transceivers, the interference at SUs from PUs is increasing with the number of primary transmitters, which degrades greatly the performance of cognitive systems. To compensate the performance loss caused by primary users' interference, as investigated in [20,21], the opportunistic relay schemes should be employed. Therefore, in [23], the outage performance of UCR-OR with multiple primary transceivers has been investigated under imperfect channel state information (CSI). Similarly, as stated in previous, although exploiting the direct path transmission can effectively compensate the performance loss of secondary systems caused by multiple primary users' interference, in [23], the direct link between cognitive source and destination was neglected.

The aforementioned literature review shows that the UCR-OR with multiple primary transceivers is very realistic cognitive radio schemes in large-scale cognitive networks. One example of such cognitive systems is wireless regional area network (WRAN) systems covering a suburb college tower and rural areas. In this case, the cognitive systems would maybe contain multiple PUs and suffer from the interference from multiple PUs. This would degrade greatly the performance of cognitive systems [24]. Therefore, for overcoming this problem, it is an effective solution to compensate the loss by exploiting the multiple relay schemes and the direct path between cognitive source and destination. However, to the best of our knowledge, in existing work about cognitive radio systems, the problem has not been resolved. This paper aims at filling this gap. Particularly, for the UCR-OR systems with multiple primary transceivers and direct path transmission between cognitive source and destination, the peak transmit power constraint and the peak interference power constraint are existent simultaneously. The peak transmit power is the available maximum power of SUs, which is determined by the battery capacity of SUs. In contrast, the peak interference power is the permissible maximum transmission power of SUs in order to guarantee the QoS of primary users. Under the peak interference power constraint, the secondary transmitters should always maintain their transmission power below a predetermined threshold. Obviously, in time-varying channels, it is impossible to satisfy this peak power constraint at all times. For this reason, in this paper, we first consider a constraint based on a stochastic concept instead of the strict peak interference power constraint. The primary systems should be allowed a certain percentage of outage so long as the outage probability of primary systems maintains below a predetermined outage constraint.

The remainder of this paper is organized as follows. In Section 2, the system model and the assumptions are presented. Based on the concept of outage constraint, by using minimum signal-to-interference-plus-noise ratio (SINR) criterion, we present the adaptive power allocation schemes for SUs in Section 3. Section 4 is the outage performance analysis. To highlight the impact of system parameters on performance of UCR-OR systems, in Section 5, the asymptotic outage probability is also derived under the case where the adaptive power allocation is not employed. The simulated results are presented in Section 6. Section 7 is the conclusions.

2 System model

As depicted in Figure 1, we consider an UCR-OR system with direct path under peak transmit power and peak interference power constraints. The UCR-OR system suffers from the interference created by multiple PUs. The secondary system is allowed to share the same spectrum band licensed to the primary system. The primary system consists of M primary source (PS_m) and primary destination (PD_m) pairs, $m \in \{1,.., M\}$, whereas, the secondary system consists of a secondary source (SS), a secondary destination (SD), and K DF secondary relay (SR) SR_k, $k \in \Theta$, $\Theta = \{1,\ldots, K\}$. This is a realistic consideration in large-scale cognitive systems where the coverage of secondary systems is much larger than that of primary systems. One example of such cognitive systems is WRAN systems covering a suburb college tower and rural areas. In this case, the cognitive systems would maybe contain multiple PUs and suffer from the interference from multiple PUs [24].

It is assumed that all primary and secondary terminals are equipped with single omni-antenna and work on half-duplex mode by using time division multiple access (TDMA). The channel coefficients (or link gains) of SS – SD, SS – SR_k, SR_k – SD, and PS_m – PD_m communication links are denoted as ϕ, g_k, h_k, and θ_m, respectively.

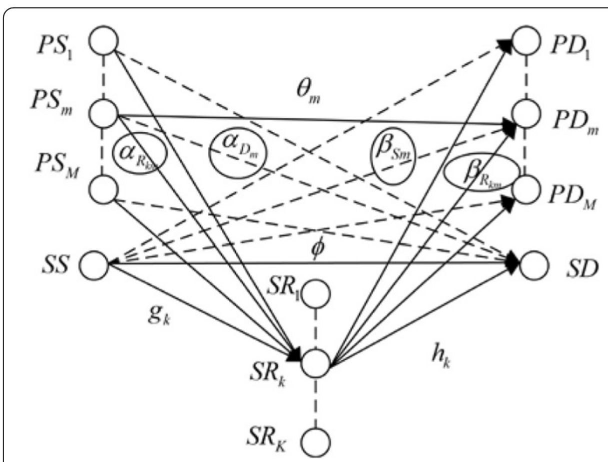


Figure 1 UCR-OR system model with the direct path between secondary source and destination.

Furthermore, the channel coefficients of the SS – PD_m and SR_k – PD_m interference links are β_{sm} and $\beta_{R_{km}}$, and the ones of the PS_m – D and PS_m – SR_k interference links are α_{Dm} and $\alpha_{R_{km}}$. We also assume that all channels in each link experience independent and identically distributed (i.i.d) Rayleigh fading. This indicates that for a given link gain X, it obeys exponential distribution with hazard rate $1/\omega_X$, denoted by $X \sim \Upsilon(1/\omega_X)$. Accordingly, as shown in Figure 1, the mean channel powers of ϕ, g_k, h_k, θ_m, β_{sm}, $\beta_{R_{km}}$, α_{D_m}, and $\alpha_{R_{km}}$ are ω_ϕ, ω_g, ω_h, ω_θ, ω_{β_S}, ω_{β_R}, ω_{α_D}, and ω_{α_R}, respectively. At each receiver node, the received signals are affected by symmetry Gaussian additive noise with identical variance N_0. Note that, for simplicity, we assume that the transmit power of all primary users is P_p, the one of all secondary relays is P_R, and the one of secondary source is P_s. The peak transmit power constraints of secondary source and relays are P_S^P and P_S^R, respectively.

A whole communication between cognitive source SS and destination SD consists of two phases. In the first phase, the cognitive source SS broadcasts its signal to all relays and destination. For a given relay SR_k, the received signal-to-interference-plus-noise radio (SINR) can be formulated as

$$\gamma_{SR_k} = \frac{P_S g_k}{\sum_{m=1}^{M} P_P \alpha_{R_{km}} + N_0} \tag{1}$$

Similarly, in this phase the received SINR at cognitive destination SD via direct link is

$$\gamma_{SD} = \frac{P_S \phi}{\sum_{m=1}^{M} P_p \alpha_{D_m} + N_0} \tag{2}$$

At the same time, in this phase, the received signal by primary receivers PD_m from PS_m, $m \in \{1,\ldots,M\}$, is corrupted by the interference from the secondary source SS. Thus, combining the received signal from primary transmitter PS_m, we also can formulate the SINR at the primary receiver PD_m as

$$\gamma_{Pm} = \frac{P_P \theta_m}{P_S \beta_{Sm} + N_0} \tag{3}$$

Since we are concerning a half-duplex two-hop DF relay transmission, in the second phase, the best relay should be selected from decoding subset DS_N that is defined as the subset of N relays able to decode the secondary source's information in the first phase. The selected best relay is given by

$$b = \arg \underbrace{\max}_{k \in DS_N} \left\{ \gamma_{DR_k} \right\} \tag{4}$$

where the instantaneous SINR γ_{DR_k} via random relay R_k is defined as

$$\gamma_{DR_k} = \frac{P_R h_k}{\sum_{m=1}^{M} P_P \alpha_{Dm} + N_0} \tag{5}$$

The corresponding instantaneous SINR via the selected best relay SR_b at the cognitive destination SD is

$$\gamma_{DR_b} = \underbrace{\max}_{k \in DS_N} \left\{ \gamma_{DR_k} \right\} \tag{6}$$

Thus, combining the received SINRs via direct link and relay link, the total received SINR at destination is

$$\gamma_{Tot} = \gamma_{SD} + \gamma_{DR_b} \tag{7}$$

Similarly, in this phase, the received expectation signals by primary receivers PD_m are also corrupted by the interference from the selected best relay SR_b. The corresponding SINR at PD_m is given by

$$\gamma_{P_{R_{bm}}} = \frac{P_P \theta_m}{P_R \beta_{R_{bm}} + N_0} \tag{8}$$

where b is defined by Equation 4.

3 Outage constraint and adaptive power allocation

Under the peak transmit power limit, it is impossible to satisfy the interference constraint at all times. For this reason, it makes sense to consider a constraint based on a stochastic concept instead of the power constant. The primary user should be allowed a certain percentage of outage. This yields that the secondary source SS has an adaptive transmit power policy. From the system model given in Figure 1, it is easy to see that, for a give primary transmission PS_m – PD_m, the outage constant is given by

$$\Pr\left\{\gamma_{Pm} \le r_{th}^P\right\} \le \varepsilon \tag{9}$$

where Pr{.} demotes probability, γ_{Pm} is defined by Equation 3, γ_{th}^P is the outage threshold of primary systems, and ε is the outage constant of PUs. Obviously, to guarantee that all primary receivers satisfy the outage constraint ε, the minimum SINR selection criterion should be employed. We have

$$P_{out}^{p_1} = \Pr\left\{ \underbrace{\min}_{m=1,\ldots,M} \left(\gamma_{Pm}\right) \le \gamma_{th}^P \right\} \le \varepsilon \tag{10}$$

By using minimum SINR criterion, Equations 3 and 10 show the relationship between the power P_s and outage constraint ε of primary receivers. It is easy to see that the outage probability $P_{\text{out}}^{p_1}$ is proportional to P_s. By taking the maximum value of Ps, we have $P_{\text{out}}^{p_1} = \varepsilon$. With this consideration, we can obtain the maximal permissible transmit power P_s of the secondary source SS. Using order statistics [25], we have

$$P_{\text{out}}^{p_1} = \Pr\left\{ \underbrace{\min}_{m=1,\ldots,M} (\gamma_{Pm}) \le \gamma_{\text{th}}^P \right\} = 1 - \Pr\left\{ \underbrace{\min}_{m=1,\ldots,M} (\gamma_{Pm}) > \gamma_{\text{th}}^P \right\} \tag{11}$$

From Equation 3, it is found that the random variables (RVs) γ_{Pm} are independent mutual. This leads to

$$\begin{aligned} P_{\text{out}}^{p_1} &= 1 - \prod_{m=1}^{M} \Pr\{\gamma_{Pm} > \gamma_{\text{th}}^P\} \\ &= 1 - \prod_{m=1}^{M} \left\{1 - \Pr\{\gamma_{Pm} \le \gamma_{\text{th}}^P\}\right\} \end{aligned} \tag{12}$$

With the definition γ_{Pm} in Equation 3 and $\theta_m \sim \Upsilon(1/\omega_\theta)$, having

$$\Pr\{\gamma_{Pm} \le \gamma_{\text{th}}^P\} = 1 - \exp\left(-\frac{N_0\gamma_{\text{th}}^P}{P_P\omega_\theta}\right)\int_0^\infty \exp\left(-\frac{P_S\gamma_{\text{th}}^P}{P_P\omega_\theta}x\right) f_{\beta_{Sm}}(x)dx \tag{13}$$

where $f_{\beta_{Sm}}(\cdot)$ is the PDF of the RV β_{Sm}. By using $\beta_{Sm} \sim \Upsilon\left(1/\omega_{\beta_S}\right)$ and taking the integral of Equation 13 with respect to β_{Sm} yields

$$\Pr\{\gamma_{Pm} \le \gamma_{\text{th}}^P\} = 1 - \exp\left(-\frac{N_0\gamma_{\text{th}}^P}{P_P\omega_\theta}\right)\left(\frac{P_S\omega_{\beta_S}}{P_P\omega_\theta}\gamma_{\text{th}}^P + 1\right)^{-1} \tag{14}$$

Combining Equations 14, 12, and 10, the outage constraint of primary receivers is written as

$$P_{\text{out}}^{p_1} = 1 - \exp\left(-\frac{MN_0\gamma_{\text{th}}^P}{P_P\omega_\theta}\right)\left(\frac{P_S\omega_{\beta_S}}{P_P\omega_\theta}\gamma_{\text{th}}^P + 1\right)^{-M} \le \varepsilon \tag{15}$$

After some mathematical manipulation, the maximum permissible transmit power P_s of secondary source under the outage constraint ε of primary receivers is given by

$$P_S = X^P \frac{P_P\omega_\theta}{P_S\omega_{\beta_S}} \tag{16}$$

Where X^P is defined as

$$X^P = \max\left(0, \left(\exp\left(-\left(\frac{N_0\gamma_{\text{th}}^P}{P_P\omega_\theta} + \frac{1}{M}\ln(1-\varepsilon)\right)\right) - 1\right)\right) \tag{17}$$

At the same time, the secondary source SS must satisfy the peak transmit power constraint, i.e., $P_S \le P_S^P$. Thus, the adaptive power allocation policy for secondary source SS is given by

$$P_S = \min\left\{ {P_S}^P, X^P \frac{P_P\omega_\theta}{P_S\omega_{\beta_S}} \right\} \tag{18}$$

In the second phase, based on the selection criterion (Equation 4), the best relay SR_b is selected from decoding subset DS to forward the received signals with power P_R. Similarly, the transmit power P_R must satisfy the outage constraint ε of primary receivers, which is given by

$$P_{\text{out}}^{p_2} = P_r\left\{ \underbrace{\min}_{m=1,\ldots,M} \left(\frac{P_p\theta_m}{P_R\beta_{R_{bm}} + N_0} \right) \le r_{\text{th}}^P \right\} \le \varepsilon \tag{19}$$

Due to the fact that the selection of the best relay is independent of the link $SR_k - PD_m$, by using the similar method as in Equations 10 to 16 and the peak transmit power P_R^P the adaptive power allocation policy for the selected best relay SR_b is given by

$$P_R = \min\left\{ P_R^P, X^P \frac{\omega_\theta P_P}{\omega_{\beta_R} r_{\text{th}}^P} \right\} \tag{20}$$

where X^P is defined by Equation 17.

Therefore, by using Equations 16 and 20, the transmit powers P_s and P_R can be determined. Due to the fact that the DF protocol is employed, the power allocation for P_s and P_R is manipulated separately. For P_s, the secondary relays and the primary destination send the local CSIs to the secondary source firstly by using feedback links. After collecting the CSIs from the secondary relays and the primary destination, with Equation 16 the transmit power P_s of the secondary source can be determined. For P_R, in our scheme, a distributed scheme is employed, which combines the best relay selection and adaptive power allocation. The basic idea is that each relay sets up an internal timer which triggers transmission. Assuming synchronization among the secondary relays, all secondary relays start their timer simultaneously, whose initial values are inversely proportional to the corresponding SINR given by Equation 5. Since the cognitive destination has the local channel state information (CSI), it can send feedback to the secondary relays. The best relay is the one with its timer reduced to zero first. When the timer of best relay has expired, the relay is expected to broadcast a 'flag' message to neighboring nodes to prevent other relays from transmission. Then, by using the collected CSI from the secondary destination and the primary destination, the selected best relay calculates the transmit power P_R according to Equation 20. Obviously, due to the distributed scheme employed, the implementation complexity of the scheme is lower than the centralized scheme.

4 Exact outage performance analyses

In this section, we investigate the outage probability of the considered UCR-OR systems. The outage probability is an effective method to quantify the system performance, which is defined as the probability that the instantaneous end-to-end SNR (or SINR) falls below a predefined threshold. Since we consider a two-hop DF system with direct path transmission between cognitive source and destination, we should start the analysis by studying the decoding subset DS_N, $N = 0,\ldots,K$, i.e., the set of N relays able to decode the information transmitted by cognitive source in $\mathrm{SS}-\mathrm{SR}_k$ links, where $k \in \Theta$. Obviously, if the decoding subset is empty, i.e., $N = 0$, there is no signal transmitted through cognitive relay links. In this case, only the direct path signals are received by cognitive destination SD. This leads to the outage probability of UCR-OR system given by

$$P_{\mathrm{out}}^{1} = \Pr\left\{\mathrm{DS}_0, \gamma_{\mathrm{SD}} \le \gamma_{\mathrm{th}}^{S}\right\} \tag{21}$$

where the direct link SINR γ_{SD} is given by Equation 2, and γ_{th}^{S} is the outage threshold at cognitive destination.

On the contrary, if the decoding subset is not empty, i.e., $N \neq 0$, the best relay among the decoding subset DS_N is selected to forward the received source signals. In this case, the cognitive destination SD receives the direct link signals and the best relay link signals, simultaneously. The cognitive destination employs the maximal ratio combiner (MRC) to combine the received signals. According to the total probability theory, the outage probability is given by

$$P_{\mathrm{out}}^{2} = \sum_{N=1}^{K} \underbrace{\underbrace{\Pr\{\mathrm{DS}_N\}}_{P_{\mathrm{out}}^{21}} \Pr\left\{\gamma_{\mathrm{SD}} + \gamma_{DR_b} \le \gamma_{\mathrm{th}}^{S} | \mathrm{DS}_N\right\}}_{P_{\mathrm{out}}^{22}} \tag{22}$$

Finally, combining Equations 21 and 22, the total outage probability of the DF UCR-OR systems can be written as

$$P_{\mathrm{out}} = P_{\mathrm{out}}^{1} + P_{\mathrm{out}}^{2} \tag{23}$$

In the following subsection, we would derive the closed-form expressions to P_{out}^{1} and P_{out}^{2}.

4.1 Detailed analyses to P_{out}^{1}

Here, we first derive the closed-form expression to P_{out}^{1}. Due to the fact that the two events $\{\mathrm{DS}_0\}$ and $\left\{\gamma_{\mathrm{SD}} \le \gamma_{\mathrm{th}}^{S}\right\}$ are independent mutually, we can formulate the outage probability P_{out}^{1} as

$$P_{\mathrm{out}}^{1} = \Pr\{\mathrm{DS}_0\} \Pr\left\{\gamma_{\mathrm{SD}} \le \gamma_{\mathrm{th}}^{S}\right\} \tag{24}$$

Since $\Pr\{\mathrm{DS}_0\}$ denotes the probability that there is no relay decoding correctly the cognitive source signals in the first phase, using the equivalent SINR (Equation 1) and the fact that γ_{SR_k} are independent mutually, we have

$$\Pr\{\mathrm{DS}_0\} = \prod_{k=1}^{K} \Pr\left\{\frac{P_S g_k}{Z_k + N_0} \le \mu_{\mathrm{th}}^{S}\right\} \tag{25}$$

where we define μ_{th}^{S} as the outage threshold over the cognitive $\mathrm{SS}-\mathrm{SR}_k$ link, and $Z_k = \sum_{m=1}^{M} P_P \alpha_{R_{km}}$. Since $\alpha_{R_{km}}$ are i.i.d Rayleigh fading channel coefficients with variable ω_{α_R}, the random variable Z_k is a chi-square RV. The corresponding PDF is given by

$$f_{Z_k}(z) = \frac{1}{\Gamma(M)} \left(\frac{1}{P_p \omega_{\alpha_R}}\right)^{M} z^{M-1} \exp\left(-\frac{z}{\omega_{\alpha_R} P_P}\right) \tag{26}$$

where $\Gamma(.)$ is the gamma function defined by (8.310.1) in [26]. For the convenience of derivation, we define

$$\Delta = \Pr\left\{\frac{P_S g_k}{Z_k + N_0} \le \mu_{\mathrm{th}}^{S}\right\} \tag{27}$$

Then, using $g_k \sim \Upsilon(1/\omega_g)$ leads to

$$\Delta = 1 - \frac{1}{\Gamma(M)} \left(\frac{1}{P_p \omega_{\alpha_R}}\right)^{M} \exp\left(-\frac{\mu_{th}^{S} N_0}{P_S \omega_g}\right) \int_0^{\infty} z^{M-1} \exp\left(-z\left(\frac{\mu_{th}^{S}}{P_S \omega_g} + \frac{1}{\omega_{\alpha_R} P_P}\right)\right) dz \tag{28}$$

Using (3.351.3) in [26] leads to

$$\Delta = 1 - \exp\left(-\frac{\mu_{\mathrm{th}}^{S} N_0}{P_S \omega_g}\right) \left(\frac{P_p \omega_{\alpha_R}}{P_S \omega_g} \mu_{\mathrm{th}}^{S} + 1\right)^{-M} \tag{29}$$

Finally, with the consideration that all channels in the cognitive $\mathrm{SS}-SR_k$ link are i.i.d fading, by substituting Equation 29 into Equation 25, we have

$$\Pr\{\mathrm{DS}_0\} = \Delta^K = \left(1 - \exp\left(-\frac{\mu_{\mathrm{th}}^{S} N_0}{P_S \omega_g}\right) \left(\frac{P_p \omega_{\alpha_R}}{P_S \omega_g} \mu_{\mathrm{th}}^{S} + 1\right)^{-M}\right)^{K} \tag{30}$$

In Equation 24, the term $\Pr\left\{\gamma_{\mathrm{SD}} \le \gamma_{\mathrm{th}}^{S}\right\}$ is outage probability over the direct link $\mathrm{SS}-\mathrm{SD}$. Observing the definitions γ_{SD} and γ_{SR_k} in Equations 2 and 1, we can find that γ_{SD} and γ_{SR_k} have similar forms. Therefore, the evaluation to $\Pr\left\{\gamma_{\mathrm{SD}} \le \gamma_{\mathrm{th}}^{S}\right\}$ can be achieved from Equation 29 through the respective parameter exchange, i.e., $\omega_{\alpha_R} -> \omega_{\alpha_D}$, $\omega_g -> \omega_\phi$, and $\mu_{\mathrm{th}}^{S} -> \gamma_{\mathrm{th}}^{S}$, which is given by

$$\Pr\{\gamma_{SD} \le \gamma_{th}^S\} = 1 - \exp\left(-\frac{\gamma_{th}^S N_0}{P_S \omega_\phi}\right)\left(\frac{P_p \omega_{\alpha_D}}{P_S \omega_\phi}\gamma_{th}^S + 1\right)^{-M} \tag{31}$$

Substituting Equations 31 and 30 into Equation 24, the evaluation to P_{out}^1 is achieved.

4.2 Detailed analysis to P_{out}^2

Equation 22 shows that the outage probability P_{out}^2 consists of two parts, i.e., P_{out}^{21} and P_{out}^{22}. The term $P_{out}^{21} = \Pr\{DS_N\}$ denotes the probability that N relays out of the K candidates are in the decoding subset DS. Thus, using the equivalent SINR (Equation 1) and the fact that γ_{SR_k} are independent mutually, we have

$$P_{out}^{21} = \sum_{DS_N} \prod_{k \in DS_N} \Pr\left\{\gamma_{SR_k} \ge \mu_{th}^S\right\} \prod_{j \notin DS_N} \Pr\left\{\gamma_{SR_j} \le \mu_{th}^S\right\} \tag{32}$$

Since we are assuming that all channels in each link experience i.i.d Rayleigh fading, the outage probability P_{out}^2 does not depend on which relay nodes are in the decoding subset DS, but on how many relay nodes belong to the decoding subset. Therefore, using the definition $\Delta = \Pr\left\{\gamma_{SR_k} \le \mu_{th}^S\right\}$ and the result given by Equation 29, it is easy to see that P_{out}^{21} is given by

$$P_{out}^{21} = C_N^K (1-\Delta)^N \Delta^{K-N} \tag{33}$$

At the same time, Equation 22 shows that, to obtain the evaluation of P_{out}^2, the term $P_{out}^{22} = \Pr\left\{\gamma_{SD} + \gamma_{DR_b} \le \gamma_{th}^S | DS_N\right\}$ is required. Using γ_{SD} and γ_{DR_b} defined by Equations 2 and 6, P_{out}^{22} can be formulated as

$$P_{out}^{22} = \Pr\left\{\left(\frac{P_S \phi}{Y + N_0} + \frac{Z}{Y + N_0}\right) \le \gamma_{th}^S\right\} \tag{34}$$

where we define $Y = \sum_{m=1}^{M} P_P \alpha_{D_m}$ and $Z = \underbrace{\max}_{k \in DS_N} \{P_R h_k\}$. Due to the correlation among the received SINR at cognitive relays and destination caused by PUs' interference, the exact closed-form expression to this cannot be calculated as the conventional analysis any more. Therefore, conditioned on Y and Z, P_{out}^{22} is given by

$$P_{out}^{22} = \int_0^\infty \int_{\frac{z}{\gamma_{th}^S} - N_0}^{\infty} \Pr\left(\phi \le \frac{y + N_0}{P_S}\left(\gamma_{th}^S - \frac{z}{y + N_0}\right) | y, z\right) f_Y(y) f_Z(z) dy dz \tag{35}$$

Since the PDF of $Y = \sum_{m=1}^{M} P_P \alpha_{D_m}$ can be obtained from Equation 26 through the respective parameter exchange $\omega_{\alpha_R} -> \omega_{\alpha_D}$, with $\phi \sim \Upsilon(1/\omega_\phi)$, we have P_{out}^{22} given by

$$P_{out}^{22} = P_{out}^{22-1} - P_{out}^{22-2} \tag{36}$$

where the two parts P_{out}^{22-1} and P_{out}^{22-2} are given by

$$P_{out}^{22-1} = \frac{1}{\Gamma(M)}\left(\frac{1}{P_P \omega_{\alpha_D}}\right)^M \int_0^\infty \int_{\frac{z}{\gamma_{th}^S} - N_0}^{\infty} y^{M-1} \exp\left(-\frac{y}{P_p \omega_{\alpha_D}}\right) f_Z(z) dy dz \tag{37}$$

$$P_{out}^{22-2} = \frac{1}{\Gamma(M)}\left(\frac{1}{P_P \omega_{\alpha_D}}\right)^M \int_0^\infty \int_{\frac{z}{\gamma_{th}^S} - N_0}^{\infty} \exp\left(-\frac{1}{\omega_\phi}\frac{y + N_0}{P_S}\left(\gamma_{th}^S - \frac{z}{y + N_0}\right)\right) y^{M-1} \exp\left(-\frac{y}{P_p \omega_{\alpha_D}}\right) f_Z(z) dy dz \tag{38}$$

We first consider the part P_{out}^{22-2}, using $\int_u^\infty x^{\nu-1}\exp(-\mu x) dx = \mu^{-\nu}\Gamma(\nu, \mu u)$ given by (3.381.3) in [26], it can be written as

$$P_{out}^{22-2} = \frac{1}{\Gamma(M)}\exp\left(-\frac{N_0 \gamma_{th}^S}{P_S \omega_\phi}\right)\left(\frac{1}{P_P \omega_{\alpha_D}}\right)^M \times \int_0^\infty \exp\left(\frac{z}{P_S \omega_\phi}\right)\left(\frac{\gamma_{th}^S}{P_S \omega_\phi} + \frac{1}{P_p \omega_{\alpha_D}}\right)^{-M} \Gamma\left(M, \left(\frac{z}{\gamma_{th}^S} - N_0\right)\left(\frac{\gamma_{th}^S}{P_S \omega_\phi} + \frac{1}{P_p \omega_{\alpha_D}}\right)\right) f_Z(z) dz \tag{39}$$

where $\Gamma(.,.)$ is the incomplete gamma function defined by (8.350.2) in [26]. Using the identity $\Gamma(n+1, x) = n! \exp(-x)\sum_{l=0}^{n}\frac{x^l}{l!}$ yields

$$P_{\text{out}}^{22-2} = \left(\frac{1}{P_P\omega_{\alpha_D}}\right)^M \exp\left(\frac{N_0}{P_p\omega_{\alpha_D}}\right)\int_0^\infty \exp\left(-\frac{z}{P_p\omega_{\alpha_D}\gamma_{\text{th}}^S}\right)\sum_{n_1=0}^{M-1}\frac{1}{n_1!} \left(\frac{z}{\gamma_{th}^S}-N_0\right)^{n_1}\left(\frac{\gamma_{\text{th}}^S}{P_S\omega_\phi}+\frac{1}{P_p\omega_{\alpha_D}}\right)^{-M+n_1} f_Z(z)dz \tag{40}$$

Using the binomial expansion, having

$$P_{\text{out}}^{22-2} = \left(\frac{1}{P_P\omega_{\alpha_D}}\right)^M \exp\left(\frac{N_0}{P_p\omega_{\alpha_D}}\right)\sum_{n_1=0}^{M-1}\frac{(-N_0)^{n_1}}{n_1!}\left(\frac{\gamma_{\text{th}}^S}{P_S\omega_\phi}+\frac{1}{P_p\omega_{\alpha_D}}\right)^{-M+n_1} \sum_{n_2=0}^{n_1}\binom{n_1}{n_2}(-1)^{n_2}\left(\frac{1}{N_0\gamma_{\text{th}}^S}\right)^{n_2}\int_0^\infty z^{n_2}\exp\left(-\frac{z}{P_p\omega_{\alpha_D}\gamma_{\text{th}}^S}\right)f_Z(z)dz \tag{41}$$

where $\binom{n_1}{n_2} = n_1!/(n_2!(n_1-n_2)!)$ is the binomial coefficient. With the definition $Z = \underbrace{\max}_{i\in \text{DS}_N}\{P_R h_i\}$, it is easy to see that the PDF of the RV Z is given by

$$f_Z(z) = \frac{N}{P_R\omega_h}\sum_{n_3=0}^{N-1}\binom{N-1}{n_3}(-1)^{n_3}\exp\left(-\frac{(n_3+1)z}{P_R\omega_h}\right) \tag{42}$$

Substituting Equation 42 into Equation 41 yields

$$P_{\text{out}}^{22-2} = \left(\frac{1}{P_P\omega_{\alpha_D}}\right)^M \exp\left(\frac{N_0}{P_p\omega_{\alpha_D}}\right)\sum_{n_1=0}^{M-1}\frac{(-N_0)^{n_1}}{n_1!} \left(\frac{\gamma_{\text{th}}^S}{P_S\omega_\phi}+\frac{1}{P_p\omega_{\alpha_D}}\right)^{-M+n_1}\sum_{n_2=0}^{n_1}\binom{n_1}{n_2}(-1)^{n_2}\left(\frac{1}{N_0\gamma_{\text{th}}^S}\right)^{n_2} \times\frac{N}{P_R\omega_h}\sum_{n_3=0}^{N-1}\binom{N-1}{n_3}(-1)^{n_3}\int_0^\infty z^{n_2} \exp\left(-z\left(\frac{1}{P_p\omega_{\alpha_D}\gamma_{\text{th}}^S}+\frac{(n_3+1)}{P_R\omega_h}\right)\right)dz \tag{43}$$

Using (3.351.3) [26], after some mathematical manipulation, the evaluation to P_{out}^{22-2} is given by

$$P_{\text{out}}^{22-2} = \left(\frac{1}{P_P\omega_{\alpha_D}}\right)^M \exp\left(\frac{N_0}{P_p\omega_{\alpha_D}}\right)\sum_{n_1=0}^{M-1}\sum_{n_2=0}^{n_1}\sum_{n_3=0}^{N-1}\binom{n_1}{n_2} \binom{N-1}{n_3}\frac{(-1)^{n_1+n_2+n_3}{N_0}^{n_1}n_2!}{n_1!} \times\frac{N}{P_R\omega_h}\left(\frac{\gamma_{\text{th}}^S}{P_S\omega_\phi}+\frac{1}{P_p\omega_{\alpha_D}}\right)^{-M+n_1}\left(\frac{1}{N_0\gamma_{\text{th}}^S}\right)^{n_2} \left(\frac{1}{P_p\omega_{\alpha_D}\gamma_{\text{th}}^S}+\frac{(n_3+1)}{P_R\omega_h}\right)^{-n_2-1} \tag{44}$$

For the part P_{out}^{22-1}, using the $\int_u^\infty x^{\nu-1}\exp(-\mu x)dx = \mu^{-\nu}\Gamma(\nu,\mu u)$ given by (3.381.3) in [26] leads to

$$P_{\text{out}}^{22-1} = \frac{1}{\Gamma(M)}\int_0^\infty \Gamma\left(M,\frac{1}{P_p\omega_{\alpha_D}}\left(\frac{z}{\gamma_{\text{th}}^S}-N_0\right)\right)f_Z(z)dydz \tag{45}$$

Similar to Equation 40, having

$$P_{\text{out}}^{22-1} = \exp\left(\frac{N_0}{P_p\omega_{\alpha_D}}\right)\sum_{m_1=0}^{M-1}\frac{1}{m_1!}\left(\frac{1}{P_p\omega_{\alpha_D}}\right)^{m_1}\int_0^\infty\left(\frac{z}{\gamma_{\text{th}}^S}-N_0\right)^{m_1} \exp\left(-\frac{1}{P_p\omega_{\alpha_D}}\frac{z}{\gamma_{\text{th}}^S}\right)f_Z(z)dz \tag{46}$$

Substituting Equation 42 into Equation 46, P_{out}^{22-1} is written as

$$P_{\text{out}}^{22-1} = \exp\left(\frac{N_0}{P_p\omega_{\alpha_D}}\right)\sum_{m_1=0}^{M-1}\frac{1}{m_1!}\left(\frac{1}{P_p\omega_{\alpha_D}}\right)^{m_1}\frac{N}{P_R\omega_h}\sum_{m_2=0}^{N-1} \binom{N-1}{m_2}(-1)^{m_2}(-N_0)^{m_1}\sum_{m_3=0}^{m_1}\binom{m_1}{m_3}\left(\frac{-1}{N_0\gamma_{\text{th}}^S}\right)^{m_3} \times\int_0^\infty z^{m_3}\exp\left(-z\left(\frac{1}{P_p\omega_{\alpha_D}\gamma_{\text{th}}^S}-\frac{(m_2+1)}{P_R\omega_h}\right)\right)dz \tag{47}$$

Using (3.351.3) in [26], we have

$$P_{\text{out}}^{22-1} = \exp\left(\frac{N_0}{P_p\omega_{\alpha_D}}\right)\frac{N}{P_R\omega_h}\sum_{m_1=0}^{M-1}\sum_{m_2=0}^{N-1}\sum_{m_3=0}^{m_1}\binom{N-1}{m_2} \binom{m_1}{m_3}(-1)^{m_1+m_2+m_3}\frac{m_3!{N_0}^{m_1}}{m_1!}\left(\frac{1}{P_p\omega_{\alpha_D}}\right)^{m_1} \left(\frac{1}{N_0\gamma_{\text{th}}^S}\right)^{m_3}\left(\frac{1}{P_p\omega_{\alpha_D}\gamma_{th}^S}+\frac{(m_2+1)}{P_R\omega_h}\right)^{-m_3-1} \tag{48}$$

Combining Equations 44 and 48, the result for P_{out}^{22} can be obtained. Then, by substituting Equations 36 and 33 into Equation 22, we can obtain the closed-form solution to P_{out}^2 that is outage probability of DF UCR-OR systems when the decoding subset is not empty.

5 Asymptotic outage performance analyses

Although in Section 4 we obtain the exact closed-form expression of outage probability for the considered UCR-OR systems, the derivations are computationally complicated and do not offer insight into the impact of system parameters on system performance. Therefore, in practice, some simplified expressions are required. To this end, we now derive the asymptotic closed-form expressions of outage probability by using the assumption that the value of the transmit power P_s is high. At the same time, for simplicity, we also assume $P_R = \lambda P_S$. Then, from the asymptotic results in high transmit power P_s, the diversity and coding gains can be achieved.

For the convenience of derivation, we combine the general expressions 21 and 22 of outage probability and rewrite as

$$P_{\text{out}}^{\text{Asy}} = \sum_{N=0}^{K} \underbrace{\Pr\{\text{DS}_N\}}_{P_{\text{out}}^{21}} \underbrace{\Pr\left\{\gamma_{\text{SD}} + \gamma_{\text{DR}_b} \le \gamma_{th}^{S} | \text{DS}_N\right\}}_{P_{\text{out}}^{22}} \quad (49)$$

where P_{out}^{21} and P_{out}^{22} are defined by Equations 33 and 34, respectively. Equation 33 indicates that the simplified expression for Δ is required firstly. Using the definition of Δ and Equation 27, we have

$$\Delta = \Pr\left\{\frac{P_S g_k}{Z_k + N_0} \le \mu_{\text{th}}^{S}\right\} \approx \Pr\left\{P_S g_k \le \mu_{\text{th}}^{S} Z_k\right\} \quad (50)$$

With $g_k \sim \Upsilon(1/\omega_g)$ and the assumption of high P_s, having

$$\Delta \approx \frac{\mu_{th}^{S}}{P_S \omega_g} \int_0^{\infty} z f_{Z_k}(z) dz \quad (51)$$

Substituting the PDF (Equation 26) of Z_k into Equation 51 leads to

$$\begin{aligned}\Delta &\approx \frac{1}{\Gamma(M)} \frac{\mu_{\text{th}}^{S}}{P_S \omega_g} \left(\frac{1}{P_p \omega_{\alpha_R}}\right)^{M} \int_0^{\infty} z^M \exp\left(-\frac{z}{\omega_{\alpha_R} P_P}\right) dz \\ &= \frac{P_p \omega_{\alpha_R}}{P_S \omega_g} M \mu_{\text{th}}^{S}\end{aligned} \quad (52)$$

Thus, substituting Equation 52) into Equation 33, the asymptotic expression of P_{out}^{21} is given by

$$P_{\text{out}}^{21} \approx C_N^K \left(\frac{P_p \omega_{\alpha_R}}{P_S \omega_g} M \mu_{\text{th}}^{S}\right)^{K-N} \quad (53)$$

Note that here, we employ the fact 1 – $\Delta \approx 1$ in high P_s.

For the term P_{out}^{22} defined by Equation 34, in high transmit power P_s, we have

$$P_{\text{out}}^{22} \approx \Pr\left\{P_S \phi \le \gamma_{\text{th}}^{S} Y - Z\right\} \quad (54)$$

With the definition $Z = \underbrace{\max}_{k \in \text{DS}_N} \{P_R h_k\}$ and $P_R = \lambda P_S$, in high transmit power P_s, the PDF of the random variable Z is given approximately by $f_Z(z) \approx \frac{N}{(\omega_h P_R)^N} z^{N-1}$. This leads to Equation 54 given asymptotically by

$$\begin{aligned}P_{\text{out}}^{22} &\approx \frac{1}{P_S \omega_\phi} \int_0^{\infty} \int_0^{\gamma_{\text{th}}^{S} y} (\gamma_{\text{th}}^{S} y - z) f_Z(z) f_Y(y) dz dy \\ &\approx \frac{1}{P_S \omega_\phi} \frac{N}{(\omega_h P_R)^N} \int_0^{\infty} \int_0^{\gamma_{\text{th}}^{S} y} (\gamma_{\text{th}}^{S} y - z) z^{N-1} f_Y(y) dz dy \\ &= \frac{1}{P_S \omega_\phi} \frac{1}{(\omega_h P_R)^N} \frac{1}{N+1} \int_0^{\infty} \left(\gamma_{\text{th}}^{S} y\right)^{N+1} f_Y(y) dy\end{aligned} \quad (55)$$

The PDF of the random variable $Y = \sum_{m=1}^{M} P_P \alpha_{D_m}$ can be obtained from Equation 26 through the respective parameter exchange $\omega_{\alpha_R} -> \omega_{\alpha_D}$. Thus, Equation 55 is written as

$$\begin{aligned}P_{\text{out}}^{22} &\approx \frac{1}{P_S \omega_\phi} \frac{1}{(P_R \omega_h)^N} \frac{1}{(P_p \omega_{\alpha_D})^M} \frac{1}{(N+1)\Gamma(M)} \left(\gamma_{\text{th}}^{S}\right)^{N+1} \int_0^{\infty} y^{M+N} \exp\left(-\frac{y}{\omega_{\alpha_D} P_P}\right) dy \\ &= \frac{1}{P_S \omega_\phi} \frac{(P_P \omega_{\alpha_D})^{N+1}}{(P_R \omega_h)^N} \frac{(M+N)!}{(N+1)\Gamma(M)} \left(\gamma_{th}^{S}\right)^{N+1}\end{aligned} \quad (56)$$

Finally, substituting Equations 56 and 53 into Equation 49 yields that in high transmit power P_s, the asymptotic expression of outage probability is given by

$$\begin{aligned}P_{\text{out}}^{\text{Asy}} &\approx \sum_{N=0}^{K} C_N^K \left(\frac{P_p \omega_{\alpha_R}}{P_S \omega_g} M \mu_{\text{th}}^{S}\right)^{K-N} \frac{1}{P_S \omega_\phi} \frac{(P_P \omega_{\alpha_D})^{N+1}}{(P_R \omega_h)^N} \frac{(M+N)!}{(N+1)\Gamma(M)} \left(\gamma_{\text{th}}^{S}\right)^{N+1} \\ &= \frac{(P_p)^{K+1}}{(P_S)^{K+1}} \frac{1}{\Gamma(M)} \frac{1}{\omega_\phi} \sum_{N=0}^{K} C_N^K \left(\frac{\omega_{\alpha_R}}{\omega_g}\right)^{K-N} \frac{(\omega_{\alpha_D})^{N+1}}{(\lambda \omega_h)^N} \frac{(M+N)!}{(N+1)} \left(M \mu_{\text{th}}^{S}\right)^{K-N} \left(\gamma_{\text{th}}^{S}\right)^{N+1}\end{aligned} \quad (57)$$

When $\mu_{\text{th}}^{S} = \gamma_{\text{th}}^{S}$, having

$$P_{\text{out}}^{\text{Asy}} \approx \varepsilon \frac{\left(\gamma_{\text{th}}^{S}\right)^{K+1}}{(P_S)^{K+1}} \quad (58)$$

Where we define

$$\varepsilon = \frac{(P_p)^{K+1}}{\Gamma(M)} \frac{1}{\omega_\phi} \sum_{N=0}^{K} C_N^K \left(\frac{\omega_{\alpha_R}}{\omega_g}\right)^{K-N} \frac{(\omega_{\alpha_D})^{N+1}}{(\lambda \omega_h)^N} \frac{(M+N)!}{(N+1)} (M)^{K-N} \quad (59)$$

Thus, in high transmit power P_s, the diversity gain is

$$G_d = \lim_{P_S -> \infty} -\frac{\log P_{\text{out}}^{\text{Asy}}}{\log P_S} = K + 1 \quad (60)$$

The coding gain is

$$G_c = \frac{\varepsilon^{\frac{-1}{K+1}}}{\gamma_{\mathrm{th}}^{S}} \tag{61}$$

It can be observed from Equations 60 and 61 that the diversity gain of the considered UCR-OR systems is determined by the number of relays, i.e., $G_d = K + 1$. The parameters of primary system only affect the coding gain, not the diversity gain. This is due to the fact that the key idea of relay cooperation is that multiple single antenna relays work together and form virtual multiple-input and multiple-output (MIMO) systems. In such virtual MIMO systems, the number of the available source-relay or relay-destination transmissions dominates the diversity gain. However, from Equation 3 to 7, we can find that the mutual interference between PUs and SUs only affects the equivalent SINR of each single path signal but not cause the increase or decrease in the number of the available source-relay or relay-destination transmissions. That is to say, the number of multiple path signals is still $K + 1$. Therefore, we have the result that the diversity gain of the UCR-OR systems is $K + 1$.

6 Simulation results and performance comparison analyses

In previous sections, we obtain the adaptive power allocation schemes of cognitive transmitters for the considered UCR-OR systems under outage and peak transmit power constraints. Based on the results, we achieve the exact evaluation to the outage probability of UCR-OR systems with the multiple PU pairs and the direct path transmission between cognitive source and destination. At the same time, to obtain the insight about the effect of system parameters on outage performance, with the assumption of high transmit power P_s, the asymptotic closed-form expression of outage probability is derived too. With these derivations, the simulated and numerical results are presented in this section, which is used to validate the derivations and to obtain the acknowledgement about the impact of system parameter on the UCR-OR systems. During the analyses, we use MATLAB to build simulations. In all case, the channels are generated by using MATLAB toolbox 'Rayleigh', which makes a fading channel. Specially, the following system parameters are employed: number of the cognitive relays $K = 10$, PUs' outage threshold $\gamma_{\mathrm{th}}^{P} = -9dB$, mean power of PUs channels $\omega_\theta = 2$, peak transmit power constraints at SUs $P_S^P = P_R^P = 10dB$, and noise variance $N_0 = 1$. Note that, for the clarity of comparison analyses, in the sequence discussion, the outage probabilities of the UCR-OR systems with and without direct path are presented simultaneously, which are marked by 'Dir link' and 'No-dir link', respectively.

By taking the channel mean powers $\omega_g = \omega_h = 2$ and $\omega_{\beta_S} = \omega_{\beta_R} = \omega_{\alpha_R} = \omega_{\alpha_D} = 0.2$ and the outage threshold $\gamma_{\mathrm{th}}^{S} = \mu_{\mathrm{th}}^{S} = -10dB$, we first investigate the adaptive power allocations for P_s and P_R. Due to the fact that the symmetric system parameters are employed, according to Equations 18 and 20, we have $P_s = P_R$. Thus, we only present the investigation on P_s. For P_R, the results are straightforward. In Figure 2, we present the power P_s versus the peak power P_P (dB) of primary transmitters under different values of M and ε. The presented figures show clearly that the transmit power P_s is changing with M and ε. We first analyze the power allocation of the UCR-OR systems with $M = 4$ and $\varepsilon = 0.01$. It is observed that $P_s = 0$ when the value of P_p is less than or equal to the specific value $P_P^* = 14dB$. When P_p is greater than $14\,dB$, the transmission power P_s increases gradually. The transmit power P_s ascends to the maximal value as P_p increases to another specific value $P_P^{**} = 17.5dB$ and then remains constant as $P_P > 17.5\,dB$. This can be explained by the fact that the outage probability of PUs is greater than the outage constraint ε when $P_P < 14\,dB$. According to the adaptive power allocation policy, we have $P_s = 0$. Then, when $P_P > 14\,dB$, the outage probability of PUs satisfies the outage constraint ε. In this case, the increasing P_p leads to an increase in the transmission rate of PUs. Accordingly, the secondary transmitters can increase their transmit powers P_s and P_R and still keep the outage probability of PUs below the given outage constraint ε. Furthermore, as P_p continuously increases, the transmit power of SUs would approach the maximal values, P_S^P and P_R^P, given by Equations 18 and 20, i.e., the performance of the secondary systems is optimal. When

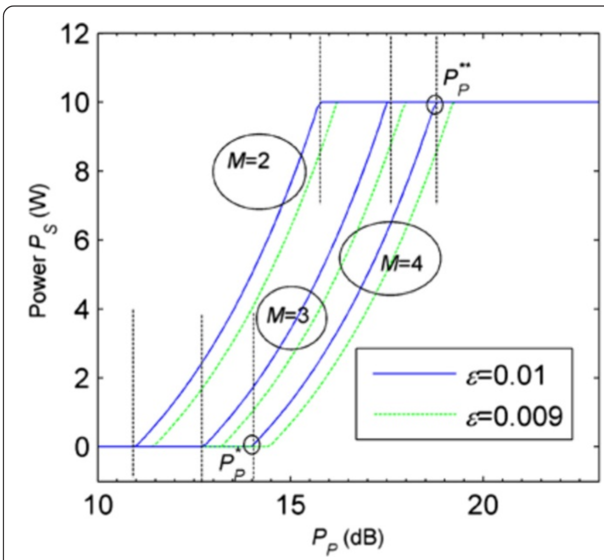


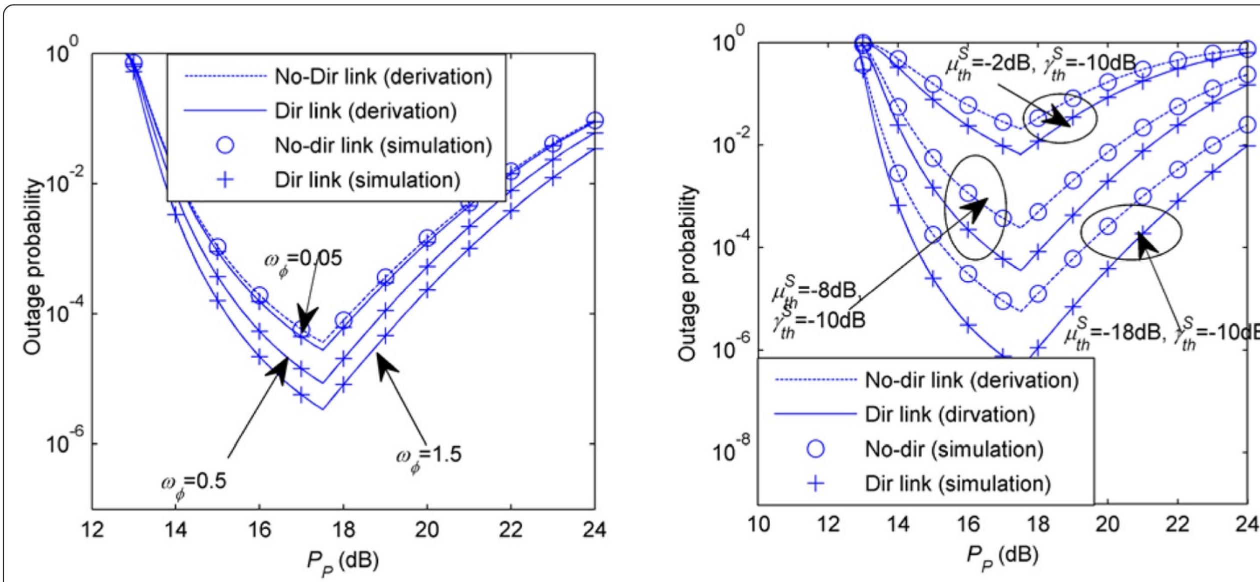
Figure 2 Adaptive power allocations for $P_s(P_R)$.

the transmit power of primary users is increased beyond the optimal value P_P^{**}, P_s cannot be further increased due to the peak transmit power constraint, i.e., $P_S = P_S^P$. At the same time, the figure also shows that the two specific values P_P^* and P_P^{**} are different for different values of M and ε.

Figure 3a,b shows the outage probability of UCR-OR systems versus the peak power P_P by taking $M = 3$ and $\varepsilon = 0.01$. In Figure 3a, with the given outage thresholds $\mu_{\text{th}}^S = \gamma_{\text{th}}^S = -10dB$, we investigate the impact of the mean channel power ω_ϕ of direct path SS – SD on the outage probability. The presented results show firstly that the simulations match well with the analytical results, which validate the derivations. At the same time, it is seen that the outage probability of the UCR-OR systems is decreasing when the value of P_p is less than 17.5 dB. However, it is increasing gradually when P_P is greater than 17.5 dB. Similar to Figure 2, this can be explained by the fact the increasing P_P leads to an increase in transmission rate of PUs, i.e., improvement in the performance of primary systems. Therefore, the secondary transmitters can increase their transmit power and still keep the outage probability of PUs below the given outage constraint ε. Furthermore, with the continuous increase of P_P, the transmit power P_S and P_R of cognitive system will approach the maximal values according to the adaptive power allocation (Equations 18 and 20), i.e., the performance of cognitive systems approaching optimal point. However, if the transmit power P_P of PUs is increased beyond the optimal value, the transmit P_S and P_R of cognitive systems cannot be further increased due to the peak power constraint. Therefore, in this case, any increase in P_P will lead to degradation in the performance of the interested UCR-OR systems.

For the impact of the direct link, we can find that the systems with direct link outperform the ones without direct link. With the increase of the direct link mean power ω_ϕ, the gap between the outage probabilities of the two systems is increasing. For example, when $\omega_\phi = 0.05$, we can find the gap of outage probabilities is very little and can be ignored. Whereas, when $\omega_\phi = 1.5$, the outage performance of UCR-OR systems is improved greatly. In this case, the direct link between cognitive source and destination should be considered, which results in the enhancement of communication reliability.

In Figure 3b, by using $\omega_\phi = 0.5$ and $\gamma_{\text{th}}^S = -10dB$, we investigate the impact of the outage threshold μ_{th}^S of the first hop. In the figure, we take $\mu_{\text{th}}^S = -2, \ -8, \ \text{and} -18$ dB, respectively. It can be clearly seen that the outage probability of the UCR-OR systems is decreasing with the decrease of the outage threshold μ_{th}^S. This is due to the fact that the number of relays in the decoding subset DS is increased as the outage threshold μ_{th}^S of the first hop is decreased. As a result, the outage performance of the UCR-OR systems is improved. At the same time, it can be also seen that the gap of the outage probabilities between the two UCR-OR systems (with and without direct path) is increasing with the decrease of the outage threshold μ_{th}^S. The observation is explained as follows. As aforementioned, the



(a) (b)

Figure 3 The impact of the cognitive direct link and the outage threshold μ_{th}^S on outage performance. (a) Impact of ω_φ and **(b)** impact of outage thresholds μ_{th}^S. The arrows denote the corresponding curves for given system parameters.

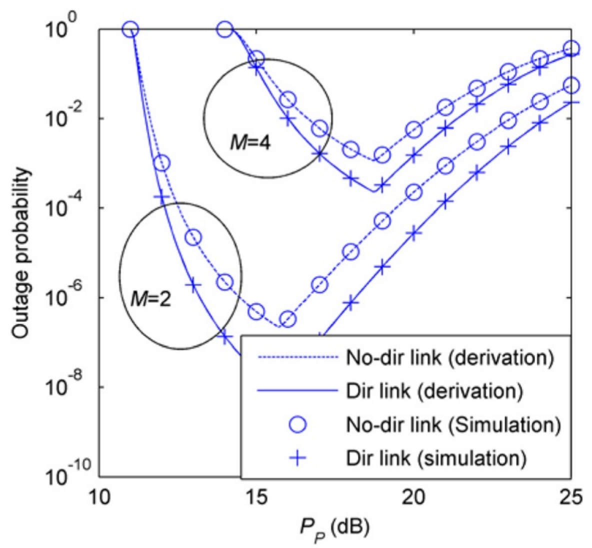


Figure 4 The impact of the number of PU pairs (*M*).

number of relays in the decoding subset DS is increased as the outage threshold μ_{th}^S of the first hop is decreased. This yields that the outage performance of the UCR-OR systems is improved greatly and is dominated by the relay link. The gap is increasing with the number of relays in the decoding subset DS.

In Figure 4, the impact of M is investigated. In the figure, we take the mean power of cognitive direct link $\omega_\phi = 0.5$, the outage constraint $\varepsilon = 0.01$, and the outage threshold $\mu_{\text{th}}^S = \gamma_{\text{th}}^S = -10dB$. The figure shows that the UCR-OR system with $M = 2$ outperforms far the one with $M = 4$.

In Figure 5, another aspect of the outage performance of the considered UCR-OR systems is presented. In the figure, by taking $\omega_{\beta_S} = \omega_{\beta_R} = \omega_{\alpha_R} = \omega_{\alpha_D} = 0.6$, $P_P = 14\,dB$, $\varepsilon = 0.01$, and $\mu_{\text{th}}^S = \gamma_{\text{th}}^S = -10dB$, we present the outage probability versus the cognitive relaying link mean power ω_g ($\omega_g = \omega_h$). In Figure 5a, with $M = 3$, the impact of the mean power of cognitive direct link is investigated. In the figure, we take $\omega_\phi = 1$, 2, and 3 dB, respectively. As obtained in Figure 2, it is found that the outage performance of the considered UCR-OR systems is improved as the mean power of direct link is increased. Besides this observation, we find that the slopes of outage probabilities are the same over the entire values of ω_g. In Figure 5b, the impact of M is investigated. It is easily seen that the value of M has great impact on the outage performance. The result is similar as the one obtained in Figure 4.

By taking $P_p = 14\,dB$, $\lambda = 1$, i.e., $P_S = P_R$, in Figure 6, we present the outage probability versus the transmit power P_s. Note that, in this case, the presented adaptive power scheme is not employed. Figure 6a,b shows that in high P_s, the asymptotic results match well with the exact ones, which corroborates the accuracy of our derivations. At the same time, Equations 60 and 61 also show that the diversity gain of secondary system is determined by the number of relays. The parameters of the primary system only affect the coding gain. These results are

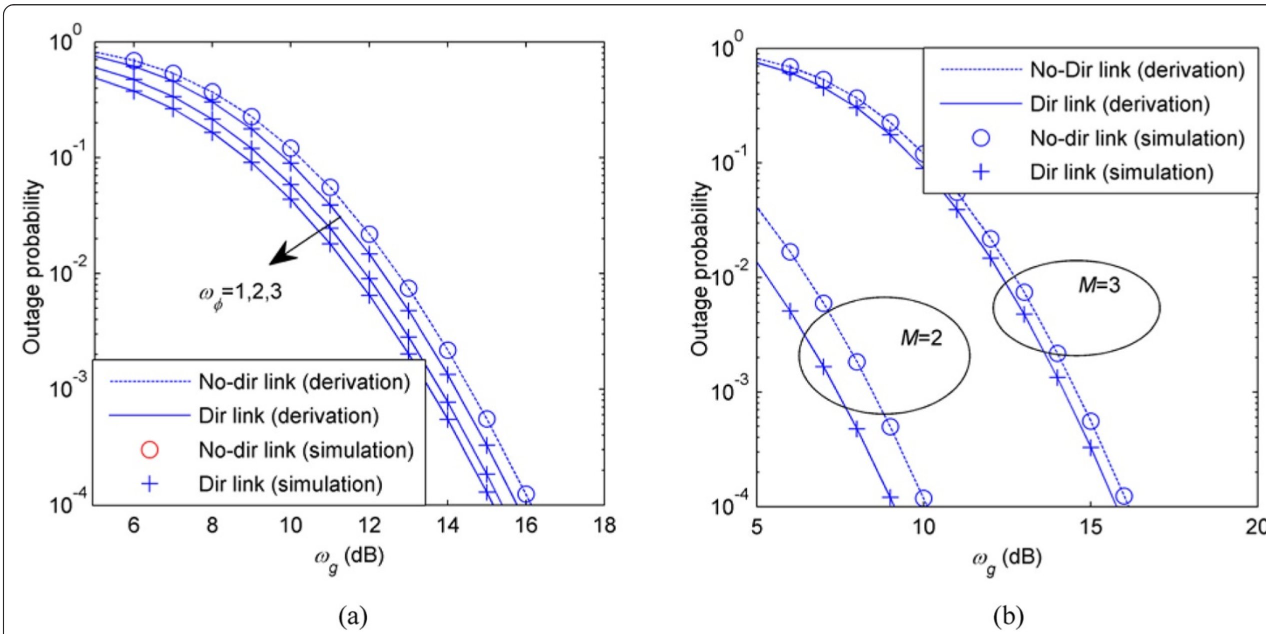


Figure 5 The outage probability versus the mean power ω_g. (a) Impact of ω_φ and **(b)** impact of *M*.

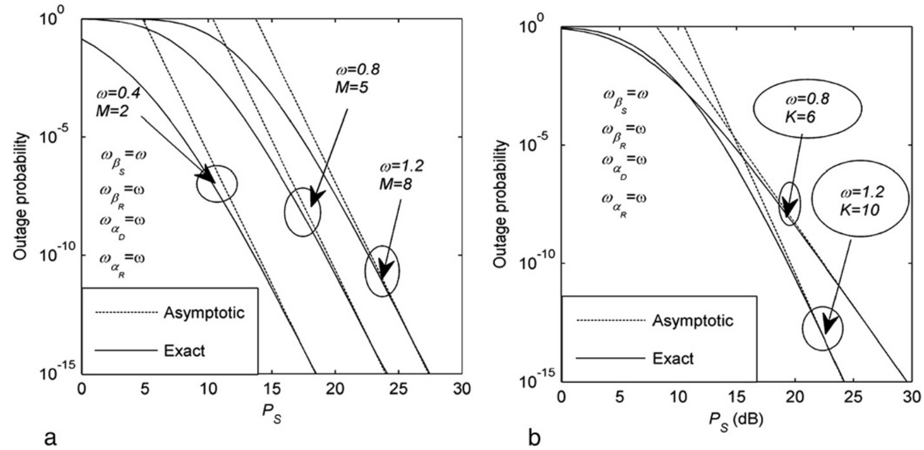


Figure 6 The asymptotic performance comparison analysis. **(a)** Fixed $K = 10$ and **(b)** Variable K ($K = 6$, 10). The arrows denote the corresponding curves for given system parameters.

validated by Figure 6a,b. For example, in Figure 6a, we take the fixed $K = 10$. This yields that that the slopes of asymptotic outage probability are identical even if the parameters of primary system are different. In Figure 6b, the slopes of asymptotic outage performance are different. This is due to the fact that the values of K are different.

7 Conclusions

In this work, we investigate the UCR-OR systems in terms of transmission power allocation and outage performance. Specially, we consider a system in which there are multiple primary user pairs and the direct path between cognitive source and destination. Under primary outage constraint and cognitive peak transmit power limit, the adaptive power allocation schemes for secondary users are achieved firstly. Then, we obtain the exact closed-form expression to the outage probability of UCR-OR systems with direct path transmission and multiple PUs' interference. Finally, to obtain the insight into the impact of system parameters on the performance of UCR-OR systems, by using the approximation of the high transmit power of SUs, the asymptotic closed-form expression of outage probability is achieved. The asymptotic results show that the diversity gain of the considered UCR-OR systems is determined by the number of relays. The parameters of primary systems only affect the coding gain but not the diversity gain. Simulated results validate the derivations firstly. At the same time, we investigate the impact of system parameters on outage performance such as the peak power P_P of primary users, the mean power ω_ϕ of cognitive direct path, and the number M of primary user pairs. Specially, for the impact of ω_ϕ and M, the simulations show that the direct path transmission can improve the performance of UCR-OR systems, and the number of primary users has very severe impact on the UCR-OR system's performance.

Competing interests

The authors declare that they have no competing interests.

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